

Monopsony in Growth Theory*

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Abstract

The secular declines in the labor share and labor supply suggest that imperfect labor markets matter for long-run growth. We show that, unlike rising markups, rising wage markdowns are consistent with balanced growth featuring a declining labor share and a constant capital-output ratio. We propose and solve a model that generates both trends. Our model embeds oligopsony and oligopoly in neoclassical growth, with superstar firms and an inferior sector providing workers' outside option. Faster superstar-sector TFP growth delivers endogenously rising markdowns. Calibrating the model to post-2000 US data, where the labor share and hours per worker fall as measured markdowns rise, we find a consumption-equivalent loss of 12–15 percent relative to optimal growth. The markdown growth estimated in existing studies implies that rising monopsony accounts for approximately one third of the post-2000 decline in the US labor share and one half of the decline in hours per worker. An extension with hand-to-mouth workers and capitalists generates balanced growth with increasing inequality. In this economy a growing minimum wage can restore efficient growth, while transfers to workers cannot.

Keywords: Monopsony, Growth, Hours Growth, Labor Share, Misallocation.

JEL codes: O40, O41, J23, J30, J42 :

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1 Introduction

The neoclassical growth model (NGM) assumes competitive labor markets (Cass, 1965; Koopmans, 1965; Acemoglu, 2009). However, two long-run macroeconomic facts suggest that imperfect labor markets can potentially also play a role in long-run economic growth. First, the secular decline in the labor share implies a long-run reduction in the share of total income accruing to labor (Bergholt et al., 2022; Karabarbounis, 2024). Second, long-run statistics on hours worked by the average individual point to a secular decline in labor supplied to the market (Huberman and Minns, 2007; Ramey and Francis, 2009; Boppart and Krusell, 2020).

Since the seminal contributions of Romer (1990) and Aghion and Howitt (1992), market power in the product market has been the focus of growth theory. Empirically, market power by so-called “superstar firms” (Autor et al., 2020) is associated with stagnant wages (Deb et al., 2022) and declining labor share. In standard growth models, a persistent increase in product-market power can drive a continuous fall in the labor share. Nevertheless—with standard neoclassical production functions—a permanent increase in market power is hardly compatible with a constant capital-output ratio, a key long-run property of any Balanced Growth Path (BGP). Growth models devote less attention to market power in the hiring of labor, even though labor-market imperfections too imply a wedge between the firm’s marginal product and the wage paid to individuals; in recent years, the existence of monopsony power in the labor market has attracted considerable attention in both theoretical and empirical work (Manning, 2021; Yeh et al., 2022; Berger et al., 2022). The starting point of our analysis is to show that—unlike monopoly power—a long-lasting increase in monopsony power is consistent with both a constant capital-output ratio and a persistent decline in the labor share.

The paper proposes and solves a multisector growth model with sector-specific TFP growth and a superior sector in which “superstar” firms enjoy both oligopsonistic and oligopolistic power. Superstar firms derive oligopoly power from barriers to entry and from knowledge of the imperfect substitutability between different goods, and oligopsonistic power from knowledge of inferior firms’ labor demand conditional on wages. Superstar firms compete à la Cournot between themselves, take the price of the inferior good as given, and do not influence the aggregate supply of hours. As in traditional growth theory, the production functions are neoclassical and the demand for goods is isoelastic. When superstar firms enjoy both oligopoly and oligopsony power, labor-market imperfections induce growth effects and the decentralized growth process is inefficient. With standard isoelastic product demand, there are no growth effects from oligopoly power alone, but only level effects, including lower labor share and lower hours worked. Instead, oligopsony power by superstar firms induces a drag on the growth rates of the labor share, hours worked, and total production. The asymmetry reflects a compatibility requirement of the balanced growth path. A rising markup is inconsistent with a constant capital-output ratio, so it cannot sustain a declining labor share in line with the Kaldor facts; a rising markdown faces no such obstacle. Monopsony power is thus the form of market power that can rationalize an ongoing decline in the labor share along a BGP. Furthermore, the decline in hours per capita is a feature of the model also when income and substitution effects from wage changes cancel each other out and preferences are consistent with the King et al. (1988) restriction. The negative effect of oligopsonistic power on growth in hours represents the growth equivalent of the classic static result of the negative effects on hours worked of a generic labor-market

wedge (Prescott, 2004).

In equilibrium, oligopsony power causes the labor wedge to increase over time, and this increase drives the growth results. The mechanism is consistent with the available empirical evidence: Yeh et al. (2022) document a US manufacturing markdown that rises sharply after 2000, Deb et al. (2022) measure a rising economy-wide markdown over 1997–2016, and an accounting exercise reported in the appendix shows that the rising labor concentration of superstar firms (Autor et al., 2020) implies markdown growth of the same magnitude over the same window. The model takes the phenomenon of superstar firms as given and investigates the growth consequences of labor-market power in those markets.

We solve our model under standard capital accumulation and labor-augmenting technological progress, and require TFP growth to be unbalanced in favor of the superstar sector. In a robustness variant with Hicks-neutral technological progress, TFP growth differentials across sectors are not necessary: a more capital-intensive superstar sector is sufficient to generate all the implications of the model.

The quantitative analysis consists of two parts. The first studies the long-run evidence for a representative advanced economy, while the second studies the transition of the United States after 2000.

The first part considers three secular trends for a representative advanced economy: positive GDP growth, declining hours, and a declining labor share. Using the averages of nine countries over 1980–2023, we ask whether these trends can coexist along a balanced growth path and what parameter values they identify. For hours per worker, the markdown channel supplies 44 percent of the income effect in excess of the constant-hours benchmark of King et al. (1988) — the dynamic counterpart of the transfer channel of Prescott (2004). Asymptotically, the presence of oligopsony lowers GDP growth by 9 percent of the competitive rate, and hours per worker fall about 60 percent faster than in the competitive economy.

The second part focuses on the United States for two reasons documented in the facts section. First, the US is the economy in which the joint decline can be clearly dated. Virtually the entire postwar fall in the labor share is concentrated after 2000, and hours per worker also begin to decline around the turn of the century. Second, the United States is the economy for which direct measurements of markdowns are available. We calibrate the transition of a fixed-factor version of the model to the 2000–2023 period, anchoring the level of the markdown to the direct estimates of Deb et al. (2022) and Yeh et al. (2022), while using a labor-share anchor as a conservative reference. The calibrated economy matches GDP growth from the start of the transition to within two percent. By contrast, the declines in the labor share and hours build up along the transition, starting at approximately 73 and 59 percent of their asymptotic rates. Using levels of markdown from the literature (Yeh et al., 2022; Deb et al., 2022), the welfare cost of monopsony is a consumption-equivalent loss between 12 and 15 percent, of which 1.6 to 1.9 percentage points reflect a growth drag that only a dynamic model captures. In addition, the markdown growth measured by Deb et al. (2022) accounts for approximately one third of the post-2000 decline in the US labor share and one half of the decline in hours per worker. Lastly, the paper shows that a one-time change in superstar firms concentration has level effects on hours worked and the labor share, without affecting their long-run growth rates.

The theory developed in this paper has implications for long-run inequality. A version of the model with capitalists and hand-to-mouth workers shows that the divergence between the capital share and

the labor share is accompanied by increasing and long-lasting income and consumption inequality. In addition, the model with heterogeneous agents has implications for the long-run dynamics of labor supply, since the hand-to-mouth individuals work harder than would be optimal for a central planner, and harder than the representative decentralized agent subject to oligopsony and oligopoly. We also discuss the role of policy in this setting. We first study the effects of a proportional tax on capitalists' income rebated to workers, showing that while it addresses the most severe effects of inequality, it depresses labor supply and GDP growth vis-à-vis the efficient levels. We also address the possibility that a growing minimum wage increases both growth of hours and growth of wages, in line with the classical intuition due to Robinson (1933).

Related Literature. This paper sits at the intersection of five strands of literature. First, the literature on *monopsony in labor markets* has expanded rapidly in recent years, both theoretically and empirically.¹ Yet, despite the prominent role of product-market power in the endogenous growth revolution,² the growth effects of labor-market imperfections have received remarkably little attention. The early literature on growth and labor-market frictions focused on the relationship between growth and unemployment,³ while the only prior attempt to study the growth consequences of monopsony, to our knowledge, is Barr and Roy (2008), where spatial mobility costs depress wages and slow human capital accumulation in an endogenous growth setting. Barr and Roy (2008) do not link monopsony to the labor share, to long-run labor supply, or to superstar firms—the three connections that organize our analysis. De Loecker et al. (2024) also study the joint secular dynamics of market power, the labor share, and labor supply in general equilibrium, from the product-market side; relative to their work, ours is a *monopsony-based balanced-growth* explanation, in which the declining labor share and hours arise along a balanced path at constant rates, together with the Kaldor fact of constant capital–output ratio. Relative to the static quantification of labor-market power in Berger et al. (2022), our model is able to capture the *dynamics* of market power: Section 6 decomposes the welfare cost of monopsony into a static component — which essentially coincides with their estimate — and a growth drag that only a growth model delivers; and relative to Boppart and Krusell (2020), the markdown channel both replaces part of the income effect needed to match declining hours and adds the labor-share trend that the preference channel alone cannot produce. Second, our paper relates to recent *growth-theoretic explanations of the labor share decline*. Existing models generate level effects on the labor share through endogenous markups, automation, or capital-embodied technological change.⁴ Relative to this work, we propose a different mechanism—monopsony rather than monopoly or capital deepening—and, more importantly, generate a balanced growth path along which the labor share declines at a *constant rate*, rather than transitions between BGPs of different levels. Third, the paper refers to the *empirical literature on the secular rise in market concentration* documented for the US and other advanced economies by Autor et al. (2020), Hsieh and Rossi-Hansberg (2023) and Kwon

¹See Manning (2021) for a survey, and Berger et al. (2022) and Yeh et al. (2022) for evidence of monopsony in the US labor market. Deb et al. (2022) study superstar firms with labor-market power in a static setting.

²Romer (1990); Aghion and Howitt (1992).

³Aghion and Howitt (1994); Mortensen and Pissarides (1998); more recently, Martellini and Menzio (2020) embed declining search frictions in a balanced growth model with constant unemployment.

⁴Akcigit and Ates (2021, 2023) build models with endogenous markups; Acemoglu (2025) studies endogenous automation; Jones and Liu (2024) characterize a BGP with constant labor share under capital-embodied technological change.

et al. (2024). A growing share of sales and employment in each major industry is accounted for by a small number of large “superstar” firms, which exhibit faster productivity growth and higher markups than the non-superstar fringe. Autor et al. (2020) show that industries experiencing larger increases in concentration also experience steeper declines in the labor share, and emphasize rising product-market power as the principal mechanism. More recent evidence indicates that superstar firms exert market power not only in the product market but also in the labor market: Yeh et al. (2022) document rising markdowns in US manufacturing, and Berger et al. (2022) provide quantitative estimates of labor-market power at the firm level. Our paper takes the existence of superstar firms as given, and asks what their oligopsony power implies for long-run growth, the labor share, and hours worked. Fourth, the model features sectoral labor reallocation driven by unbalanced TFP growth, placing it in the tradition of the *structural change* literature.⁵ Relative to this strand, we solve a fully decentralized economy with imperfections in both product and labor markets, multiple firms in the superior sector, and endogenous labor supply. Finally, we contribute to the *misallocation* literature, which Jones (2022) identifies as a central research avenue in growth economics.⁶ Whereas this literature typically traces TFP differentials to micro-level distortions, in our framework the source of misallocation is monopsony power by superstar firms, which slows the reallocation of factors toward the more productive sector relative to the competitive benchmark.

The paper proceeds as follows. Section 2 documents the stylized facts on a sample of nine advanced economies between 1980–2023 and the subperiod evidence that dates the joint US decline after 2000. Section 3 presents the challenge posed by a rising markup, but not a rising markdown, in obtaining a declining labor share alongside a constant capital-output ratio on a BGP. Section 4 presents the basic structure of the economy, the superstar sector, production technologies, and preferences. It then solves for the balanced growth path with superstar oligopolists and oligopsonists, proving its stability, as well as for the competitive equilibrium, and compares the two solutions, highlighting the growth effects of oligopsony power. Section 5 solves the model with hand-to-mouth workers and capitalists and derives the implications for inequality. Within this framework, we also analyze two types of policy intervention: taxation of capitalists’ income and the minimum wage. Section 6 presents the quantitative analysis: the long-run exercise on the representative advanced economy, and the US-calibrated transition exercise. Section 7 summarizes and concludes.

2 The Decline in Labor Share and Hours Worked

We document two facts that are well known in the literature, even though they are not typically considered jointly and part of a unified theory. The first fact concerns the long run dynamics of the labor share. The second fact is the long-run dynamic of labor supply. We organize the evidence using the latest available data from the Penn World Table (Feenstra et al., 2015). We focus on nine advanced economies over the period 1980–2023, the window over which the Penn World Table covers the labor share, hours, and GDP for all nine countries: We use this common sample throughout the

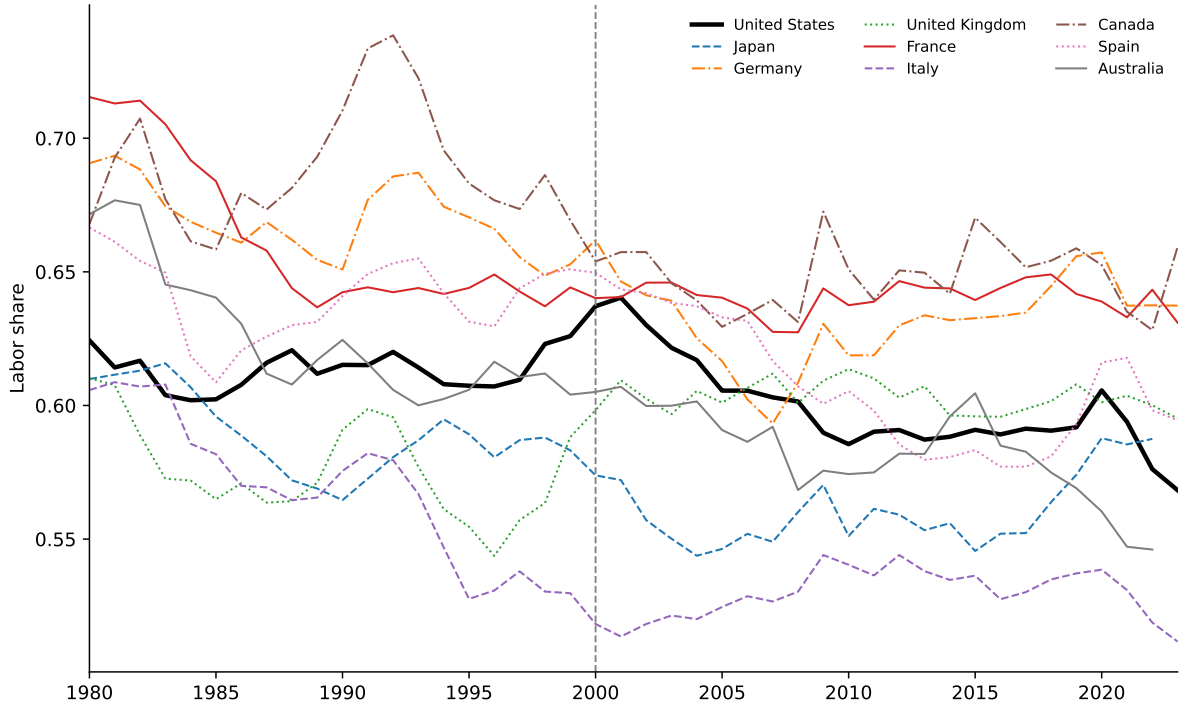
⁵Kongsamut et al. (2001); Ngai and Pissarides (2007); Acemoglu and Guerrieri (2008). The asymptotic BGP of our model mirrors the long-run equilibrium of Acemoglu and Guerrieri (2008).

⁶Seminal contributions include Restuccia and Rogerson (2008) and Hsieh and Klenow (2009); Hsieh et al. (2019) quantify the role of reduced talent misallocation in US TFP growth.

paper. The decline of hours worked is a secular phenomenon, documented over more than a century by Huberman and Minns (2007), Ramey and Francis (2009), and Boppart and Krusell (2020); we take that longer record as background and do not re-estimate it here.⁷

Using the same dataset, Table 1 reports also the average annual growth rates in GDP per capita and in GDP per worker for the nine countries. In the remainder of our analysis, we will consider GDP growth for the “representative advanced economy” as the growth rates reported in Table 1 for 1980 to 2023 for the nine-country sample, so as to use the same sample and time frame as for labor share and hours.

Figure 1: Dynamics of the Labor Share, Nine Advanced Economies, 1980–2023



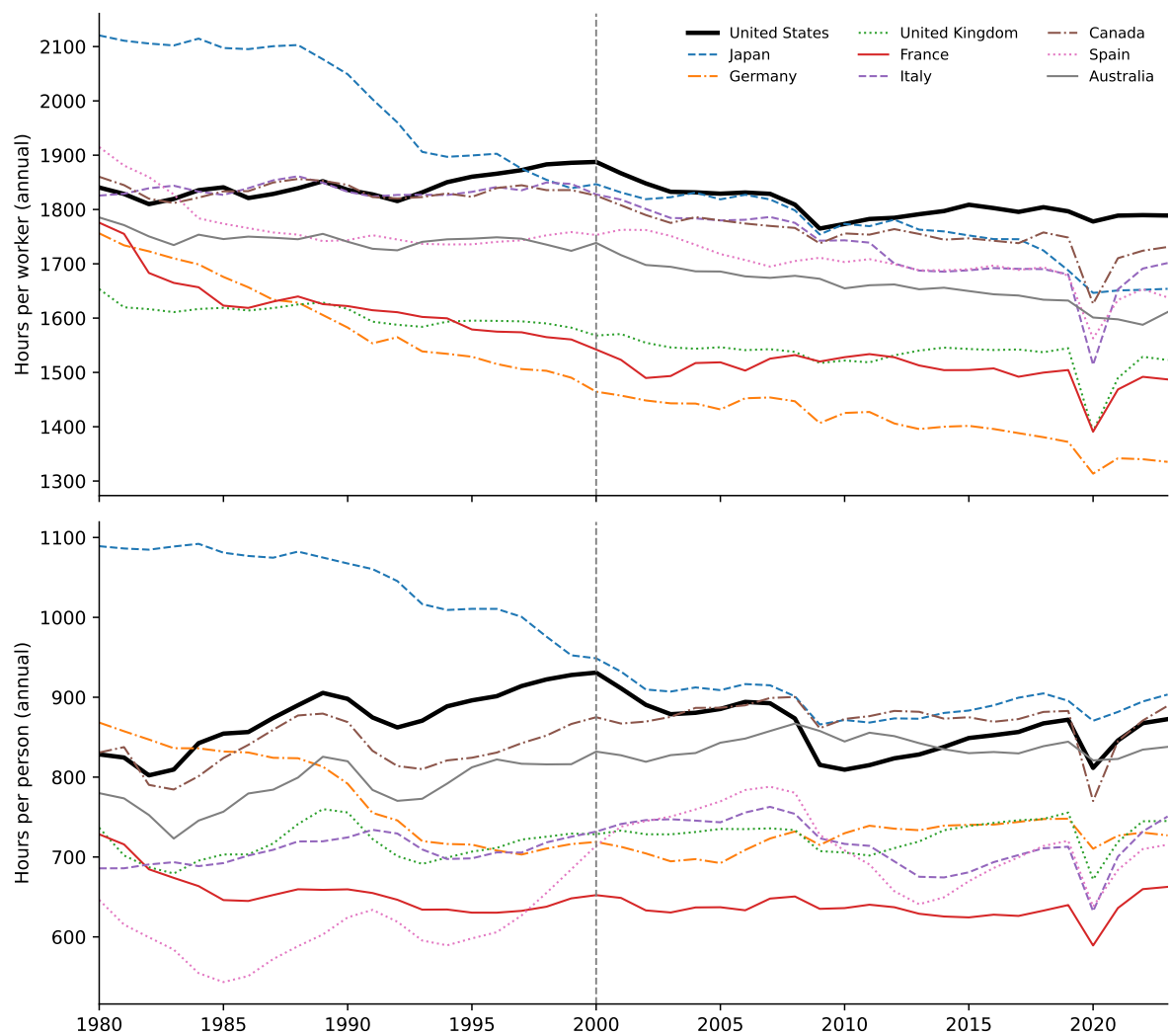
Source: Penn World Table (Feenstra et al., 2015). The United States in bold; the dashed vertical line marks 2000, the break date documented in Table 2.

2.1 The Decline of the Labor Share

The secular decline in the labor share has been first recognized in the seminal paper by Karabarbounis and Neiman (2014). Since then, the empirical literature on the measure of the labor share has been very vivid across the two sides of the Atlantic, and in most developing countries (Cette et al., 2019; Brooks et al., 2021). At the empirical methodological level, there is strong debate about

⁷Following Karabarbounis (2024) for the labor share and Boppart and Krusell (2020) for hours worked, it is possible to focus on slightly different advanced countries and on a longer period. In summarizing the evidence, the paper focuses on consistency and reports statistics on the nine countries listed in Table 1. Adding single advanced economies does not change the nature of the stylized facts.

Figure 2: Dynamics of Hours Worked, Nine Advanced Economies, 1980–2023



Source: Penn World Table (Feenstra et al., 2015). Top panel: hours per worker; bottom panel: hours per person. The United States in bold; the dashed vertical line marks 2000.

measurement issues linked to the role of housing, self employment and proprietary income, and the role of government. Atkeson (2020)- for example- argues that the estimates of the falling labor share are largely due to changes in measurement details by the Bureau of Labor Statistics. More recently, Karabarbounis (2024) reviews the large work on the decline in the labor share and reports unambiguous evidence of its decline. Figure 1 uses data from the Penn World Table and reports the long run trend in the labor share. Quantitatively, Table 1 — in line with Karabarbounis (2024) — shows that since 1980 the labor share has been declining at a rate of -0.18 percent per year in log terms for the representative advanced economy, a value that we will consider in the calibration exercise. In the percentage-point units common in the labor-share literature, a share falling at -0.18 percent per year of its own level, from an average level of 0.61, loses about 0.11 percentage points per year — roughly five points of national income over the four decades of the sample, in line with Karabarbounis and Neiman (2014). As we document in Section 2.3, this full-sample average masks sharp time variation: in the United States, virtually the entire postwar decline of the labor share is concentrated after 2000. Several possible explanations for the secular fall in the labor share in the US and in most countries have been proposed, and relate to technology (Acemoglu and Restrepo, 2022), cost of capital (Kaymak and Schott, 2023), product market power by firms and rising concentration (Autor et al., 2020; Deb et al., 2022), the joint dynamics of market power in product and labor markets (De Loecker et al., 2024), changes in labor market institutions and firms’ market power (Yeh et al., 2022), and globalization. While it is difficult to give weights to all the possible concurrent factors, this paper examines the relationship between balanced growth and declining labor share through the lenses of increasing market power by superstar firms, with special emphasis on the hiring of labor.

Table 1: Stylized Facts: Labor Share, Hours, and GDP Growth, Nine Advanced Economies, 1980–2023

Country	Labor Share (%/yr) ^a	Hours/Worker (%/yr)	Hours/Person (%/yr)	GDP/Capita (%/yr)	GDP/Worker (%/yr)
United States	−0.130***	−0.076***	−0.027	+1.658***	+1.610***
Japan	−0.174***	−0.603***	−0.621***	+1.265***	+1.282***
Germany	−0.200***	−0.596***	−0.335***	+1.492***	+1.231***
United Kingdom	+0.128***	−0.205***	+0.080**	+1.654***	+1.370***
France	−0.172***	−0.340***	−0.164***	+1.250***	+1.074***
Italy	−0.297***	−0.288***	+0.018	+0.832***	+0.527***
Canada	−0.193***	−0.192***	+0.149***	+1.270***	+0.928***
Spain	−0.245***	−0.239***	+0.525***	+1.553***	+0.789***
Australia	−0.353***	−0.231***	+0.254***	+1.735***	+1.249***
<i>Average^b</i>	−0.182	−0.308	−0.013	+1.412	+1.118
^a , All columns report OLS trends of the natural logarithm on calendar year, in percent per year.					
^b , The Average row is the simple mean of country trends.					
Stars denote significance of the underlying slope: *** $p \leq 0.01$, ** $p \leq 0.05$, * $p \leq 0.10$.					
<i>Source:</i> Penn World Table (Feenstra et al., 2015); authors’ calculations.					

2.2 Labor Supply in The Long Run

Traditional growth theory used to take the long run stability of total labor and leisure as a key stylized fact. Prescott (1986) argued that leisure shows no secular trend while the real wage has grown steadily. In terms of preferences, the stability of leisure and labor supply has been obtained

by representative agent models in which the income and substitution effects of wage increases cancel each other out, the well known King et al. (1988) restriction. Recently, however, the stability of hours worked over the long run has been challenged first by Huberman and Minns (2007) and Ramey and Francis (2009), and more recently by Boppart and Krusell (2020).

Indeed, hours worked per worker have been declining in most countries when we consider a time horizon of more than a century, similarly to what is reported in the top panel of Figure 2 for the nine countries we focus on in our analysis. Hours worked per person (bottom panel of Figure 2) have declined too for most countries, even if post World War II data for the US suggest a stable level of hours worked per person. Yet, even the US features a marked secular decline over two centuries (Boppart and Krusell, 2020).⁸ Table 1 reports trend estimates for hours worked and shows that for the average country of our sample — in a way fully coherent with the evidence reported in Boppart and Krusell (2020) — hours worked per worker decline at a rate of approximately -0.31 percent per year over 1980–2023, significantly and in every country. With respect to hours per person, the nine-country average shows essentially no trend (-0.01 percent per year), the outcome of offsetting movements across countries and over time (Table 2): the decline in hours per worker is therefore the more robust fact. For this reason, it is the main labor-supply measure in the quantitative analysis. Interestingly, in the United States hours per worker *rose* between 1980 and 2000, and the postwar decline resumes only after 2000 (Section 2.3).

2.3 The Timing of the Trends: Subperiod Evidence

The averages in Table 1 conceal substantial variation over time. Table 2 splits the working sample, 1980–2023, into two windows — 1980–2000 and 2000–2023 — and reports log-linear trends for GDP, both hours margins, and the labor share, for the United States (Panel A) and for the nine-country average (Panel B).⁹

Two features stand out in Panel A. First, the 1980–2000 window is exceptional for the United States: the labor share is flat (the point estimate is positive and insignificant), hours per worker *rise* at $+0.16$ percent per year, and hours per person rise strongly ($+0.63$ percent per year, the participation boom). Second, the entire postwar decline of the US labor share is concentrated after 2000: the trend is -0.346 percent per year over 2000–2023, against $+0.079$ over 1980–2000 and a near-flat record before 1980 (footnote 9). The renewed decline of both hours margins and the halving of GDP-per-capita growth (from 2.25 to 1.05 percent per year) arrive in the same window.

Panel B shows that the United States is the exception, not the rule. In the nine-country average, hours per worker decline at the same rate in both windows (-0.30 percent per year, the smooth secular pattern of Boppart and Krusell, 2020), while the labor-share decline is *concentrated before 2000* (-0.26 against -0.09 percent per year), driven by continental Europe and Japan, whose earlier

⁸Ramey and Francis (2009) argues that the decline in hours worked per person are mainly accounted for by an increase in schooling by young workers. Conversely, prime age individuals between the ages of 25 and 54 are working the same number of hours now as in 1900, as a combination of a rise in female hours worked and reduction in male hours.

⁹The pre-1980 record is not re-estimated in this paper; on the secular decline of hours over more than a century see Huberman and Minns (2007), Ramey and Francis (2009), and Boppart and Krusell (2020). For reference, Penn World Table data for the United States over 1950–1980 show a nearly flat labor share (-0.048 log percent per year, marginally significant) with hours per worker declining at -0.362 .

experience reflects factors with a literature of its own (Cette et al., 2019). The United States displays the opposite timing on both margins: flat labor share and rising hours before 2000, joint decline after. This division of the evidence shapes the quantitative strategy of Section 6. The nine-country averages over the full window — smooth trends with no break — discipline the *asymptotic* exercise of Section 6.1, which concerns the long-run coexistence of the three trends and makes no dating claim. The United States record — where the joint decline of the labor share and hours is dated and post-2000 — disciplines the *transition* exercise of Section 6.2, whose calibration targets the 2000–2023 window. We run the transition exercise on the United States for two reasons. First, most of the aggregate literature on market power and growth focuses on the US (Autor et al., 2020; Deb et al., 2022). Second, as we document in Section 6.2, it is the economy for which direct measurements of the markdown exist, and they display precisely this timing, declining before 2000 and rising after.

Table 2: The Two Windows: 1980–2000 vs. 2000–2023 (log-linear trends, percent per year)

	1980–2000	2000–2023
<i>Panel A. United States^a</i>		
GDP per capita	+2.250*** (0.082)	+1.050*** (0.070)
GDP per worker	+1.776*** (0.054)	+1.145*** (0.038)
Hours per worker	+0.160*** (0.029)	−0.172*** (0.034)
Hours per person	+0.633*** (0.073)	−0.267*** (0.100)
Labor share	+0.079 (0.049)	−0.346*** (0.052)
<i>Panel B. Nine-country average^b</i>		
GDP per capita	+2.096	+0.677
GDP per worker	+1.805	+0.516
Hours per worker	−0.300	−0.301
Hours per person	−0.009	−0.140
Labor share	−0.262	−0.087
^a , OLS trends of the natural logarithm on calendar year, in percent per year, standard errors in parentheses; ***/**/* denote $ t > 2.58/1.96/1.64$. Same estimator as Table 1.		
^b , Simple average of the country-level trends of the nine economies of Table 1.		
Pre-1980 trends are discussed in footnote 9.		
Source: Penn World Table (Feenstra et al., 2015); authors' calculations.		

3 Markup, Markdown, and the Kaldor Facts: An Asymmetry

This section establishes a preliminary theoretical result that motivates the modeling choices of the paper. We show that in a standard NGM, a permanent decline in the labor share can in principle be generated by either a permanently rising markup or a permanently rising markdown. The two mechanisms, however, are not qualitatively equivalent. A rising markup is incompatible with a constant capital–output ratio in balanced growth, whereas a rising markdown preserves this Kaldor fact.

To formalize these claims, consider the recent neoclassical growth models with market power in product and labor market, proposed - among others - by Deb et al. (2022). In these models the aggregate production function- obtained by solving for the symmetric equilibrium in output and labor varieties- is an equilibrium object. It mimics a standard neoclassical production function with labor-augmenting technological progress in which production takes place according to $Y_t = F(K_t, N_t, A_t)$, where F is neoclassical (exhibiting constant returns to scale in capital K_t and labor N_t), satisfies

Inada's conditions and Uzawa's theorem (so that it can be rewritten as a function of K_t and effective labor $A_t N_t$), and A_t grows at rate g . Appendix A provides the micro structure underlying the aggregate model. We define capital and output per unit of efficiency as $x_t = \frac{K_t}{A_t N_t}$ and $\tilde{y}_t = \frac{Y_t}{A_t N_t}$, with intensive-form production function $\tilde{y}_t = f(x_t) = F(x_t, 1, 1)$. Along a balanced growth path, a constant and positive capital-output ratio requires a time-invariant $x_t = x > 0$,¹⁰ and hence a constant $\tilde{y}_t = \tilde{y}$. We denote by $\varrho_t = \frac{p_t}{MC_t}$ the markup of the aggregate price p_t over marginal cost MC_t , and by $\mu_t = \frac{MPN_t}{w_t}$ the markdown of the marginal product of labor over the wage w_t . Two natural mechanisms for generating a declining labor share emerge from the literature: a permanently rising markup ($\dot{\varrho}_t/\varrho_t = g_\varrho > 0$) or a permanently rising markdown ($\dot{\mu}_t/\mu_t = g_\mu > 0$). At this level of generality we do not specify the underlying market structure giving rise to either.

Under a rising markup, the labor share in intensive form is

$$\alpha_t^\varrho = \frac{1}{\varrho_t} \frac{f(x_t) - x_t f'(x_t)}{f(x_t)}, \quad (1)$$

where $\varrho = 1$ in perfect competition. Holding $x_t = x > 0$ constant, a rising markup at rate g_ϱ would drive $\alpha_t^\varrho \rightarrow 0$ at the constant rate $-g_\varrho$, generating the permanently declining labor share empirical regularity. However, such path is inconsistent with canonical (CRRA) preferences and neoclassical production technology. The Euler equation for the representative consumer with constant elasticity of the marginal utility of consumption γ reads, when hours are constant,

$$\gamma \left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g \right) = \frac{f'(x_t)}{\varrho_t} - \delta - \rho, \quad (2)$$

where $c_t = C_t/N_t$ is per capita consumption, $\tilde{c}_t = c_t/A_t$ is consumption per unit of efficiency, δ is the depreciation rate, and ρ is the discount rate. In a BGP - if it exists - we have that $d\tilde{c}_t/dt = 0$. For a constant markup, equation (2) is satisfied for a unique stationary x , and the capital-output ratio K_t/Y_t in the solution is thus constant. However, the same mechanism required to deliver a declining labor share in (1) - a permanently rising ϱ_t - enters the right-hand side of (2) and forces x_t to decline and converge to zero. This - in turn - implies that the capital-output ratio converges to zero, in contradiction with empirical evidence documenting its stability (Jones, 2016), which is accepted as one of the Kaldor's facts.¹¹

The same tension does not arise when the decline in the labor share is driven by a rising markdown. In that case the analogous equations (when $\varrho_t = 1$) are

$$\alpha_t^\mu = \frac{1}{\mu_t} \frac{f(x_t) - x_t f'(x_t)}{f(x_t)}, \quad (3)$$

$$\gamma \left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g \right) = f'(x_t) - \delta - \rho. \quad (4)$$

¹⁰The fact that x_t must be constant follows from rewriting

$$\frac{K}{Y} = \frac{K}{F(K, N, A)} = \frac{K}{ANF(K/AN, 1, 1)} = \frac{x}{f(x)}.$$

If $K/Y > 0$, then $x > 0$ too because of Inada's conditions, by de L'Hôpital rule $\lim_{x \rightarrow 0} x/f(x) = \lim_{x \rightarrow 0} 1/f'(x) = 0$.

¹¹Ball and Mankiw (2023) study dynamic inefficiency in a similar model with time invariant markups.

A rising μ_t drives $\alpha_t^\mu \rightarrow 0$ along a BGP with constant x . Crucially, μ_t does not enter the Euler equation (4): the markdown affects only the labor market, leaving the equilibrium condition for capital – and hence the capital–output ratio – intact. We summarize these observations as our first result.

Result 1 (Markup–Markdown Asymmetry). *In a neoclassical growth model with labor-augmenting technological progress and canonical preferences:*

- (i) *A permanently rising markup generates a permanent decline in the labor share but is incompatible with a constant capital–output ratio in balanced growth.*
- (ii) *A permanently rising markdown generates a permanent decline in the labor share and is compatible with a constant capital–output ratio in balanced growth.*

The intuition for the asymmetry is straightforward. A markup over the marginal cost of production distorts the return to all factors, including capital, and therefore alters the steady-state condition for capital accumulation. A markdown of the wage over the marginal product of labor distorts only the labor margin, leaving the capital margin – and the Kaldor fact on the capital–output ratio – undisturbed.

Result 1 requires two qualifications. It rules out a permanently growing *firm-level* markup as the sole long-run driver of the declining labor share in the canonical one-sector model; it does not rule out markups that rise during a transition and eventually converge, nor a rising *aggregate* markup generated by reallocation toward high-markup producers. Indeed, the model of this paper features exactly such a compositional increase in the aggregate markup with constant firm-level markups (Section 4.4). A *permanent* decline in the labor share driven by markups alone would, however, require the existence of firms with unboundedly high markups.

Consider now the second stylized fact, the permanent decline in hours worked. In addition to the Euler equation, a model of declining hours implies an extra condition on labor supply along the BGP (see Appendix A). In the influential framework of Boppart and Krusell (2020), a permanent decline in hours is delivered through preferences in which the income effect of rising wages dominates the substitution effect. Raising market power markdown offers an alternative mechanism. In a static setting, Prescott (2004) shows that any wedge between marginal product and wages depresses the level of hours. A positive growth in markup and markdown (g_μ), and (g_ϱ) represents the dynamic extension of such classic result, and represents our second result.

Result 2 (Market Power and Hours Growth). *A permanently rising markdown generates a permanent decline in hours worked along the BGP, even under preferences satisfying the King et al. (1988) restriction (income and substitution effects offsetting each other). A permanently rising markup depresses hours growth through the same wedge, but — as emphasized in Result 1(i) — is incompatible with balanced growth.*

The joint combination of Results 1 and 2 makes a methodological case for organizing the rest of the paper around monopsony power rather than monopoly power. A model built on a rising markdown can in principle deliver the two facts documented in Section 2 – a declining labor share and declining hours – and also be consistent with a constant capital output ratio along a balanced growth path without requiring auxiliary restrictions on preferences. The remainder of the paper develops such a model from explicit microfoundations and quantifies its implications.

4 A Model of Superstar Monopsonistic Growth

This section develops the model. We first describe the technology, market structure, and household preferences (Sections 4.1–4.2). We then characterize the problems of the two types of firms (Sections 4.3–4.4) and of the representative household (Section 4.5). Section 4.6 defines the symmetric monopsonistic equilibrium, establishes the existence of a unique asymptotic balanced growth path, and derives the key growth rates. Section 4.7 contrasts these results with the competitive benchmark.

4.1 Technology and Market Structure

The economy produces a single final consumption good by competitively combining two intermediate goods, indexed $j \in \{s, i\}$, with superscripts denoting a *superior* and an *inferior* sector. The final good is produced according to a constant-elasticity-of-substitution technology

$$Y_t = \left[\zeta^{\frac{1}{\sigma}} (y_t^s)^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} (y_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution between the two intermediaries and y_t^j is the output of sector j . We denote by p_t^j the price of intermediary j and normalize the price of the final good to one at all dates, so that

$$1 \equiv P_t = [\zeta (p_t^s)^{1-\sigma} + (1 - \zeta) (p_t^i)^{1-\sigma}]^{\frac{1}{1-\sigma}}.$$

The two intermediaries are produced with Cobb–Douglas technologies that combine capital K_t^j and labor N_t^j with the same elasticity of output to capital $\alpha \in (0, 1)$, but with different productivity levels A_t^j :

$$y_t^j = (K_t^j)^\alpha (A_t^j N_t^j)^{1-\alpha}, \quad A_t^j = A^j e^{g^j t}, \quad j \in \{s, i\},$$

with $A^s > A^i$ and $g^s > g^i$, so that the superior sector has a higher initial level and grows faster.¹²

The two intermediaries differ not only in technology but also in market structure. The inferior good i is produced by a competitive fringe. The superior good s is produced by m identical *superstar* firms indexed $f \in \{1, \dots, m\}$, all sharing the same productivity A_t^s , so that total sectoral output is $y_t^s = \sum_{f=1}^m y_t^{f,s}$ with $y_t^{f,s} = (K_t^{f,s})^\alpha (A_t^s N_t^{f,s})^{1-\alpha}$. The m superstar firms hold property rights over the superior technology and consequently enjoy both oligopoly power in the product market and oligopsony power in the labor market, in the spirit of Berger et al. (2022) and Robinson (1933). We take this market power as given and study its growth consequences.¹³ The capital market is competitive.

¹²The assumption $A^s > A^i$ is inessential; the higher growth rate of sector s implies that A_t^s eventually overtakes A_t^i regardless of initial conditions. With Hicks-neutral technological progress, $g^s = g^i$ with a higher capital intensity in the superior sector would deliver the same qualitative results (see Section 4.10).

¹³Autor et al. (2020) show that superstar firms display higher patenting activity and larger R&D investment than ordinary firms. The oligopoly problem extends naturally to the case in which $A_t^{f,s}$ differs across firms, provided they share the same growth rate g^s .

4.2 Preferences

The economy is populated by a continuum of measure one of identical, infinitely-lived agents; we abstract from population growth. Each agent is endowed with one unit of time and supplies $h_t \leq 1$ hours to labor. Preferences are time-separable and additively separable in consumption and hours, belonging to the MaCurdy (1981) class within the Boppart and Krusell (2020) family:

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right] dt, \quad \text{if } \gamma \neq 1,$$

with $u(C_t) = \log C_t$ when $\gamma = 1$. The parameter $\gamma > 0$ is the inverse elasticity of marginal utility of consumption; to be consistent with the Boppart and Krusell (2020) class, we will typically have $\gamma > 1$. The parameter θ is the Frisch elasticity of labor supply. With these preferences, $\epsilon_{w_t, n_t} = \theta$, so the firm chooses the wage along a λ -constant labor supply schedule. Agents allocate their hours between the two sectors,

$$n_t^i + n_t^s = h_t, \quad h_t \leq 1,$$

where $n_t^s = \sum_{f=1}^m n_t^{f,s}$. They own all firms in the economy and save in safe assets that yield rights to the capital stocks in production.

4.3 Final Good Sector and the Problem of the Inferior Intermediary

The final-good firm produces competitively, taking the prices p_t^s and p_t^i as given and solving

$$\Pi_t^f = \max_{y_t^s, y_t^i} Y_t - p_t^s y_t^s - p_t^i y_t^i. \quad (5)$$

The first-order conditions yield the demand functions for the two intermediaries:

$$y_t^s = \zeta (p_t^s)^{-\sigma} Y_t, \quad (6)$$

$$y_t^i = (1 - \zeta) (p_t^i)^{-\sigma} Y_t. \quad (7)$$

The inferior intermediary is produced by a competitive fringe that takes prices and wages as given, solving

$$\Pi_t^i = \max_{K_t^i, N_t^i} p_t^i y_t^i - w_t^i N_t^i - R_t^i K_t^i, \quad (8)$$

where w_t^i is the wage in the inferior sector and R_t^i the rental rate of capital. Standard Cobb–Douglas factor demands give

$$w_t^i = p_t^i \frac{(1 - \alpha) y_t^i}{N_t^i}, \quad R_t^i = p_t^i \frac{\alpha y_t^i}{K_t^i}. \quad (9)$$

4.4 The Superstar Oligopolist and Oligopsonist

The core of the model is the problem of the superstar firms in the superior sector. These firms enjoy market power on two margins simultaneously. On the product side, the m superstar firms compete

à-la-Cournot facing a downward-sloping demand from the final good sector with constant elasticity σ . On the factor side, each superstar firm is a price taker in the capital market but an oligopsonist in the labor market: each firm internalizes the fact that hiring an additional worker forces it to pay a higher wage, both to that worker and to all its existing workforce. This subsection sets up the problem of a representative superstar firm, derives the markdown of wages over the marginal product of labor that this two-sided market power generates, and characterizes the symmetric Cournot–Nash equilibrium that defines the rest of the model.

We index superstar firms by $f \in \{1, \dots, m\}$. The choices of firm f at time t are denoted $N_t^{f,s}$ (labor), $K_t^{f,s}$ (capital), $y_t^{f,s}$ (output), and $W_t^{f,s}$ (the wage it pays). To express how firm f 's decisions interact with those of its rivals, we collect the analogous variables for the other $m - 1$ firms in the superior sector:

$$N_t^{-f,s} \equiv \sum_{l \neq f} N_t^{l,s}, \quad y_t^{-f,s} \equiv \sum_{l \neq f} y_t^{l,s}, \quad K_t^{-f,s} \equiv \sum_{l \neq f} K_t^{l,s}.$$

Total sectoral output and labor are then $y_t^s = y_t^{f,s} + y_t^{-f,s}$ and $N_t^s = N_t^{f,s} + N_t^{-f,s}$. In Cournot fashion, firm f takes the decisions of its rivals $\{N_t^{-f,s}, y_t^{-f,s}, K_t^{-f,s}\}$ as given when choosing its own. The symmetric equilibrium, in which all firms make identical choices, will be characterized below.

The market power of firm f derives from its full knowledge of two equilibrium relationships in the rest of the economy.

The first is the *labor-supply schedule* faced by firm f . Workers in the economy can either work for the superstar firms in sector s or for the competitive fringe in sector i . The wage in the inferior sector is the marginal product of labor there, $w_t^i = p_t^i(1 - \alpha)(K_t^i)^\alpha(N_t^i)^{-\alpha}(A_t^i)^{1-\alpha}$. By the household's wage-arbitrage condition (derived in Section 4.5), any wage paid by a superstar firm must equal the wage available in the inferior sector. Using $N_t^i = H_t - N_t^{f,s} - N_t^{-f,s}$, where H_t is total hours supplied in the economy, the labor-supply schedule faced by firm f is

$$w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}) = p_t^i(1 - \alpha)(K_t^i)^\alpha \left(H_t - N_t^{f,s} - N_t^{-f,s} \right)^{-\alpha} (A_t^i)^{1-\alpha}. \quad (10)$$

Equation (10) is upward-sloping in $N_t^{f,s}$: as firm f hires more labor, fewer workers remain available for the inferior sector, the marginal product of labor in that sector rises, and so does the outside option of firm f 's workforce. This is the classic monopsonistic mechanism of Robinson (1933), generalized here to the case of m firms in the spirit of Berger et al. (2022).

The second fundamental condition is the inverse demand for the superior good s . Substituting $y_t^s = y_t^{f,s} + y_t^{-f,s}$ into the demand function (6) from the final-good sector yields

$$p_t^s = Y_t^{\frac{1}{\sigma}} \left(y_t^{f,s} + y_t^{-f,s} \right)^{-\frac{1}{\sigma}} \zeta^{\frac{1}{\sigma}}, \quad (11)$$

which is downward-sloping in $y_t^{f,s}$ with constant elasticity σ .

While superstar firms have full knowledge of the labor-supply schedule of their workforce (Eq. 10) and of their product demand (Eq. 11), they maximize profits under three conjectures, which we state as assumptions and defend individually.

Assumption 1. *Firm f takes the factor inputs of the other superstar firms, $K_t^{-f,s}$ and $N_t^{-f,s}$ — and with them their output $y_t^{-f,s}$ — as given.*

Assumption 2. Firm f takes the inferior-sector price p_t^i and capital K_t^i as given. It internalizes the direct effect of its hiring on inferior-sector employment, $N_t^i = H_t - N_t^{f,s} - N_t^{-f,s}$, and hence on the fringe marginal product of labor; it ignores the further equilibrium responses of the fringe's price and capital.

Assumption 3. Firm f takes total hours H_t as given: it does not internalize the effect of the wage it pays on the household's aggregate labor-supply decision.

Assumptions 1–3 share a single logic: each firm internalizes the effects that are *direct* in its own actions and ignores responses that operate through the decisions of *other* agents. Assumption 1 is standard Cournot. Assumption 2 applies the same structure to the labor side: the firm's hiring removes workers from the fringe one for one — a direct, first-order effect on the marginal product that prices its workers' outside option — while the adjustments of the fringe's price and capital are responses of other agents, which the firm treats as it treats its rivals' quantities. Assumption 3 states that the firm, while large within its sector, is atomistic relative to the household's economy-wide participation margin. While the behavior of oligopsonistic superstar firms depends on the above assumptions; their equilibrium implication — a constant markup together with an endogenously rising markdown — is precisely the configuration that Section 3 showed a balanced growth path with a declining labor share requires.

A natural objection to Assumption 2 is its asymmetry: on the product side the firm internalizes the price of its own good, while on the labor side it does not internalize p_t^i . Suppose instead that the firm internalized the response of p_t^i to its own hiring, holding aggregate output and K_t^i fixed. From the final-good demand (7), $d \ln p_t^i / d \ln y_t^i = -1/\sigma$, and with $d \ln y_t^i / d \ln N_t^i = 1 - \alpha$, the perceived labor-supply schedule $w = p_t^i \times \text{MPL}^i$ becomes steeper: its elasticity with respect to N_t^i rises in absolute value from α to $\alpha + (1 - \alpha)/\sigma$. The markdown retains the same functional form with α replaced by $\alpha + (1 - \alpha)/\sigma > \alpha$: full internalization *strengthens* monopsony power, and since the functional form is unchanged, the asymptotic growth-rate structure of the model is unaffected — only levels rescale. Assumption 2 is therefore conservative: it bounds the markdown from below within the conjectural family.

Taking $\{H_t, N_t^{-f,s}, y_t^{-f,s}, K_t^{-f,s}, R_t^s, p_t^i, K_t^i\}$ as given, firm f chooses labor, capital, and the wage to maximize period-by-period profits subject to the labor-supply constraint (10):

$$\begin{aligned} \max_{N_t^{f,s}, K_t^{f,s}, W_t^{f,s}} \quad & \Pi_t^{f,s} = p_t^s(y_t^{f,s}; y_t^{-f,s}) y_t^{f,s} \left(K_t^{f,s}, N_t^{f,s}, A_t^s \right) - W_t^{f,s} N_t^{f,s} - R_t^s K_t^{f,s} \\ \text{s.t.} \quad & W_t^{f,s} \geq w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}). \end{aligned} \tag{12}$$

With respect to the wage, the firm optimally sets a *limit wage*: paying any more than necessary would simply transfer rents to workers, while paying any less would leave the firm short of labor. The labor-supply constraint thus binds with equality, and the firm's effective wage choice collapses to its labor choice.

Because the wage paid to all workers depends on the firm's labor input, the marginal cost of hiring an additional worker exceeds the wage that worker is paid. Differentiating total labor costs $w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}) N_t^{f,s}$ with respect to $N_t^{f,s}$, and using the Cobb–Douglas form of the inferior-sector

marginal product, the marginal cost of labor to firm f is

$$\frac{\partial \left[w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}) N_t^{f,s} \right]}{\partial N_t^{f,s}} = w_t^{f,s} \left[1 + \alpha \frac{N_t^{f,s}}{N_t^i} \right] = w_t^{f,s} \left[1 + \frac{1}{\epsilon_{N^{f,s}, w^{f,s}}} \right]. \quad (13)$$

The expression $\epsilon_{N^{f,s}, w^{f,s}} = (dN^{f,s}/dw^{f,s})(w^{f,s}/N^{f,s}) = N_t^i/(\alpha N_t^{f,s})$ is the firm-level elasticity of labor supply, in line with Manning (2021). The bracketed term in equation (13) is the *markdown* of the wage with respect to the marginal cost of labor,

$$\mu_t^{f,s} = 1 + \frac{1}{\epsilon_{N^{f,s}, w^{f,s}}}, \quad (14)$$

and plays the central role in what follows.¹⁴ The intuition behind the markdown is straightforward: a firm with monopsony power restricts hiring below the competitive level in order to keep its wage bill low, so that in equilibrium the marginal product of labor exceeds the wage. The static analogue of this mechanism is depicted in the top panel of Figure 3. The bottom panel illustrates how an increase in superior-sector productivity A^s shifts the marginal revenue product of labor outward, raising both the firm's employment and the markdown.

On the product side, the firm internalizes that selling an additional unit of output requires lowering the price. From the inverse demand (11), the marginal revenue from increasing $y_t^{f,s}$ is

$$\frac{\partial \left[y_t^{f,s} \cdot p_t^s(y_t^{f,s} + y_t^{-f,s}) \right]}{\partial y_t^{f,s}} = \left(1 - \frac{1}{\sigma} \frac{y_t^{f,s}}{y_t^{f,s} + y_t^{-f,s}} \right) p_t^s.$$

The bracketed term is below one whenever the firm's market share is positive and shrinks toward one as $\sigma \rightarrow \infty$, in which case the firm becomes a price taker. The restriction $\sigma > 1$ is standard in monopoly theory.

Combining the marginal revenue of output with the marginal cost of labor, the firm's optimal labor choice equates marginal revenue product to marginal cost:

$$\underbrace{p_t^s \left(1 - \frac{1}{\sigma} \frac{y_t^{f,s}}{y_t^{f,s} + y_t^{-f,s}} \right)}_{\text{MR}} \underbrace{\frac{\partial y_t^s}{\partial N_t^{f,s}}}_{\text{MRP}} = \underbrace{p_t^i \frac{\partial y_t^i}{\partial N_t^i}}_{w_t^i} \underbrace{\left[1 + \alpha \frac{N_t^{f,s}}{N_t^i} \right]}_{\mu_t} \quad (15)$$

Equation (15) is the central first-order condition of the model. The left-hand side combines two ingredients: the marginal revenue from selling an additional unit of y_t^s (the markup-adjusted price) and the marginal product of labor in producing it. The right-hand side combines the wage paid (which equals the marginal product of labor in the inferior sector by the wage-arbitrage condition) and the markdown that reflects the firm's awareness of how its hiring affects the wage it must pay to all its workers. The two distortions are conceptually distinct: the markup on the left is an oligopoly

¹⁴For a general inferior-sector technology $G(K_t^i, N_t^i, A_t^i)$, the markdown takes the form $\mu_t^{f,s} = 1 + (n_t^s/mn_t^i) \cdot [-G_{NN}/G_N]$; see Appendix C.

distortion of the product market, while the markdown on the right is an oligopsony distortion of the labor market.

The choice of capital is influenced only by product-market power, since the capital market is competitive:

$$p_t^s \frac{\partial y_t^{f,s}}{\partial K_t^{f,s}} = \left(1 - \frac{1}{\sigma} \frac{y_t^{f,s}}{y_t^{f,s} + y_t^{-f,s}} \right) R_t^s. \quad (16)$$

For equations (15) and (16) to admit an interior solution, we need labor to enter production with strictly diminishing returns.

Assumption 4. *Labor enters both technologies with strictly diminishing returns: the elasticity of output to capital satisfies $\alpha \in (0, 1)$, so that the elasticity of output to labor is less than one.*

Assumption 4 is more than a regularity condition: it is essential for the existence of monopsony power. Indeed, the key component of the assumption is that the inferior sector operates a technology with diminishing returns to labor, which gives rise to an upward sloping labor supply, giving bite to oligopsony power.

We focus on the symmetric Cournot–Nash equilibrium in which all m superstar firms make identical choices:

$$N_t^{f,s} = \bar{N}_t^s, \quad K_t^{f,s} = \bar{K}_t^s, \quad y_t^{f,s} = \bar{y}_t^s \quad \forall f, \quad (17)$$

with sectoral totals $N_t^s = m\bar{N}_t^s$, $K_t^s = m\bar{K}_t^s$, and $y_t^s = m\bar{y}_t^s$. Under symmetry, the rivals' aggregates simplify to $N_t^{-f,s} = (m-1)\bar{N}_t^s$ and $y_t^{-f,s} = (m-1)\bar{y}_t^s$, and the firm's market share in its sector is $y_t^{f,s}/y_t^s = 1/m$. Substituting into equations (15) and (16) yields the symmetric first-order conditions:

$$p_t^s \left(\frac{\sigma m - 1}{\sigma m} \right) \frac{\partial y_t^s}{\partial N_t^s} = p_t^i \frac{\partial y_t^i}{\partial N_t^i} \left[1 + \alpha \frac{N_t^s}{m N_t^i} \right], \quad (18)$$

$$p_t^s \left(\frac{\sigma m - 1}{\sigma m} \right) \frac{\partial y_t^s}{\partial K_t^s} = p_t^i \frac{\partial y_t^i}{\partial K_t^i}. \quad (19)$$

The term $\sigma m/(\sigma m - 1)$ is the constant oligopoly markup, which converges to one as either σ or m goes to infinity — recovering the competitive limit. Equation (18) can be written compactly as the product of markup and markdown,

$$\frac{p_t^s \partial y_t^s / \partial N_t^s}{w_t^s} = \underbrace{\frac{m\sigma}{m\sigma - 1}}_{\text{markup}} \underbrace{\left(1 + \frac{1}{\epsilon_{N_t^s, w_t^s}} \right)}_{\text{markdown}}, \quad (20)$$

making it transparent that the marginal product of labor in the superior sector exceeds the wage by a factor that combines both forms of market power. While our model with CES implies a constant markup of the superstar firms, one can potentially build models with oligopolistic markups that grow over time, as in Autor et al. (2020). Yet, as shown in Section 3, such models are inconsistent with a capital output ratio along the BGP, at the same time, because of the compositional shift, the aggregate markup of our economy does increase over time. While the markup depends on exogenous parameters of the model, the markdown depends on the endogenous ratio N_t^s/N_t^i , a mechanism that

drives most of the results in the paper. The markdown also depends negatively on the exogenously set effective-number index m .

The markdown $\mu_t = 1 + \alpha N_t^s / (m N_t^i)$ makes explicit what generates wage-setting power in this model, and it is worth stating because it differs from the notion of monopsony most commonly measured in the empirical literature. Power here derives from the *compression of the workers' outside option*: as employment in the competitive fringe shrinks relative to superstar employment, the alternative employer of the superstar workforce becomes scarce, and each firm's hiring moves the fringe marginal product more strongly. Two remarks on the limiting properties are in order. For a *fixed* employment ratio, $\mu_t \rightarrow 1$ as $m \rightarrow \infty$: coordination among many firms vanishes, as in Cournot. Along the growth path, however, N_t^s / N_t^i grows without bound, and the markdown rises for any fixed finite m . More generally, if m is allowed to vary, the markdown continues to rise whenever the employment ratio grows faster than m . The model thus features monopsony without (necessarily) firm size: the source of power is not the firm's share of the aggregate labor market, as in Berger et al. (2022) or Yeh et al. (2022), but the disappearance of the workers' alternative — a notion closer to declining outside options, complementary to concentration-based monopsony. This distinction is also why the model-implied product-market concentration is not comparable to census concentration ratios, a point we return to in Section 6.2.

Three remarks on m conclude the description of the market structure. First, m is the number of symmetric firms in the Cournot microfoundation. In the quantitative implementation, we use the same parameter as a continuous index of the *effective number* of firms, or equivalently of their effective coordination: a non-integer value describes conduct between the adjacent integer Cournot cases, much as an inverse Herfindahl index defines an equivalent number of firms. This interpretation is distinct from both the economy-wide share of the superstar sector and census concentration ratios. Second, for fixed σ and any constant finite m , the parameter affects the levels of the markup and the markdown but not their asymptotic growth rates. A one-time permanent change in m therefore produces a transitional growth adjustment and permanent level effects, without changing long-run growth. The integer specification $m = 2$ reported in Section 6.2 confirms that the qualitative conclusions do not depend on using a continuous effective-number index.¹⁵ Third, the model has no entry into the superstar sector, while superstar profits are positive and growing along the equilibrium path. The entry barrier is the property right over the superior technology: the profits are rents on an excludable technology, consistent with the evidence that superstar firms display disproportionate patenting and R&D activity (Autor et al., 2020). Endogenous entry would leave the asymptotic mechanism intact provided that the effective number of firms converges to a finite constant. A full model of entry would require an entry technology — for example, a license or an R&D race for the excludable technology — and we leave it to future work.

The inferior-sector firms equate the marginal revenue products of labor and capital to factor prices in standard fashion: $p_t^i(\partial y_t^i / \partial N_t^i) = w_t^i$ and $p_t^i(\partial y_t^i / \partial K_t^i) = R_t^i$. When both sectors face the same wage and rate of return (which holds in equilibrium when capital depreciation is symmetric across sectors), taking the ratio of the labor and capital first-order conditions across sectors yields

¹⁵In that exercise the markup target is held fixed, so σ adjusts and the model is recalibrated to the same growth targets. The targeted growth moments are therefore unchanged by construction, while levels and welfare losses change as reported in Section 6.2.

the *fundamental factor allocation*:

$$\left[1 + \alpha \frac{N_t^s}{mN_t^i}\right] \frac{N_t^s}{K_t^s} = \frac{N_t^i}{K_t^i}. \quad (21)$$

The pure neoclassical benchmark, which would obtain in a fully competitive economy, would set $N_t^s/K_t^s = N_t^i/K_t^i$: the labor–capital ratio would be equalized across sectors. Equation (21) shows that whenever $\alpha > 0$, the superstar firms operate with a distorted factor mix — specifically, they use *too little* labor relative to capital when compared to the inferior sector. This intersectoral misallocation of factors is the central inefficiency generated by monopsony power, and it is what generates the consumption-equivalent loss we quantify in Section 6. The superstar firms earn strictly positive profits along the equilibrium path, which the household receives as a rebate (see Section 4.5).

4.5 Household Problem

The representative household owns all firms and supplies labor to both sectors. Given the symmetric equilibrium across superstar firms, it is without loss of generality to write the household problem in terms of a single asset position $\mathbb{A}_t^s$ in the superior sector (proportional to total capital $K_t^s = m\bar{K}_t^s$) and a single labor supply $n_t^s = \sum_{f=1}^m n_t^{f,s}$. The household takes as given wages w_t^s, w_t^i , rates of return r_t^s, r_t^i , and aggregate superstar profits $\Pi_t^s = \sum_{f=1}^m \Pi_t^{f,s}$, and solves

$$\begin{aligned} & \max_{\{C_t, \mathbb{A}_t^s, \mathbb{A}_t^i, n_t^s, n_t^i\}_{t \geq 0}} \int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right] dt \\ & \text{s.t.} \quad C_t + \dot{\mathbb{A}}_t^s + \dot{\mathbb{A}}_t^i = w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \Pi_t^s \\ & \quad n_t^s + n_t^i = h_t, \quad n_t^s, n_t^i \geq 0, \quad \mathbb{A}_t^s, \mathbb{A}_t^i \geq 0. \end{aligned} \quad (22)$$

To ensure finite utility along the equilibrium path, we impose

Assumption 5. $-\rho + (1-\gamma)g^s < 0$.

The Hamiltonian is standard and is reported in Appendix B. The first-order conditions deliver an Euler equation, arbitrage conditions for labor and capital, and a transversality condition. The Euler equation reads

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma}, \quad (23)$$

where the capital-arbitrage condition implies

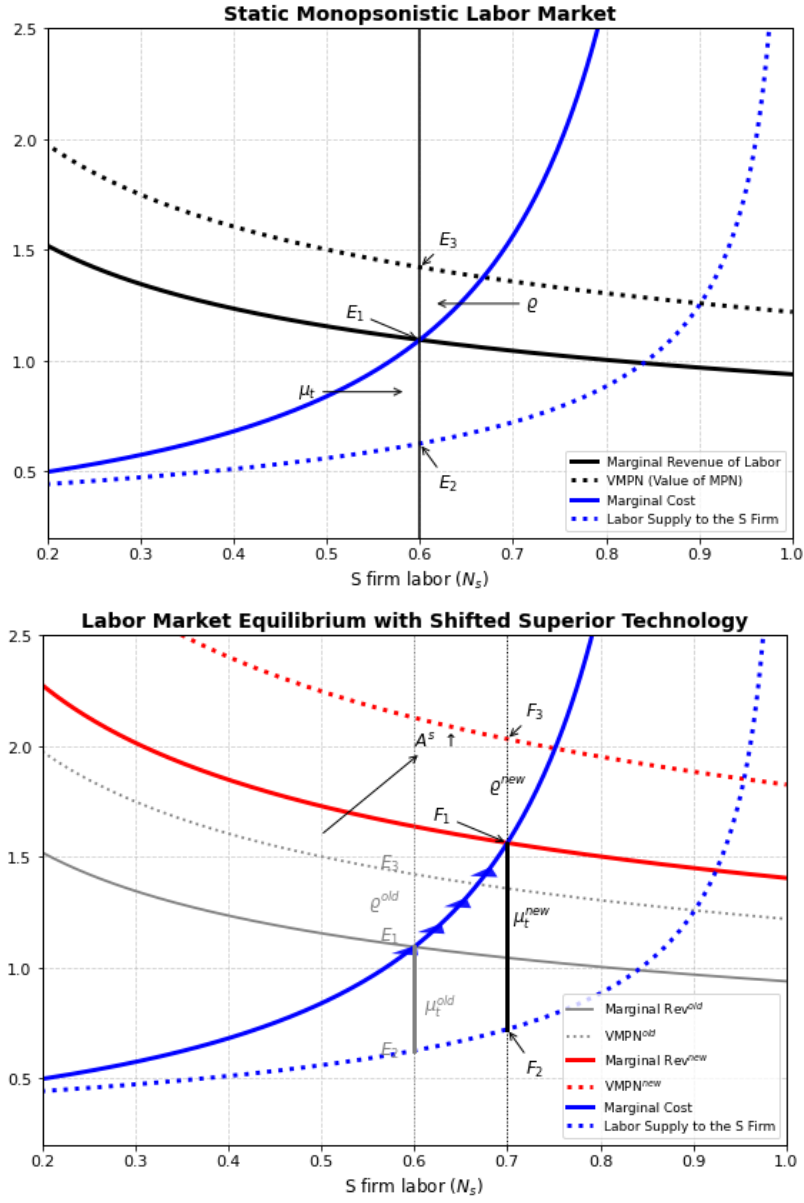
$$r_t^s = r_t^i.$$

and we denote $r_t = r_t^s = r_t^i$. The intratemporal labor-supply conditions for each sector,

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^s, \quad (24)$$

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^i, \quad (25)$$

Figure 3: Monopsonistic Labor Market with the Superior Technology



imply the *fundamental wage-arbitrage condition* $w_t = w_t^s = w_t^i$: along any equilibrium path, the wage paid by the oligopsonists must equal the wage in the inferior sector. The flow budget constraint

$$C_t + \dot{\mathbb{A}}_t^s + \dot{\mathbb{A}}_t^i = w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \Pi_t^s$$

and the transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\rho t} u'(C_t) \mathbb{A}_t^s = \lim_{t \rightarrow \infty} e^{-\rho t} u'(C_t) \mathbb{A}_t^i = 0$$

close the system.

4.6 Superstar Monopsonistic Equilibrium

Market clearing requires that asset holdings equal capital stocks net of depreciation, that labor supplied to each sector equal labor demanded, and that rates of return be equalized net of depreciation, so that

$$r_t = R_t^s - \delta^s = R_t^i - \delta^i, \quad (26)$$

$$\mathbb{A}_t^s = K_t^s, \quad \mathbb{A}_t^i = K_t^i, \quad n_t^s = N_t^s, \quad n_t^i = N_t^i, \quad h_t = H_t. \quad (27)$$

Definition 1. *Given a path for productivity $\{A_t^s, A_t^i\}_{t=0}^\infty$ and a number of superstar firms m , a symmetric superstar monopsonistic equilibrium (SSMPE) is a set of sequences for allocations, prices, and consumption such that: (i) the symmetric superstar firm's choices $\bar{n}_t^s, \bar{K}_t^s, \bar{y}_t^s$ solve the oligopolistic-oligopsonistic problem (18)–(19); (ii) final-good prices and quantities satisfy (5)–(7); (iii) the inferior firm's choices satisfy (8)–(9); (iv) the household's allocations satisfy (22), (23), and (24)–(25); (v) wages and rates of return satisfy the arbitrage conditions (20), (16), (26), (12); (vi) all markets clear, (27).*

The equilibrium can be reduced to a system of nine equations in nine unknowns; the full system is reported in Appendix B. The economic content of the model is captured by the equation that determines the intersectoral allocation of labor:

$$\frac{\sigma m - 1}{\sigma m} p_t^s (1 - \alpha) (K_t^s)^\alpha (n_t^s)^{-\alpha} (A_t^s)^{1-\alpha} = p_t^i (1 - \alpha) (K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \left[1 + \frac{\alpha}{m} \frac{n_t^s}{n_t^i} \right]. \quad (28)$$

The left-hand side is the marginal revenue product of labor in the superior sector net of the constant oligopoly markup; the right-hand side is the marginal product of labor in the inferior sector scaled by the markdown. The Euler equation (23) and the standard CES, Cobb–Douglas, and market-clearing conditions close the system.

Since the two sectors feature unbalanced productivity growth, in the limit the superior sector absorbs all factors, and the model admits no finite balanced growth path. We therefore characterize the *asymptotic* BGP in which all variables grow at constant rates. To study the dynamics, we transform the system into intensive form by defining

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s}, \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i}, \quad c_t = \frac{C_t}{A_t^s n_t^s},$$

with details in Appendix B.

Definition 2. An (asymptotic) balanced growth path (BGP) is an SSMPE along which aggregate consumption and GDP grow at constant rates.

Proposition 1. The SSMPE admits a unique asymptotic balanced growth path with the following features:

1. consumption and output per capita grow at the constant rate $g_C = g_Y = g_{y^s} > g_{y^i}$, with the superior sector absorbing the entire final-good production ($y_t^s/Y_t \rightarrow \zeta^{-\frac{1}{\sigma-1}}$, $y_t^i/Y_t \rightarrow 0$);
2. the relative labor allocation in the superior sector grows indefinitely, $\frac{1}{m} \frac{n_t^s}{n_t^i} \rightarrow \infty$, but total hours grow at the constant rate $g_h = g_{n^s} > g_{n^i}$;
3. the growth rate of wages is strictly below the growth rate of output, $g_w < g_Y$;
4. the interest rate r_t and capital per efficiency unit in the superior sector converge to constants ($r_t^s \rightarrow r$, $x_t^s \rightarrow x^s$), while $x_t^i \rightarrow \infty$.

The asymptotic price of the superior intermediary converges to a constant ($p_t^s \rightarrow \zeta^{\frac{1}{\sigma-1}}$, $g_{p^s} \rightarrow 0$), while the inferior price grows at a constant rate.

The key analytical result is that the markdown grows without bound along the BGP.

Proposition 2. The markdown of the superstar firms has a constant asymptotic growth rate

$$g_\mu = g_{n^s} - g_{n^i}.$$

The proof follows directly from differentiating equation (14).¹⁶ Reducing the asymptotic system to a linear system in growth rates (zeroing $g_{p^s} = g_{x^s} = 0$ and imposing $g_C = g_Y$, $g_h = g_{n^s}$; see Appendix B.1) yields the central analytical results of the paper:

$$g_h = \frac{1-\gamma}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \cdot \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)} (g^s - g^i), \quad (29)$$

$$g_C = \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \cdot \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)} (g^s - g^i), \quad (30)$$

$$g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)} (g^s - g^i). \quad (31)$$

It is useful to clarify the growth interpretation of these results. Equilibrium growth rates depend on the exogenous productivity growth rates g^s and g^i in interaction with all the structural parameters of the model (θ , γ , α , σ), in the spirit of semi-endogenous growth theory; but TFP growth itself is exogenous by design, and an R&D microfoundation of the differential $g^s - g^i$ is left to future work. Moreover, market power shapes growth rates through its *presence*, not its *intensity*: the rising markdown enters the asymptotic growth rates of consumption and hours through g_μ , while the intensity of oligopsony, indexed by m , affects levels only — a distinction Section 6 illustrates directly. To gauge the effect of monopsony power, we next compare these results with those of the corresponding competitive economy.

¹⁶Specifically, $\frac{\dot{\mu}_t}{\mu_t} = \frac{\alpha(g_{n^s}(t) - g_{n^i}(t))}{(1/m)(n_t^i/n_t^s) + \alpha}$, which converges to $g_{n^s} - g_{n^i}$ as $n_t^i/n_t^s \rightarrow 0$. The bottom panel of Figure 3 provides graphical intuition.

4.7 The Competitive Economy

In the competitive benchmark, all firms are price takers in product markets and wage takers in labor markets. The household problem and the inferior sector are unchanged. The superior sector now operates competitively, so superstar profits vanish ($\Pi_t^s = 0$). We denote competitive growth rates with an asterisk.

The competitive economy is a decentralized version of the unbalanced growth equilibrium of Acemoglu and Guerrieri (2008). The system of equilibrium conditions coincides with that of the SSMPE except for the intersectoral allocation equation, which loses both the markup and the markdown:

$$p_t^s(1 - \alpha)(K_t^s)^\alpha(n_t^s)^{-\alpha}(A_t^s)^{1-\alpha} = p_t^i(1 - \alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha}. \quad (32)$$

The full system in growth rates is reported in Appendix B.1, and yields the optimal growth rates

$$g_C^* = \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s, \quad g_h^* = g_{n^s}^* = \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s, \quad g_{n^s}^* - g_{n^i}^* = (1 - \alpha)(\sigma - 1)(g^s - g^i). \quad (33)$$

Proposition 3. *The competitive economy admits a unique asymptotic BGP with the same qualitative features as Proposition 1, with one notable exception: the growth rate of wages coincides with the growth rate of output, $g_w = g_Y$.*

Comparing equations (29)–(31) with (33), the two distortions in equation (28) relative to (32)—the markup and the markdown—play asymmetric roles. The constant oligopoly markup affects only the level of the steady-state capital–efficiency ratio x^s and has no impact on growth rates, while the rising markdown μ_t depresses the growth rates of both hours and consumption. We formalize and quantify this misallocation in the next subsection.

4.8 Growth Misallocation and Consumption Equivalent Loss

We can formally show that imperfect labor markets induce negative growth effects in the SSMPE. The following proposition summarizes the main result.

Proposition 4. *The balanced growth path of the SSMPE is inefficient: both output per capita and total hours grow more slowly than in the corresponding optimal growth problem.*

The proof is immediate from comparing the balanced growth path for hours and consumption in the decentralized monopsonistic equilibrium of equations (29) and (30) with the optimal ones in equation (33). The growth expressions can be rewritten as

$$g_C = g_C^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu, \quad g_h = g_h^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu, \quad (34)$$

where $g_\mu = g_{n^s} - g_{n^i} > 0$ as reported in equation (31). Equation (34) immediately shows that, since $g_\mu > 0$ along the decentralized equilibrium, monopsony power distorts both hours and consumption growth, generating a “growth drag” ($g_h < g_h^*$ and $g_C < g_C^*$). The growth rates of both consumption and total hours are thus negatively affected by the growth of the markdown, and imperfect labor markets introduce an additional negative channel in the long-run dynamics of labor supply.

The total growth misallocation depends on the preference parameters γ and θ as well as on the technological parameters α and σ . We first discuss the role of the technological parameters. Assumption 4 highlighted that diminishing returns to capital—and thus $0 < \alpha < 1$ —is an essential prerequisite for the existence of monopsony. In addition, equation (34) shows that misallocative effects on output are present as long as $\alpha < 1$, that is, as long as labor has a productive role in the superior technology. The role of σ operates through its effect on the growth of the markdown g_μ : a higher value of σ implies higher markdown growth ($\partial g_\mu / \partial \sigma > 0$). The parameter σ in the model has a dual role, since it reflects both the elasticity of substitution between the two intermediaries and the oligopoly markup of the superstar firms. The model requires $\sigma > 1/m$, since the markup is $\varrho = \frac{\sigma m}{\sigma m - 1}$. A higher σ implies a lower markup and lower oligopoly power. Yet one can easily establish that there is no *direct* growth effect of monopoly power in the model: a simple monopoly model without monopsony yields $g_C^m = g_C^*$, where g_C^m is the growth rate of consumption in the version of the model with only monopoly power in the product market and no monopsony power in the labor market. In the SSMPE, however, a higher σ implies lower growth through an *indirect* channel that operates via the markdown μ_t . The mechanism is the following. A higher σ implies higher substitutability between the goods produced in the superior and inferior sectors. Higher substitutability translates into faster growth of the markdown g_μ and thus larger monopsony power, which in turn leads to higher overall distortion and lower growth.

An Alternative Explanation for Declining Hours

The preference parameters θ and γ also play an important role in the growth process, and in particular in the associated long-run decline in hours per person and hours per worker. In the recent literature surveyed in Section 2, much of the attention has been devoted to the income effect of long-run wage increases. Boppart and Krusell (2020) propose a class of preferences coherent with the observed long-run decline in hours worked along a balanced growth path. With respect to the MaCurdy (1981) preferences specified in our model, the Boppart and Krusell (2020) class requires the elasticity of marginal utility of consumption to be strictly larger than one (i.e. $\gamma > 1$). In other words, $\gamma > 1$ is a necessary condition for obtaining a neoclassical growth model with declining labor supply, under the assumption of competitive labor markets. This is apparent from equation (33), in which the growth rate of hours is zero when $\gamma = 1$.

In the theory of labor supply, the MaCurdy (1981) utility function with $\gamma = 1$ implies a set of preferences in which the income and substitution effects of wage changes cancel each other out, as discussed by King et al. (1988). The SSMPE delivers an important result in this respect, summarized in the following proposition.

Proposition 5. *In the asymptotic balanced growth path of the SSMPE, the growth rate of hours is negative even when the income and substitution effects of wage changes cancel each other out (i.e. $\gamma = 1$).*

To see this, set $\gamma = 1$ in the monopsonistic growth rate of hours to obtain

$$g_h = -\frac{1}{\frac{1}{\theta} + 1} \frac{(1 - \alpha)(\sigma - 1)}{1 + \alpha + \sigma(1 - \alpha)} (g^s - g^i) < 0 .$$

The result is driven by the additional income effect generated by returns to capital and the profit rebate Π_t^s . Prescott (2004) used a closely related static mechanism to explain steady-state differences in hours worked across the two sides of the Atlantic. Our result can be seen as the dynamic, growth-theoretic counterpart of that classic labor-supply result.

Labor Share Dynamics

The model also has qualitative implications for the second macro fact outlined in Section 2, namely the dynamics of the aggregate labor share. In the competitive equilibrium, the labor share is time-invariant and equals

$$\alpha_{L,t}^* = \frac{w_t^{*s} n_t^{*s} + w_t^{*i} n_t^{*i}}{Y_t^*} = 1 - \alpha ,$$

where the optimal decentralized wages w_t^{*s} and w_t^{*i} coincide with the marginal product of labor in each sector. There is thus no labor share dynamics in the optimal growth problem. By contrast, in the SSMPE the labor share is time-varying and its level depends on the state of the economy. The decentralized labor share is

$$\alpha_{L,t} = \frac{w_t^s n_t^s + w_t^i n_t^i}{Y_t} ,$$

and Proposition 1 delivers the following result for its dynamics.

Proposition 6. *The growth rate of the labor share is negative in the SSMPE, and the capital share (which includes profits) tends to one.*

The result follows directly from the fact that, along the asymptotic balanced growth path, the growth rate of the labor share equals the negative of the growth rate of the markdown:

$$\frac{\dot{\alpha}_L}{\alpha_L} = -g_\mu < 0 .$$

Firm Coordination and Market Concentration

The model contains two distinct objects that should not be conflated. The first is the effective-number index m , which governs conduct within the superstar sector. For fixed σ , a lower m raises both the markup and the markdown. When m is constant, however, it is absent from the asymptotic growth rates in equations (29)–(31): it changes levels, not long-run growth. More generally, a time-varying m_t affects markdown growth while it changes. The quantitative experiment in Section 6.2 considers a one-time permanent fall in m ; it generates a temporary growth adjustment and permanent level effects, after which the economy returns to its previous long-run growth rates.

The second object is the economy-wide importance of the superstar sector. We measure it by its value share, denoted $\hat{C}_{m0}$ at time zero in the calibration,

$$\text{conc}_t^s \equiv \frac{p_t^s y_t^s}{Y_t} = 1 - \frac{p_t^i y_t^i}{Y_t} , \quad \lim_{t \rightarrow \infty} \text{conc}_t^s = 1 .$$

This concentration measure rises endogenously because the superstar sector grows faster than the competitive fringe and eventually dominates production. It is conceptually different from m : concentration can rise through the expansion of the superstar sector even when its effective coordination is unchanged.

Neither object is directly comparable to census CR4 or CR20 ratios. Those statistics measure the employment share of the largest four or twenty firms within an industry, whereas m is a conduct index and conc_t^s is the value share of the model's entire superstar sector. Appendix F uses CR4 and CR20 only in a separate accounting exercise that translates their changes into an implied growth rate of the markdown; their levels are not calibration targets.

Consumption Equivalent Loss

To estimate the misallocation effects of the superstar model, we perform a Lucas-style consumption-equivalent-loss (CEL) exercise. The exercise rests on the numerical simulation of the model and is carried out in Section 6; here we describe the logic. We simulate a path of length T of the competitive economy and denote the resulting sequences by $\{c_t^*, h_t^*\}_{t=0}^T$. Similarly, the path of the monopsonistic equilibrium is $\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T$. The corresponding utility level is

$$\tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T) = \sum_{t=0}^T \beta^t \left[\frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} - \psi \frac{\tilde{h}_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} \right],$$

where $\beta = \frac{1}{1+\rho dt}$ is the discount factor and dt is the length of the period. We approximate TFP growth over a period dt as $A_{t+1}^s - A_t^s \approx dt g^s A_t^s$. The consumption equivalent loss asks what fraction λ of the competitive consumption path the representative agent would be willing to give up in order to obtain the same utility as the misallocated economy. Formally, we seek a λ such that

$$\sum_{t=0}^T \beta^t \left[\frac{[(1-\lambda)c_t^*]^{1-\gamma}}{1-\gamma} - \psi \frac{(h_t^*)^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} \right] = \tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T) < U(\{c_t^*, h_t^*\}_{t=0}^T).$$

The results are reported in the numerical Section 6.

4.9 Stability of the SSMPE

We now investigate whether the SSMPE approaches the BGP. We focus on allocations in the neighborhood of the BGP, thus investigating only local (saddle-path) stability. To prove stability, we cannot use the formulation of the system above with x_t^s and x_t^i : since capital per unit of efficiency is unbounded ($x_t^i \rightarrow \infty$), we cannot write a Jacobian for the dynamical system evaluated at the BGP. We thus introduce variables capturing the share of capital and labor in the two sectors, in line with Acemoglu and Guerrieri (2008),

$$K_t = K_t^s + K_t^i, \quad \kappa_t = \frac{K_t^i}{K_t}, \quad \nu_t = \frac{n_t^i}{h_t}, \quad x_t = \frac{K_t}{A_t^s h_t}, \quad c_t = \frac{C_t}{A_t^s h_t}.$$

Along the BGP, κ_t , ν_t , and h_t all converge to zero, while x_t and c_t converge to positive finite values $\bar{x}$ and $\bar{c}$. We can study the behavior of the system in a neighborhood of these values. For the purposes

of the proof, the production functions of the intermediaries are Cobb–Douglas and the depreciation rates are symmetric ($\delta^s = \delta^i = \delta$). The full system of equations is presented in Appendix D.

Theorem 1. *If capital depreciates at the same rate in both sectors ($\delta^s = \delta^i$), the SSMPE exhibits local saddle-path stability.*

The details of the proof are in Appendix D. The key idea is to rewrite the system of equations as

$$\dot{z} = \Phi(z),$$

where $z = (c, h, \nu, x)$ and Φ is the transition of the dynamical system. Four equations is the minimum number of equations we can reduce to: the Euler equation, the aggregate resource constraint, and the consumption–leisure tradeoff together with the monopsonistic labor supply, both differentiated with respect to time. We can substitute out κ since the equation stemming from equality of the rates of return to capital is time-invariant; for the opposite reason, we cannot dispense with the consumption–leisure and labor-demand equations, since they are time-dependent. Because we have endogenous labor supply, we end up with one more equation than Acemoglu and Guerrieri (2008). In our setting, we have three predetermined variables: h_0 , κ_0 , and ν_0 . One of these, κ_0 , follows naturally from $K_0 = K_0^s + K_0^i$ being given. The two remaining predetermined values arise because two of our differential equations come from differentiating intertemporal constraints: once C_0 is chosen, these equations determine h_0 and ν_0 . We therefore need to show, first, that the BGP is hyperbolic—that is, that all eigenvalues of the Jacobian of the transition evaluated at the BGP have non-zero real part—and, second, that three eigenvalues are negative while one is positive.

We show that the determinant of the Jacobian of the transition evaluated at the BGP equals the determinant of the following matrix (where $*$ denotes a finite, non-zero constant):

$$\hat{J}_\Phi = \begin{pmatrix} 0 & 0 & * & \frac{\bar{c}}{\gamma} \frac{\sigma-1}{\sigma} \zeta^{\frac{1}{\sigma-1}} \alpha (\alpha-1) \bar{x}^{\alpha-2} \\ * & g_h & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * \end{pmatrix},$$

where asterisks denote finite, possibly non-zero values. We then have

$$\det J_\Phi = \det \hat{J}_\Phi = - \left(-\frac{1}{1+\theta\alpha} \right) g_h (g_{n^i} - g_{n^s}) \frac{c}{\gamma} \frac{\sigma-1}{\sigma} \zeta^{\frac{1}{\sigma-1}} \alpha (\alpha-1) x^{\alpha-2} < 0,$$

so the BGP is hyperbolic. Since the determinant is negative, either one or three eigenvalues are negative. We then compute the characteristic polynomial of the Jacobian, whose roots are the eigenvalues. Letting λ denote the variable of the polynomial and $*$ again denote a finite, non-zero constant, we find

$$P(\lambda) = \det(J_\Phi - \lambda I) = \begin{pmatrix} * - \lambda & 0 & * & * \\ * & g_h - \lambda & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} - \lambda & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * - \lambda \end{pmatrix}.$$

It is then straightforward to verify that

$$P(\lambda) = (g_h - \lambda)(g_{n^i} - g_{n^s} - \lambda) \det \begin{pmatrix} * - \lambda & * \\ -\frac{1}{1+\theta\alpha} & * - \lambda \end{pmatrix} .$$

Since both g_h and $g_{n^i} - g_{n^s}$ are negative, two of the eigenvalues must be negative. The only configuration consistent with a negative determinant of the Jacobian is then that three eigenvalues are negative and one positive, which establishes stability.

4.10 Implications of Different Capital Intensities

In our baseline model, the results are driven by unbalanced TFP growth between the superior superstar sector and the inferior competitive sector. Autor et al. (2020) provide evidence on the productivity growth differential between superstar and ordinary firms.

Our analysis, however, suggests that an alternative mechanism may also be at work, namely different capital intensities between superstar and fringe firms. Indeed, Autor et al. (2020) also document higher capital intensity in superstar firms. While the model and the SSMPE were solved under the assumption of symmetric capital intensity across sectors ($\alpha^i = \alpha^s$) with asymmetric labor-augmenting technological progress, the model can equally be solved under the alternative case of $\alpha^s > \alpha^i$ with Hicks-neutral TFP growth, as in Acemoglu and Guerrieri (2008). Under Hicks-neutral technological progress, all the growth results presented above hold as long as

$$\frac{g^s}{1 - \alpha^s} > \frac{g^i}{1 - \alpha^i} .$$

Hence, to obtain a model with declining labor share, declining hours worked, and a growth drag, it is sufficient to assume $g^s = g^i$ together with $\alpha^s > \alpha^i$. Since Autor et al. (2020) document both a productivity growth differential and higher capital intensity for superstar firms, both specifications are empirically grounded. We adopt biased TFP growth ($g^s > g^i$ with symmetric capital intensities) as the baseline throughout the paper — it is the specification the calibration of Section 6 disciplines directly through the productivity growth rates — and regard the Hicks-neutral case with $\alpha^s > \alpha^i$ as a robustness variant.

5 A Worker-Capitalist Model and Policy

One concern related to the decline of the labor share is the rise in inequality across individuals and the shifting distribution of income across segments of the population. In a representative agent (RA) framework such mechanisms are muted: while payments shift from labor to profits, the RA owns both labor and capital, and receives all capital income in the form of interest and dividends. As discussed in Section 4.8, from the labor-supply perspective the rebate of profits to the RA generates an additional income effect beyond that of a pure wage increase. This is one of the primary sources of growth misallocation in the SSMPE, closely related to the classic labor-supply effect of public spending highlighted by Prescott (2004).

This section explores how an economy populated by separate *workers* and *capitalists* changes the predictions of our model. Workers are the only suppliers of labor; they cannot save and consume

their labor income period by period. Capitalists do not work but save in a safe asset that gives them full claims to capital and profits, so that they earn both the marginal product of capital and oligopoly rents. The setup is in the spirit of Straub and Werning (2020), who build on the classic paper by Judd (1985) on optimal taxation.

The decentralized worker-capitalist equilibrium. There is a unit measure of workers and a unit measure of capitalists. Workers solve a static problem period by period:

$$\max_{C_t^w, h_t} \frac{(C_t^w)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1}, \quad \text{s.t.} \quad C_t^w = w_t^s n_t^s + w_t^i n_t^i, \quad n_t^s + n_t^i = h_t,$$

where the budget constraint shows that workers finance period- t consumption C_t^w exclusively out of labor income from the two sectors. The first-order conditions deliver a standard consumption-leisure tradeoff and the wage-arbitrage condition,

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} w_t, \quad w_t^{f,s} = w_t^i = w_t \quad \text{for all } f \in \{1, \dots, m\}.$$

Capitalists derive utility from consumption only and solve a standard forward-looking problem

$$\max_{\{C_t^c\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{(C_t^c)^{1-\gamma} - 1}{1-\gamma} dt, \quad \text{s.t.} \quad C_t^c + \dot{A}_t^s + \dot{A}_t^i = r_t^s A_t^s + r_t^i A_t^i + \Pi_t^s,$$

which delivers a standard Euler equation and the return-equalization condition,

$$\gamma \frac{\dot{C}_t^c}{C_t^c} = r_t - \rho, \quad r_t^{f,s} = r_t^i = r_t \quad \text{for all } f.$$

The production side of the economy is identical to the SSMPE of Section 4, with the same symmetric Cournot equilibrium $K_t^s = m \bar{K}_t^s$, $n_t^s = m \bar{n}_t^s$, $y_t^s = m \bar{y}_t^s$. The equilibrium system of nine ODEs is therefore essentially identical to that of the representative-agent SSMPE, with one substitution: the household Euler equation (23) is replaced by the workers' consumption-leisure condition and budget constraint,

$$C_t^w = w_t h_t, \quad \psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} w_t,$$

and the capitalists' Euler equation now governs capital accumulation. The full intensive-form system is presented in Appendix B.2.

Because the production side of the economy is unchanged, the growth rate of the markdown, the growth of the relative labor allocation, and the growth of the inferior intermediate price all coincide with the RA solution of Section 4.6:

$$g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i), \quad g_{p^i} = 2 \frac{1-\alpha}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i).$$

The growth rate of wages is likewise unchanged from the RA monopsonistic case, since it depends only on the inferior price and parameters: $g_w = g^s - g_\mu < g^s$. What differs from the SSMPE is the equilibrium growth rate of labor supply. Since workers finance consumption entirely out of labor

income and do not receive a profit rebate, the growth of C_t^w is proportional to the growth of wages, and the equilibrium growth rate of hours reads

$$g_h = \frac{1-\gamma}{\frac{1}{\theta} + \gamma} g_w = \frac{1-\gamma}{\frac{1}{\theta} + \gamma} \left[g^s - \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)} (g^s - g^i) \right]. \quad (35)$$

Capitalists, who earn both the rate of return on capital and oligopoly profits, see their consumption grow at $g_{C^c} = g^s + g_h$, while workers' consumption grows only at $g_{C^w} = g_w + g_h$. Since $g_w < g^s$, the divergence is permanent.

Proposition 7. *In the balanced growth equilibrium of the worker-capitalist economy, consumption growth is unbalanced between the two types of agents and, asymptotically, capitalists absorb the entire share of aggregate consumption.*

The result follows immediately from $g_{C^c} - g_{C^w} = g^s - g_w = g_\mu > 0$, so that $C_t^c / (C_t^w + C_t^c) \rightarrow 1$.

The utilitarian efficient benchmark. To evaluate the welfare properties of the decentralized worker-capitalist equilibrium, we introduce a utilitarian planner who assigns a welfare weight $\omega > 0$ to capitalists.¹⁷ The planner solves

$$\max_{\{C_t^c, C_t^w, h_t\}_{t=0}^\infty} \int_0^\infty e^{-\rho t} \left[\frac{(C_t^w)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \omega \frac{(C_t^c)^{1-\gamma} - 1}{1-\gamma} \right] dt$$

subject to the aggregate resource constraint, the CES aggregator, and the time constraint $n_t^s + n_t^i = h_t$. The planner uses the two CRS technologies directly and abstracts from the number of firms in the superior sector. The first-order conditions imply that the two agents consume a constant fraction of output, $\omega^{1/\gamma} C_t^{*w} = C_t^{*c}$, and that the consumption-leisure tradeoff takes the standard neoclassical form $\psi(h_t^*)^{1/\theta} = (C_t^{*w})^{-\gamma} \partial Y_t / \partial h_t$. The optimal growth rates therefore read

$$g_h^* = \frac{1-\gamma}{\frac{1}{\theta} + \gamma} g^s, \quad g_{C^w}^* = g_{C^c}^* = g^s + g_h^* = \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s, \quad (36)$$

which coincide exactly with the optimal growth rates of the representative-agent model. Two conclusions follow. First, the inequality result is dramatic: while the decentralized monopsonistic market implies ever-increasing consumption inequality between the two groups, the utilitarian planner delivers balanced consumption growth with a constant relative weight determined by ω . Second, comparing the labor-supply rates in equations (35) and (36) reveals a striking and counterintuitive result.

Proposition 8. *In the decentralized worker-capitalist monopsonistic economy, the growth rate of labor supply differs from the utilitarian optimum as follows:*

1. *If preferences are in the Boppart and Krusell (2020) class ($\gamma > 1$), workers supply more hours than is optimal: $g_h > g_h^*$.*

¹⁷We assume the planner cannot make capitalists work. This affects the levels but not the growth rates of the solution.

2. If preferences satisfy the King et al. (1988) restriction ($\gamma = 1$), hours are constant in both the efficient and decentralized monopsonistic equilibrium.

Algebraically, the deviations from the optimum can be expressed compactly as

$$g_h = g_h^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu, \quad g_{C^c} = g_{C^c}^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu, \quad g_{C^w} = g_{C^w}^* - \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g_\mu. \quad (37)$$

Proposition 8 highlights the role of profits in shaping labor supply. In the SSMPE with representative agents, the rebate of profits to the household generated an additional income effect that depressed hours growth *below* the optimal level. In the worker-capitalist economy this channel is shut down: workers decide their labor supply on the basis of labor income alone and never see the profits. The result is that workers now supply *more* hours than is socially optimal — the labor-supply distortion has flipped sign. This experiment confirms that the labor-supply channel emphasized in Section 4.8 is indeed driven by the profit-rebate income effect, and constitutes the long-run growth counterpart of Prescott (2004).

Redistributive taxation. The analysis of the worker-capitalist economy raises a natural policy question: can a redistributive tax restore the efficient allocation? Consider a proportional tax τ on capitalists' total income Ω_t^c , whose proceeds are rebated to workers. Capitalists' total income equals output net of labor payments and capital depreciation,

$$\Omega_t^c = Y_t - w_t h_t - \delta^s K_t^s - \delta^i K_t^i,$$

which combines returns to capital and oligopoly profits. The only modification to the model is the workers' budget constraint, which becomes

$$C_t^w = (1 - \tau)w_t h_t + \tau(Y_t - \delta^s K_t^s - \delta^i K_t^i). \quad (38)$$

Equation (38) implies that, asymptotically, workers consume a constant fraction τ of output net of depreciation.¹⁸ Since the production side is unchanged, the growth rates of the markdown, the inferior price, and the wage are identical to the no-tax case. The transfer of profits to workers, however, reintroduces the additional income effect that was active in the representative agent SSMPE. The resulting growth rates with the tax read

$$g_h^\tau = g_h^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu, \quad g_{C^w}^\tau = g_{C^c}^\tau = g_C^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu. \quad (39)$$

Comparing equation (39) with the no-tax outcome of equation (37) yields a clear picture: the tax pushes the growth rate of hours from *above* the optimum (Proposition 8) to *below* the optimum, mirroring the SSMPE distortion. The tax thus eliminates the growth inequality between workers and capitalists at the cost of overshooting the efficient labor supply.

Proposition 9. *A proportional tax on capitalists' income, rebated to workers:*

¹⁸By Walras's law, capitalists' consumption is pinned down by the aggregate resource constraint and does not enter the system of equations explicitly.

1. *eliminates the growth inequality between workers' and capitalists' consumption;*
2. *depresses the growth rate of hours and total output below the utilitarian optimum, reintroducing the labor-supply distortion of the representative-agent SSMPE.*

A tax levied on profits only would have the same qualitative effects, since asymptotically the marginal product of labor accrues to profits.

Minimum wage. In the presence of monopsony, a binding minimum wage can in principle restore efficient outcomes, in the spirit of the classic Robinson (1933) argument. We show that along the balanced growth path of the worker-capitalist economy, a minimum wage that grows at the rate g^s achieves the efficient growth rates of hours and consumption.

To formalize the argument, consider an exogenously specified minimum wage $\bar{w}_t$. The inferior sector and the household problem are unchanged. Each superstar firm chooses labor, capital, and the wage to maximize profits subject to $W_t^{f,s} \geq \bar{w}_t$; the firm's problem and the conditions under which the minimum wage is binding are reported in Appendix B.2.¹⁹ When the minimum wage binds, each superstar firm sets $w_t^{f,s} = \bar{w}_t$, so that the firm becomes effectively a pure oligopolist: the wage is exogenously given and the firm has no scope for monopsonistic markdown.

The consumption-leisure condition of workers under a binding minimum wage reads

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} \bar{w}_t.$$

Combining this condition with the utilitarian growth rates from equation (36), the minimum wage growth rate that delivers the efficient allocation is

$$g_{\bar{w}}^* = \frac{1}{\theta} g_h^* + \gamma g_{C^w}^* = g^s.$$

Proposition 10. *A minimum wage growing at the rate of superior-sector productivity, $g_{\bar{w}}^* = g^s$, achieves the utilitarian-efficient growth rates of hours worked and consumption in the worker-capitalist monopsonistic economy.*

Two remarks are in order. First, although the minimum wage achieves the efficient growth rate of hours, it actually *depresses* hours growth relative to the no-policy decentralized equilibrium of Proposition 8. This is a direct consequence of the fact that, in the worker-capitalist economy, monopsony causes workers to *work too much* rather than too little — the conventional intuition of monopsony reducing labor input is inverted here because the profit-rebate channel is shut down. Second, the result for the worker-capitalist economy contrasts with what a minimum wage would do in the representative agent SSMPE of Section 4. In the RA case, where workers also receive profits, a growing minimum wage would deliver the classic Robinson (1933) outcome: it would *raise* both the growth of labor supply and the growth of wages relative to the decentralized monopsonistic equilibrium.

Proposition 10 must also be read against the recent quantitative literature on minimum wages under monopsony, in particular Berger et al. (2025), who find limited scope for welfare gains in an

¹⁹The minimum wage need not be binding at every date: when the inferior-sector marginal product of labor exceeds $\bar{w}_t$ even at $N_t^i = H_t$, the constraint is slack. For the analysis below we focus on the case in which $\bar{w}_t$ binds.

estimated oligopsony model with heterogeneous employers: modest optimal levels, and losses from high minimum wages driven by the reallocation of employment across firms of different productivity. We view our results as complementary to theirs. The exercise of Berger et al. (2025) concerns employment and welfare in levels, while Proposition 10 concerns the *growth rate* of the wage floor along a balanced path — the classic Robinson (1933) logic applied to trends. Proposition 10 should therefore be interpreted narrowly: a wage floor indexed to superior-sector productivity corrects the dynamic margin of the markdown distortion, while the exercise we perform above is silent on the static, cross-sectional costs that discipline the level of the minimum wage in Berger et al. (2025).

6 Quantifying the Growth Misallocation and the Consumption Equivalent Loss

We are now in a position to quantify the misallocative effects of the distortions induced by market power in the superior sector. The quantitative analysis has two parts, and they mirror the two levels at which Section 2.3 organized the evidence.

The first part (Section 6.1) is a *long-run* exercise on the representative advanced economy. It asks whether the three secular trends of Table 1 — positive GDP growth, declining hours, declining labor share — can coexist on a balanced growth path, and what their coexistence identifies. The data inputs are the nine-country averages over the full 1980–2023 window: by Panel B of Table 2, the panel trends are smooth across subperiods, so full-window averaging is the appropriate object, and the exercise makes *no dating claim*. Within this part we also decompose the income effect required to match the hours decline and quantify the asymptotic growth drag of oligopsony relative to the competitive benchmark.

The second part (Section 6.2) is a *transition* exercise on the United States. It asks how much of the dated, post-2000 joint decline of the labor share and hours the mechanism delivers when disciplined by direct measurements of the markdown, and at what welfare cost. The data inputs are the US 2000–2023 trends of Panel A of Table 2.

The two parts are calibrated on separate data and are connected by an explicit convergence assumption, which we state here once: *the calibrated US economy converges to a country specific asymptotic path, whose growth rates are the observed US 2000–2023 trends*. We do not assume that the United States converges to the representative advanced economy. Treating observed sample-average trends as asymptotic rates involves an approximation the model itself can evaluate: along the calibrated transition, growth rates at the start of the sample differ from their asymptotic values, and Table 5 reports exactly how far — the economy’s position on the path, to which we return below. While Section 6.1 relies on the version of the model with capital, Section 6.2 confronts the well-known instability problems that affect shooting algorithms in growth models with capital and forward-looking consumption (Judd, 1998; Miranda and Fackler, 2002).²⁰ In our model these problems are exacerbated by the complexity of the joint labor and capital allocation across the two sectors. To circumvent the instability, we calibrate a version of the model in which capital is replaced by a fixed factor of production, which can be interpreted as “materials” or “land”. In this version

²⁰The shooting algorithm is unstable because small errors in guessing initial conditions are magnified through the integration of the differential equations, leading to slow or failed convergence.

there are no savings and consumption absorbs period- t output; the model can be solved and simulated for any exogenous productivity path $\{A_t^s, A_t^i\}_{t=0}^T$. The details of the model and the few differences with respect to the full model with capital are reported in Appendix E. The simulations in Section 6.2 refer to the model with the fixed factor.

6.1 Asymptotic Calibration and Growth Drag

In the first quantitative exercise, we assume that the “representative advanced economy” features a long-run growth behavior summarized by the three moments reported in Section 2, and moves along the asymptotic superstar equilibrium defined by the model. The three long-run facts concern growth in GDP, (negative) growth in hours worked, and (negative) growth of the labor share. Table 3, in the columns labeled “data”, reports the target moments on both margins: the nine-country averages of log-linear trends over the full 1980–2023 window from Table 1. Following the evidence of Section 2.3, the *per-worker* margin is our main target: it is the hours fact that is robust across countries and subperiods. The per-capita margin is reported for completeness, and we explain below why it is fragile in this exercise.

We denote the empirical estimates of the three per-capita moments by $\hat{g}_C^{pc}$, $\hat{g}_h^{pc}$, and $\hat{g}_{\alpha_L}^{pc}$, and the corresponding per-worker moments by $\hat{g}_C^{pw}$, $\hat{g}_h^{pw}$, and $\hat{g}_{\alpha_L}^{pw}$. The quantitative goal is to find values of the parameters γ, g^s, θ that match the three theoretical moments derived in Section 4.6. We take θ from the literature (λ -constant elasticity of labor supply from Chetty et al., 2011), so two parameters remain: g^s and γ .

Consider first the monopsonistic SSMPE. For given θ and the three empirical moments, the theoretical growth rates derived in Section 4.6 can be inverted to back out $\hat{\gamma}$ and $\hat{g}^s$ in closed form:

$$\hat{\gamma}^j = 1 - \left(1 + \frac{1}{\theta}\right) \frac{\hat{g}_h^j}{\hat{g}_C^j} + \frac{\hat{g}_{\alpha_L}^j}{\hat{g}_C^j}, \quad j = pw, pc \quad (40)$$

$$\hat{g}^{s,j} = \frac{\hat{g}_h^j \left(\frac{1}{\theta} + \hat{\gamma}^j\right) - \hat{g}_{\alpha_L}^j}{1 - \hat{\gamma}^j}, \quad j = pw, pc \quad (41)$$

Finally, the SSMPE implies that the asymptotic growth rate of the markdown is the negative of the asymptotic growth rate of the labor share,

$$\hat{g}_\mu = -\hat{g}_{\alpha_L}, \quad (42)$$

which is independent of whether one uses per-capita or per-worker data. Given the closed-form expressions (40)–(42), the SSMPE matches the three moments exactly in both cases, as confirmed in Table 3. The inversion also provides an overidentifying implication: combining (40) and (41), the implied technology growth satisfies $\hat{g}^s = \hat{g}_C - \hat{g}_h$ on both margins, and the data deliver the same value, $\hat{g}^s = 1.43$ percent per year, per worker and per capita alike.

We now turn to the columns labeled “NGM” in Table 3, and we stress how they should be read. The NGM is not a competing model but the *nested* case of the SSMPE at $g_\mu = 0$: equations (40) and (41) still match GDP and hours growth, but the labor share is asymptotically constant, as outlined in Section 4.8, regardless of the capital elasticity. The comparison is therefore an identification statement rather than a test the SSMPE could fail: the GDP and hours moments identify (γ, g^s)

in either model, and the labor-share moment identifies g_μ — the object the NGM restricts to zero. Preferences in which the income effect of wage growth dominates the substitution effect, as in Boppart and Krusell (2020), account for the joint decline of hours with positive growth; a model with a trending labor share, such as the SSMPE, is needed for the third moment, and the interesting quantitative question — taken up in Section 6.2 — is whether independently measured markdown growth can support the required g_μ .

Table 3 also performs an additional exercise concerning the income effect and the size of γ in the MaCurdy (1981) preferences. Boppart and Krusell (2020) show that, within an NGM, matching the declining trend of labor supply requires $\gamma > 1$. The larger the downward trend of hours worked, the larger the income effect needed. In the SSMPE, however, profits are rebated to the representative consumer, and this profit channel generates an additional income effect that depresses hours worked — the dynamic counterpart of the Prescott (2004) static result. Since the SSMPE has a built-in profit channel arising from market imperfections, it relies less on the preference parameter γ to match the observed decline in hours. We measure the extra income effect attributable to misallocation as

$$\Delta IE = \frac{\gamma^{NGM,j} - \gamma^{SSMPE,j}}{\gamma^{NGM,j} - 1}, \quad j = pw, pc,$$

where $\gamma^{NGM,j}$ and $\gamma^{SSMPE,j}$ are the values of γ calibrated in the two models. In the definition of ΔIE , the value $\gamma = 1$ corresponds to the traditional preferences used in the business-cycle literature since King et al. (1988), under which hours are constant; ΔIE thus measures how much of the income effect *in excess of* the constant-hours benchmark the markdown channel replaces. For hours per worker, Table 3 delivers $\gamma^{SSMPE} = 1.21$ against $\gamma^{NGM} = 1.37$: the markdown channel replaces 44 percent of the excess income effect. The per-capita exercise is not informative. Indeed, the panel hours-per-person trend is essentially zero (Table 1), so $\gamma^{NGM,pc} = 1.01$ — the denominator of ΔIE vanishes — and the SSMPE inversion returns $\gamma^{SSMPE,pc} = 0.88 < 1$, with the labor-share term dominating a near-zero hours term. When the hours trend vanishes, the inversion in (41) divides by $1 - \hat{\gamma} \approx 0$: the per-capita margin simply does not carry enough information to separate the income effect from the markdown channel, which is the quantitative counterpart of the choice of the per-worker margin as the headline hours fact (Section 2).

The third quantitative exercise linked to the asymptotic equilibrium estimates the size of the “growth drag” due to market imperfections. The SSMPE delivers a growth drag for both GDP and hours, the analytical expressions for which were derived in Section 4.8. Table 4 quantifies this drag using the calibrated values from Table 3. The *presence* of oligopsony lowers the asymptotic growth rate of GDP by 9.4 percent of the competitive rate on both margins; hours per worker decline about 61 percent faster than in the competitive economy (−0.31 against −0.19 percent per year). On the per-capita margin the competitive economy features mildly *rising* hours, so a proportional drag is not defined against a near-zero base; the level statement — oligopsony turns mildly rising hours per person into flat-to-declining ones — is reported in the table note. This result suggests that the presence of oligopsony affects asymptotic *growth rates*, through the drag $-g_\mu$; for a constant effective-number index m , its intensity has *level* effects only, a distinction the transition experiment of Figure 5 illustrates directly.

The exercises of this subsection are statements about asymptotic growth rates, and nothing in the closed forms above pins where an actual economy stands relative to its asymptote. That position is a

Table 3: Calibration of the Asymptotic Balanced Growth Path, Representative Advanced Economy (1980–2023)

	<i>Model SSMPE^a</i>		<i>NGM ($g_\mu = 0$)^b</i>		<i>Data</i>	
	per worker	per capita	per worker	per capita	per worker	per capita
<i>Moments (per year)</i>						
GDP growth	0.0112	0.0141	0.0112	0.0141	0.0112	0.0141
Growth of hours	-0.0031	-0.0001	-0.0031	-0.0001	-0.0031	-0.0001
Growth of labor share	-0.0018	-0.0018	0.0000	0.0000	-0.0018	-0.0018
<i>Parameters^c</i>						
γ	1.210	0.884	1.373	1.013		
g^s	0.014	0.014	0.014	0.014		
θ	2.84	2.84	2.84	2.84		
<i>Share of income effect due to misallocation^d (per worker)</i>						
	43.6%					
^a SSMPE is the calibrated asymptotic Superstar Monopsonistic Equilibrium; it matches the three moments exactly.						
^b The NGM is the model at $g_\mu = 0$: it matches GDP and hours growth, and implies a constant labor share.						
^c γ and θ are the MaCurdy (1981) preference parameters; g^s the growth of the superior technology.						
Data: nine-country averages of log-linear trends, 1980–2023, Table 1.						
^d $(\gamma^{NGM} - \gamma^{SSMPE})/(\gamma^{NGM} - 1)$; reported for the headline per-worker margin only — for the per-capita margin $\gamma^{NGM} \approx 1$ and the ratio is uninformative (see text).						
Source: Authors’ calculations.						

Table 4: Asymptotic Growth Drag, Representative Advanced Economy (1980–2023)

	<i>Model SSMPE^a</i>		<i>Competitive Economy^b</i>	
	per worker	per capita	per worker	per capita
<i>Moments (per year)</i>				
GDP growth	0.0112	0.0141	0.0123	0.0156
Growth of hours	-0.0031	-0.0001	-0.0019	0.0013
Growth of labor share	-0.0018	-0.0018	0.0000	0.0000
<i>Growth drag^c (percent of the competitive rate)</i>				
GDP growth	-9.42% (pw) -9.42% (pc)			
Hours	+60.63% (pw); per capita: sign reversal, see text			
^a SSMPE: asymptotic Superstar Monopsonistic Equilibrium at the calibrated parameters of Table 3.				
^b Competitive economy with the SSMPE parameters ($g_\mu = 0$).				
^c Loss in the asymptotic growth rate between the SSMPE and the competitive economy. In the per-capita margin the competitive economy has mildly <i>rising</i> hours (+0.133% per year) while the SSMPE has flat-to-declining hours (−0.013%); the proportional drag is not defined against a near-zero base.				
Source: Authors' calculations.				

property of the transition, and we quantify it for the United States in Section 6.2 (Table 5), using the fixed-factor version of the model introduced there. One caveat is worth stating already: the lockstep relation $g_{\alpha_L} = -g_\mu$ of equation (42) holds asymptotically but not along the transition path, where both rates are smaller in absolute value; statements that rely on it are asymptotic statements, and we flag them as such throughout.

6.2 Calibration of the Growth Transition in the US and Consumption Equivalent Loss

In the fourth quantitative exercise we focus on the US economy and solve numerically for the transition along the asymptotic path. The ultimate goal of this section is to quantify the consumption equivalent loss for the US induced by the growth misallocation of the SSMPE. To address the numerical instability discussed at the beginning of the section, we calibrate a version of the model with a fixed factor of production, interpreted as “materials”.²¹ The logic of the exercise is to calibrate the model to relevant features of the US economy at the beginning of the 21st century, and then simulate the dynamic transition toward the asymptotic SSMPE. Following the subperiod evidence in Section 2.3, we calibrate the model over 2000–2023. During this period, every variable entering the mechanism moves in the direction predicted by the model. For the level of the markdown we rely on the direct measurements of Yeh et al. (2022) and Deb et al. (2022). An accounting exercise based on the concentration data of Autor et al. (2020), reported in Appendix F, provides an independent cross-check on the *growth rate* of the markdown; we return to it when quantifying how much rising monopsony explains.

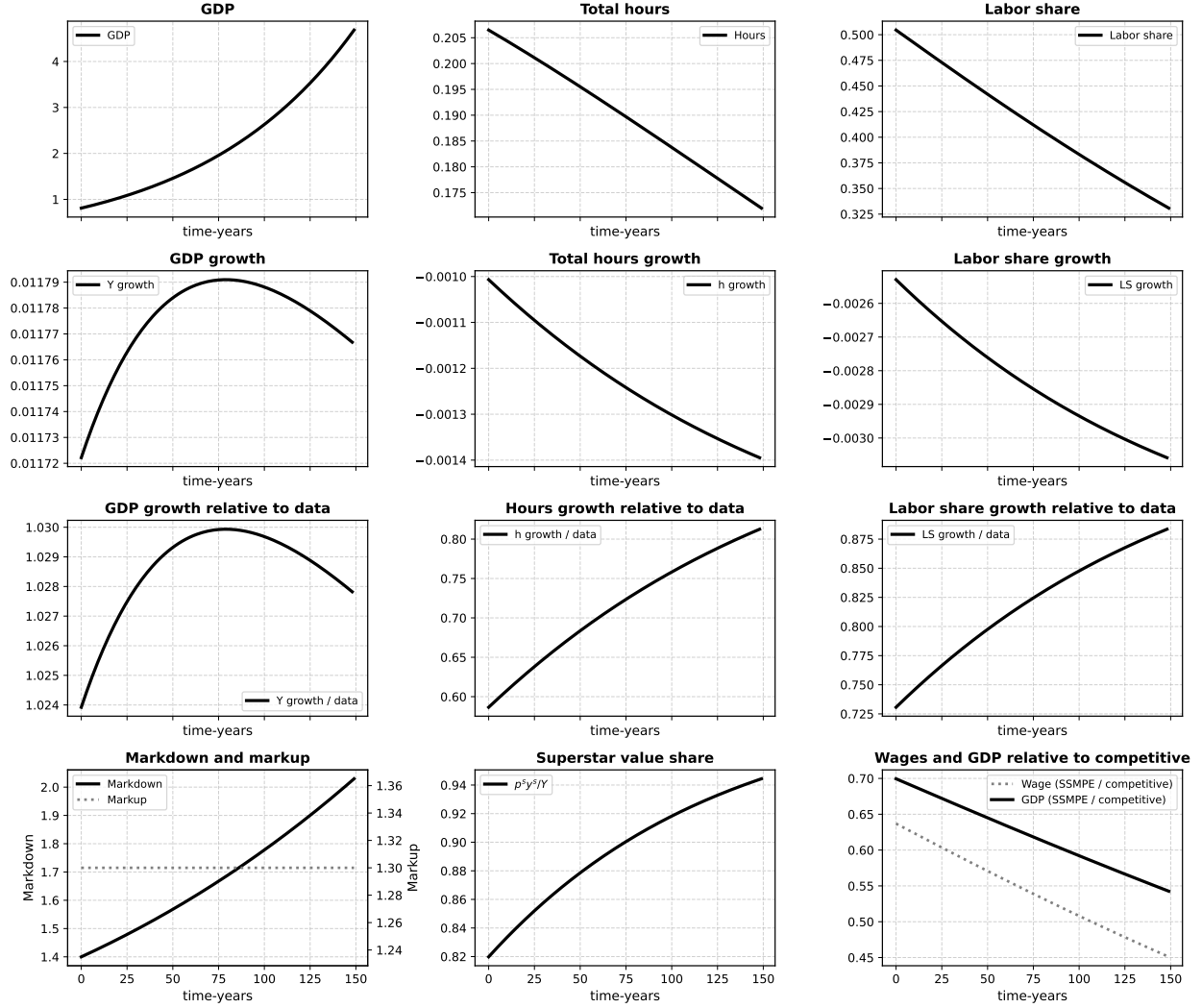
US Calibration and Consumption Equivalent Loss

Calibrating the transitional dynamics for the US economy requires four steps.

Step 1: asymptotic moments. We assume that the US economy features asymptotic growth moments consistent with the model. The empirical moments $\hat{g}_C^{j,us}$, $\hat{g}_h^{j,us}$, $\hat{g}_{\alpha_L}^{j,us}$ describe the asymptotic behavior of the economy, and the calibrated US economy converges to such a stable asymptotic path for given initial conditions. Following the logic of Table 9, this assumption pins down θ , γ , g^s , and g^i . Following the dating of Section 2.3, the growth targets are the US log-linear trends over 2000–2023 reported in Panel A of Table 2: GDP growth of +1.145 percent per year (per worker) and +1.050 (per capita), hours growth of -0.172 and -0.267 , and labor-share growth of -0.346 . Table 9 shows the implied preference and technology parameters. The calibrated $\gamma^{US,pc}$ is slightly above one, reflecting the post-2000 decline of hours per person. For hours per worker the inversion delivers $\gamma^{US,pw} = 0.90 < 1$: over the post-2000 window the observed decline in hours per worker is *more* than fully accounted for by the markdown drag, with the income and substitution effects mildly favoring rising hours. In decomposition terms, the targeted hours-per-worker trend of -0.172 percent per year splits into $+0.100$ from the income and substitution channels and -0.272 from the markdown drag. In this window the model therefore does not rely on the preference parameter to generate falling hours: the markdown channel does the work.

²¹The equations of the model with a fixed factor are reported in Appendix E.

Figure 4: Transitional Dynamics of the Calibrated US Economy (2000–2023 calibration)



Step 2: externally set parameters and normalization. The remaining parameters in the second panel of Table 9 are set as follows. The combination of σ and m determines the markup $\sigma m/(\sigma m - 1)$. We target an initial markup of 1.30 from Autor et al. (2020); this target identifies the product σm , but not σ and m separately. We therefore normalize the effective-number index at $m = 1.25$ and back out the corresponding σ . Section 4.4 explains the conduct interpretation of a continuous m . As a robustness exercise, we instead impose the integer value $m = 2$, preserve the markup target by adjusting σ , and recalibrate the model to the same growth targets.²² The discount rate ρ is set to a standard value yielding a yearly long-run interest rate, and the initial level of inferior-sector technology is normalized to $A^i = 1$. The parameter α is particularly important for the transitional dynamics. With the standard benchmark $\alpha = 0.33$, the speed of convergence to the asymptotic SSMPE is far too slow. In the model with fixed materials, however, the elasticity of output with respect to the fixed factor need not coincide with the elasticity of output to capital. We set $\alpha = 0.2$, which delivers a reasonable transition speed.

Step 3: identification of the remaining parameters. The core of the calibration concerns the four remaining parameters A^s, ψ, T, ζ . Ideally, we would match four time-zero moments of the US economy: the average markdown $\hat{\mu}_0$, the level of the labor share (Karabarbounis, 2024) $\hat{\alpha}_{L0}$, the level of hours worked $\hat{h}_0$, and the model's superstar-sector value share $\hat{C}_{m0}$. However, the analytics of the model imply that these moments are not independent. In particular, the equilibrium of the model delivers

$$\begin{aligned}\hat{\mu}_0 &= 1 + \frac{\alpha}{m} \frac{n_0^s}{n_0^i} \\ \frac{\sigma m}{\sigma m - 1} \frac{n^s}{n^i} \left(1 + \frac{\alpha}{m} \frac{n^s}{n^i} \right) &= \frac{\hat{C}_{m0}}{1 - \hat{C}_{m0}} \\ \frac{\sigma m}{\sigma m - 1} \frac{n^s}{n^i} \left(1 + \frac{\alpha}{m} \frac{n^s}{n^i} \right) + 1 &= \frac{(1 - \alpha)}{\hat{\alpha}_{L0}} \left(1 + \frac{n^s}{n^i} \right).\end{aligned}$$

The three conditions show that only one of the three moments — markdown, labor share, and concentration — can be matched independently. We therefore report three calibrations, corresponding to three alternative numerical anchors, in the three columns of Table 8. The first two are our primary calibrations. They anchor the markdown level to the two direct measurements available in the literature, $\hat{\mu}_0 = 1.40$ (Deb et al., 2022) and $\hat{\mu}_0 = 1.53$ (Yeh et al., 2022), set $n_0^s/n_0^i = m(\hat{\mu}_0 - 1)/\alpha$, and leave the labor share and concentration to be determined endogenously. The third calibration, consistent with the paper's focus on the declining labor share (Karabarbounis, 2024), reverses these roles: it matches $\hat{\alpha}_{L0}$ to the average US labor share over 2000–2023 (0.5996) and leaves $\hat{\mu}_0$ and $\hat{C}_{m0}$ endogenous. The markdown implied by the labor-share anchor is $\hat{\mu}_0 = 1.216$ — *below* both direct measurements. Relative to the measured degree of monopsony, the labor-share anchor therefore provides a conservative reference. Because the choice of anchor affects only the time-0 allocation, all growth implications — the labor-share trend, the hours trend, and the growth drag — are identical

²²At $m = 2$, σ adjusts from 3.47 to 2.17: under the primary anchor $\hat{\mu}_0 = 1.40$ the implied time-0 labor share is 0.483 (against 0.505) and the consumption-equivalent loss at $T = 100$ is 10.5 percent (against 12.4); under $\hat{\mu}_0 = 1.53$ the loss is 13.0 percent (against 14.7); under the labor-share anchor the implied markdown is 1.174 (against 1.216) and the loss 6.1 percent (against 8.9). The targeted growth moments are unchanged by construction, while the level and welfare results retain the same qualitative ordering.

across the three columns. What changes is the level block. Under the measured-markdown anchors, the implied time-0 labor shares (0.50 and 0.45) fall short of the data (0.60). This is the mirror image of the conservative markdown implied by the labor-share anchor, and it is associated with a larger welfare cost of monopsony. Accordingly, each calibration uses one time-0 moment to identify the level block, while the remaining two are non-targeted implications that provide the empirical test of the model.

The concentration measure requires a qualification under all three calibrations. The model-implied concentration (0.68–0.87 across columns) is not comparable with the census-based CR4 average of Autor et al. (2020) (0.275): the two-sector abstraction compresses all competitive employment into a single fringe, so the superstar share of value in the model is mechanically larger than the sales share of the four largest firms within a census industry. We report the model concentration for completeness, but we do not treat the discrepancy as a target failure — concentration in the model and CR4 in the census measure different objects.

Step 4: identification of (A^s, ψ, T, ζ) . The four parameters A^s, ψ, T, ζ are not separately identified: only two nonlinear combinations can be pinned down from data, namely

$$(i) \quad \frac{\zeta}{1 - \zeta} \left(\frac{A^s}{A^i} \right)^{\sigma-1}, \quad (ii) \quad \frac{T^{\alpha(1-\gamma)}}{\psi},$$

with details in the appendix. The final step is therefore to choose A^s and ψ for given T and ζ such that, through combinations (i) and (ii) and the equations above, the model exactly matches initial hours worked $\hat{h}_0$ and the time-0 anchor of each column of Table 8. Since ψ is chosen to match the level of hours, hours at time zero are reproduced by construction under every anchor; the informative hours statistic in Table 8 is the gap between the monopsonistic and the competitive economy.

Once these four steps are completed, the model is fully calibrated. One might worry that since some objects — such as the level of output at time zero — depend on the exact values of A^s, ψ, T, ζ rather than on their identifiable combinations, the consumption equivalent loss might not be well-defined. The next proposition shows that this concern does not arise.

Proposition 11. *Given combinations of ζ and A^s , and of T and ψ , that reproduce the moments from the data (labor share and hours worked), the consumption equivalent loss can be computed and does not depend on the individual values of these parameters — only on the combinations that can be identified from the data.*

The proposition follows from the fact that the consumption equivalent loss compares utility in the monopsonistic and competitive economies, so that levels — such as the level of output — cancel out.

The calibration is run over 150 years and the results are plotted in Figure 4. The model replicates the three growth facts during the transition: the calibrated economy exhibits positive GDP growth alongside negative growth in both hours and the labor share. The match is exact in the asymptotic equilibrium and gradually improves along the transition. From the start of the transition, simulated GDP growth is within two percent of the observed rate, while the growth rates of the labor share and of total hours are smaller in absolute value than their asymptotic values: the mechanism accelerates through the sample. Figure 4 also reports the dynamics of the implied moments.

Table 5 identifies the US position on the path: it reports the growth rates of the simulated economy at selected dates along the transition against their asymptotic values. Nothing in the asymptotic closed forms of Section 6.1 pins this position; it is determined by the time-0 *levels* that the calibration matches — hours and the anchored moment of Table 8 — not by the growth targets. The labor-share and hours declines are, at the beginning of the transition, 73 and 59 percent of their asymptotic rates, and build up over time. GDP growth displays the mirror image: because $g_Y = g^s + (1 - \alpha)g_h$ up to reallocation terms that vanish asymptotically, hours that decline more slowly than asymptotically imply GDP growth slightly *above* its asymptotic value — 102 percent at $t = 0$ — converging from above as the hours drag builds up. The three rows reflect the same mechanism, and the size of the GDP gap (2 percent, against 27 and 41 percent for the labor share and hours) measures the accuracy of treating the observed 2000–2023 trends as asymptotic rates.

Table 5: Position on the Path: Model Growth Rates Along the Calibrated US Transition vs. Their Asymptotic Values

(%/yr) ^a	$t = 0$	$t = 25$	$t = 50$	$t = 100$	asymptotic ^b	$t = 0$ share ^c
GDP growth	+1.172	+1.176	+1.178	+1.179	+1.145	102%
Hours growth	−0.101	−0.109	−0.117	−0.130	−0.172	59%
Labor-share growth	−0.253	−0.265	−0.276	−0.293	−0.346	73%
^a , Growth rates of the simulated monopsonistic economy along the calibrated transition (primary calibration, $\hat{\mu}_0 = 1.40$, of Table 8, per-worker margin), at the indicated dates. ^b , The asymptotic column reports the calibration targets, which the model attains in the limit. ^c , Share of the asymptotic rate realized at $t = 0$: the economy’s position on the path at the start of the transition (2000). The labor-share and hours declines build up along the transition; GDP growth correspondingly lies slightly above its asymptotic value and converges from above, since $g_Y = g^s + (1 - \alpha)g_h$ up to reallocation terms that vanish in the limit — a smaller hours drag along the path implies faster GDP growth. Convergence is verified numerically (g_Y attains its asymptotic value to the fourth decimal by $t \approx 1,500$). Source: Authors’ calculations.						

The final panel of Figure 4 reports the wage and GDP levels relative to the first-best competitive economy: both gaps widen over time, motivating the consumption equivalent loss calculation that follows.

The consumption equivalent loss (CEL), derived analytically in Section 4.8, is reported in the bottom panels of Table 8. Under the primary, measured-markdown anchors, the growth misallocation of the SSMPE implies a CEL of 12.4 percent at $\hat{\mu}_0 = 1.40$ and 14.7 percent at $\hat{\mu}_0 = 1.53$; under the conservative labor-share anchor the CEL is 8.9 percent (8.7 percent on the per-person margin). The result is robust to the time horizon of the simulation, and the labor-share anchor provides a lower bound for the welfare cost.

The CEL of the SSMPE combines two conceptually distinct losses: the *static* misallocation of a given markdown, the object measured by the static literature on labor market power, and the *growth drag* of a markdown that rises along the path, the component only a growth model can deliver. Table 6 separates the two. We construct a frozen-wedge economy in which the markdown is held at its time-0 value μ_0 forever while the markup distortion is unchanged; the frozen-wedge economy coincides with the full SSMPE at $t = 0$ by construction. The CEL of the frozen-wedge economy relative to the competitive benchmark is the static component; the residual, defined multiplicatively, is the growth-drag component. Under the primary anchors the total losses of 12.4 and 14.7 percent split into $10.9 + 1.7$ and $13.0 + 1.9$ (static + growth drag); under the conservative labor-share anchor the split is $7.8 + 1.2$. The static component is directly comparable to — and essentially coincides

with — the 7.6 percent welfare cost of monopsony estimated in the static framework of Berger et al. (2022): the model reproduces the static literature’s number as its static component, and prices the dynamics of market power on top of it.

Table 6: Decomposition of the Consumption-Equivalent Loss: Static Misallocation vs. Growth Drag (US, 2000–2023)

CEL component (%) ^a	$\hat{\mu}_0 = 1.40$	$\hat{\mu}_0 = 1.53$	labor share
<i>Panel A. Per worker (headline margin)</i>			
Total CEL	12.38	14.66	8.90
Static component (markdown frozen at μ_0) ^b	10.92	13.03	7.83
Growth-drag component (rising markdown) ^c	1.65	1.88	1.16
<i>Panel B. Per person</i>			
Total CEL	12.07	14.26	8.73
Static component (markdown frozen at μ_0)	10.66	12.69	7.70
Growth-drag component (rising markdown)	1.58	1.80	1.11
^a , Horizon $T = 100$ years, $dt = 1$; anchors and parameters as in Table 8.			
^b , The static component compares the competitive economy with a frozen-wedge economy in which the markdown is held at its time-0 value μ_0 along the entire path, with the markup distortion unchanged; the frozen-wedge economy coincides with the full SSMPE at $t = 0$ by construction.			
^c , Defined residually and multiplicatively, $(1 - \text{CEL}_{tot}) = (1 - \text{CEL}_{stat})(1 - \text{CEL}_{growth})$: the additional loss attributable to the <i>rise</i> of the markdown along the growth path.			
<i>Source: Authors’ calculations.</i>			

How much does rising monopsony explain?

The calibration matches the entire post-2000 labor-share decline by construction: it requires markdown growth of $g_\mu = 0.346$ percent per year. Table 7 evaluates how much of the required markdown growth is supported by *independent* estimates. The economy-wide estimate of Deb et al. (2022) delivers +0.114 percent per year. The accounting exercise of Appendix F, which converts the employment concentration ratios of Autor et al. (2020) into implied markdowns, delivers +0.121 percent per year for the post-2000 window — remarkably close to the fully independent Deb et al. (2022) estimate, and faster than before 2000 (+0.077), the same timing as the manufacturing series of Yeh et al. (2022). Of the accounting exercise we use only the growth rate: the implied *level* (≈ 1.017) is not a measure of wage-setting power and has no counterpart in the model (Section 4.4). Because the model’s labor-share and hours responses are linear in g_μ , attribution is a simple ratio: *measured markdown growth accounts for about one third of the post-2000 decline in the US labor share, and for about half of the decline in hours per worker* (52–55 percent; 31–33 percent of the hours-per-person decline). The remaining two thirds of the labor-share decline are left to forces outside the model. This is the main quantitative result of the paper: it is smaller than full attribution, and no measured series contradicts it.

As a final exercise, we simulate the transitional dynamics under an unanticipated shock that reduces the effective-number index m by 20 percent, representing tighter coordination within the superstar sector.²³ Figure 5 reports the resulting dynamics. The markdown-and-markup panel displays the one-off increase induced by the fall in m . Figure 5 illustrates the distinction between level and growth effects emphasized by Lucas (1988). At the shock date, the growth rates of GDP and

²³Formally, we re-run the transition of Figure 4 assuming an unexpected mid-path reduction in m , from 1.25 to 1.00 at $t = 75$.

Table 7: How Much Does Rising Monopsony Explain? Measured vs. Required Markdown Growth (US, 2000–2023)

	Deb et al. ^a	CR20 accounting ^b
Measured markdown growth $\hat{g}_\mu$ (%/yr)	0.114	0.121
Required to match the labor-share trend ($-g_{\alpha_L}$, %/yr)	0.346	0.346
Share of the labor-share decline explained^c	33%	35%
Implied hours drag $-\hat{g}_\mu/\Delta$ (%/yr, per worker) ^d	−0.090	−0.095
Observed hours-per-worker trend (%/yr)	−0.172	−0.172
Share of the hours-per-worker decline explained	52%	55%
^a , Deb et al. (2022): economy-wide markdown 1.37 → 1.40 over 1997–2016. ^b , CR20-employment accounting exercise, 2000–2012 (Appendix F). ^c , The calibration of Table 8 matches the entire post-2000 labor-share decline by construction, requiring $g_\mu = 0.346$ %/yr; responses are linear in g_μ , so shares are ratios of measured to required growth. ^d , $\Delta = 1/\theta + \alpha + \gamma(1 - \alpha) = 1.272$ at the per-worker calibration; Δ does not depend on the anchor. On the per-person margin the corresponding share is 31 percent. The residual decline is attributed to forces outside the model, including the income/substitution channel, mildly positive post-2000 (+0.100 %/yr). Source: Authors' calculations.		

of the labor share display a one-period adjustment, the counterpart of the downward jump in levels, and then return to their previous paths. A one-time tightening of coordination therefore produces marked negative *level* effects on the labor share and hours worked, but does not alter their long-run growth rates: once m is again constant, it is absent from the asymptotic growth rates, which depend on the growth of the markdown g_μ (Section 6.1).

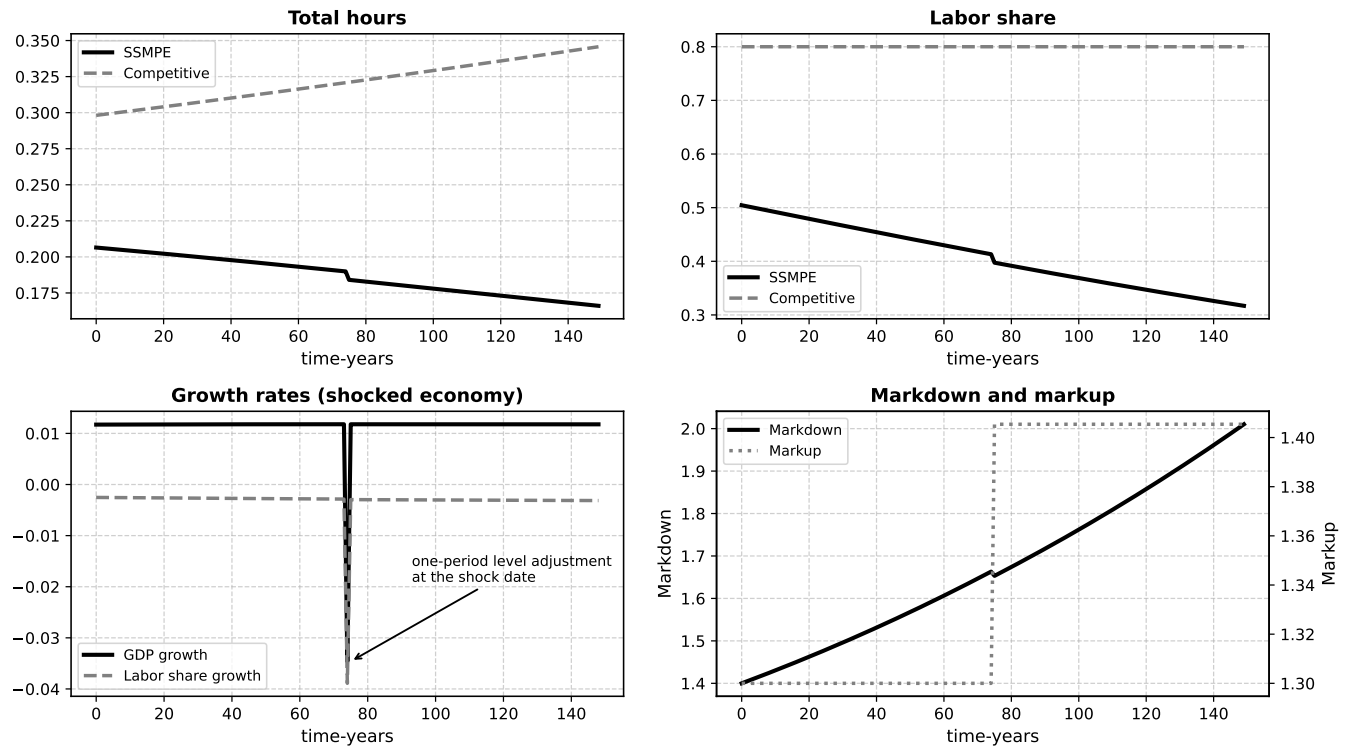
7 Conclusion

Long-run dynamics of labor supply and imperfect labor markets are becoming more relevant in macroeconomics and in growth theory. Models of growth should address the long-run decline in hours per worker and the fall in the labor share observed in most advanced economies. This paper takes first steps in this direction, proposing and solving a standard growth model with declining hours worked and declining labor share driven by oligopsonistic power by superstar firms. In a broader perspective, the paper calls for expanding the intersection between growth theory and labor economics. Market power by firms, well established in economics of growth, should be extended to factor markets, incorporating empirically relevant features of real life firms. This research avenue is likely to uncover novel and interesting policy implications. The paper studied the growth and distributive effects of proportional taxation and rising minimum wage in economies with declining hours worked and labor share. In such economies, both policies can be welfare improving.

Much remains to be done. Once monopsonistic power by firms is recognized as part of growth theory, it should be studied how its existence shapes incentives for innovation and technology adoption. Imperfect labor markets have implications also for the empirics of growth. Almost seventy years ago, Solow (1957) introduced growth accounting under the assumption of perfect labor and capital markets. In his remarkable and influential analytical derivation, Solow argued that the empirical exercise should be extended to the case of imperfect labor markets. These concerns should now be taken seriously by national and international statistical offices.

Figure 5: Dynamic Simulation of the Calibrated Economy with a Drop in the Effective-Number Index

Dynamics with a drop in the effective-number index m



Note: the effective-number index m falls from 1.25 to 1.00 at $t = 75$, representing tighter coordination within the superstar sector. Growth rates computed from the shocked series.

Table 8: Re-Anchored Calibration of the US Economy: Three Anchors and Implied Moments (2000–2023)

Moment / statistic	Anchor (targeted moment in bold) ^a		
	$\hat{\mu}_0 = 1.40^b$	$\hat{\mu}_0 = 1.53^b$	labor share ^c
Panel A. Time-0 allocation (common to both margins)			
Markdown μ_0	1.4000	1.5300	1.2163
Labor share α_{L0}	0.5045	0.4546	0.5996
Concentration $p^s y^s / Y^d$	0.8198	0.8682	0.6813
Employment ratio n^s / n^i	2.5000	3.3125	1.3521
Implied A^s	6.9217	8.6347	4.5539
Panel B. Per worker (headline margin) ^e			
Total hours h_0 (share of time)	0.2065	0.2065	0.2065
Hours gap vs. competitive (%)	−30.74	−36.17	−20.65
Consumption gap vs. competitive (%)	−30.05	−34.39	−21.82
CEL, $T = 100$ (%) ^f	12.38	14.66	8.90
CEL, $T = 500$ (%)	12.44	14.73	8.94
CEL, $T = 1000$ (%)	12.44	14.73	8.94
Implied ψ	4.1697	3.8380	4.7732
Memo: γ	0.9003	0.9003	0.9003
Memo: hours-growth decomposition ^g	−0.172 = +0.100 − 0.272 %/yr		
Panel C. Per person			
Total hours h_0 (share of time)	0.0983	0.0983	0.0983
Hours gap vs. competitive (%)	−28.65	−33.90	−19.02
Consumption gap vs. competitive (%)	−28.37	−32.52	−20.53
CEL, $T = 100$ (%)	12.07	14.26	8.73
CEL, $T = 500$ (%)	12.12	14.32	8.77
CEL, $T = 1000$ (%)	12.12	14.32	8.77
Implied ψ	11.7466	10.5550	14.0330
Memo: γ	1.0135	1.0135	1.0135
Memo: hours-growth decomposition ^g	−0.267 = −0.013 − 0.254 %/yr		
Data counterparts, US 2000–2023			
Labor share (PWT, period mean)	0.5996		
Hours, per worker / per person	0.2065 / 0.0983		
Measured markdown	1.40 (Deb et al., 2022) − 1.53 (Yeh et al., 2022)		
CR4 sales concentration (2012)	0.2750		
^a , Targets are US log-linear trends and period means over 2000–2023 (Penn World Table); the growth targets coincide with the 2000–2023 column of Panel A of Table 2. The growth block does not depend on the anchor ($\sigma = 3.4667$, $g^s = 0.0128$, $g^i = 0.0072$, $g_\mu = -g_{\alpha_L} = 0.346$ %/yr per worker), so every growth moment is identical across columns. Fixed: $\alpha = 0.2$, $m = 1.25$, $\theta = 2.84$, markup = 1.3, $\rho = 0.05$, $T = 1$, $\zeta = 1/8$.			
^b , Columns 1–2 set $n_r = m(\hat{\mu}_0 - 1)/\alpha$ directly, since $\mu = 1 + (\alpha/m)n_r$; A^s and ψ then adjust.			
^c , Column 3 pins n_r by the labor-share equation at the period-mean labor share.			
^d , Model concentration is not comparable with census CR4; see the discussion in the text.			
^e , Because ψ is free, the level of hours is matched by construction in all columns; the informative statistics are the gaps vs. the competitive economy at $t = 0$ and the consumption-equivalent loss.			
^f , CEL: consumption-equivalent loss vs. the competitive economy over a horizon of T years, $dt = 1$.			
^g , Targeted hours growth split into the income/substitution channel $(1 - \gamma)g^s/\Delta$ and the markdown drag $-g_\mu/\Delta$, with $\Delta = 1/\theta + \alpha + \gamma(1 - \alpha)$; both are anchor-independent.			
Source: Authors' calculations.			

Table 9: Re-Anchored Calibration of the US Economy: Parameter Values and Identification (2000–2023)

Parameter	Notation	Pinned by	$\hat{\mu}_0 = 1.40$	$\hat{\mu}_0 = 1.53$	labor share
<i>Step 0. Preset parameters and normalizations (common to all anchors)</i>					
Cobb–Douglas weight	α	preset	0.20	0.20	0.20
Effective-number index	m	normalization	1.25	1.25	1.25
Frisch elasticity	θ	Chetty et al. (2011)	2.84	2.84	2.84
Monopolistic markup	$\sigma m / (\sigma m - 1)$	preset	1.30	1.30	1.30
Discount rate	ρ	preset	0.05	0.05	0.05
Fixed materials supply	T	normalization	1	1	1
CES share	ζ	normalization	0.125	0.125	0.125
Time-0 productivity, inferior	A^i	normalization	1	1	1
CES elasticity	σ	markup and m	3.4667	3.4667	3.4667
<i>Step 1. Growth block: (g^s, γ, g^i) invert the three growth targets^a</i>					
Superior productivity growth (pw)	g^s	$g_C - (1 - \alpha)g_h$	0.0128	0.0128	0.0128
Superior productivity growth (pc)	g^s	$g_C - (1 - \alpha)g_h$	0.0126	0.0126	0.0126
Utility curvature (pw)	γ	g_h/g_C and g_{α_L}	0.9003	0.9003	0.9003
Utility curvature (pc)	γ	g_h/g_C and g_{α_L}	1.0135	1.0135	1.0135
Inferior productivity growth (pw)	g^i	g^s and $g_\mu = -g_{\alpha_L}$	0.0072	0.0072	0.0072
Inferior productivity growth (pc)	g^i	g^s and $g_\mu = -g_{\alpha_L}$	0.0071	0.0071	0.0071
<i>Step 2. Time-0 allocation: the anchor pins n_r; A^s and ψ follow^b</i>					
Employment ratio	$n_r = n^s/n^i$	anchor	2.5000	3.3125	1.3521
Time-0 productivity, superior	A^s	n_r equilibrium at $t=0$	6.9217	8.6347	4.5539
Disutility of work (pw)	ψ	hours level h_0	4.1697	3.8380	4.7732
Disutility of work (pc)	ψ	hours level h_0	11.7466	10.5550	14.0330
^a , The three asymptotic growth equations are inverted for (g^s, γ, g^i) from the three growth targets of the 2000–2023 column of Panel A of Table 2. No anchor enters Step 1, so the growth block is identical across the three columns. σ solves markup = $\sigma m / (\sigma m - 1)$ given the markup 1.3 and $m = 1.25$.					
^b , Columns 1–2 set $n_r = m(\hat{\mu}_0 - 1)/\alpha$, since $\mu = 1 + (\alpha/m)n_r$; column 3 solves the labor-share equation at the period-mean labor share 0.5996. Given n_r , A^s is backed out so that n_r is the time-0 equilibrium, and ψ from the level of total hours. Each target pins exactly one parameter; n_r and A^s are margin-independent.					
<i>Source:</i> Authors' calculations.					

Declaration of generative AI and AI-assisted technologies

During the preparation of this work the authors used Claude (Claude Code, Anthropic), an AI agent, to assist with the revision of the manuscript: drafting and editing portions of the text under the authors' direction, preparing and formatting tables and figures, writing and executing the code that produces the calibration and simulation results, and checking the reference list. All analyses, derivations, and text produced with this assistance were reviewed, verified, and edited by the authors, who take full responsibility for the content of the publication.

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A NGM with Markups and Markdowns

This appendix provides a minimal microfounded version of the one-sector neoclassical growth model of Section 3. The reduced-form markup ϱ_t and markdown μ_t that enter the equations of Section 3 are obtained as equilibrium objects of a model featuring monopolistic competition over differentiated intermediate varieties (Dixit and Stiglitz, 1977) and monopsonistic competition in the labor market associated with differentiated labor (Berger et al., 2022; Deb et al., 2022). The derivation grounds the labor share and Euler equation of Section 3 in explicit firm optimization.

Consider a model in which a single homogeneous final good Y_t is produced competitively by aggregating a continuum of intermediate varieties $j \in [0, 1]$ according to the CES technology

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y_{j,t}$ is the output of variety j and $\varepsilon > 1$ is the elasticity of substitution across varieties. The price of the final good is normalized to one. Profit maximization of the final-good producer yields the standard isoelastic demand for each variety,

$$y_{j,t} = p_{j,t}^{-\varepsilon} Y_t, \quad 1 = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \quad (43)$$

Each variety j is produced by a single firm using capital $K_{j,t}$, labor $N_{j,t}$, and labor-augmenting technology A_t , with a neoclassical constant-returns-to-scale production function

$$y_{j,t} = F(K_{j,t}, A_t N_{j,t}), \quad (44)$$

identical across varieties. Productivity A_t grows at the constant rate g . Capital is supplied competitively at the common rental rate R_t , and depreciates at rate δ .

A representative household has preferences

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right] dt. \quad (45)$$

We assume that jobs at different varieties are imperfect substitutes from the worker's perspective. Total hours are aggregated through the CES index

$$N_t = \left(\int_0^1 N_{j,t}^{\frac{\eta+1}{\eta}} dj \right)^{\frac{\eta}{\eta+1}},$$

with $\eta > 0$ the firm-level elasticity of labor supply. Cost minimization by the household across varieties then yields the firm-level labor-supply schedule and aggregate wage index

$$N_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^\eta N_t, \quad w_t = \left(\int_0^1 w_{j,t}^{1+\eta} dj \right)^{\frac{1}{1+\eta}}, \quad (46)$$

and the wage bill aggregates as $\int_0^1 w_{j,t} N_{j,t} dj = w_t N_t$.

A typical intermediate firm j chooses capital, labor, and the wage to maximize period- t profits, taking as given the rental rate R_t , the aggregate wage index w_t and the aggregate final-good output Y_t . The firm maximizes

$$\max_{K_{j,t}, N_{j,t}, w_{j,t}, p_{j,t}} p_{j,t} y_{j,t} - w_{j,t} N_{j,t} - R_t K_{j,t}$$

subject to (44), (43), and (46). The firm optimally sets the wage to bind the labor-supply constraint with equality. The first-order conditions for labor and capital then deliver

$$p_{j,t} \left(1 - \frac{1}{\varepsilon}\right) F_N(K_{j,t}, A_t N_{j,t}) A_t = w_{j,t} \left(1 + \frac{1}{\eta}\right), \quad (47)$$

$$p_{j,t} \left(1 - \frac{1}{\varepsilon}\right) F_K(K_{j,t}, A_t N_{j,t}) = R_t. \quad (48)$$

The bracketed term on the left of equation (47) is the monopoly markdown of marginal revenue over price; the bracketed term on the right is the monopsony markdown of marginal cost of labor over the wage. Both ε and η are structural parameters: in order to generate rising markups or markdowns in view of the discussion in Section 3, in the remainder of this appendix we treat the elasticities as time-varying and denote them by ε_t and η_t .

We define

$$\varrho_t \equiv \frac{\varepsilon_t}{\varepsilon_t - 1}, \quad \mu_t \equiv 1 + \frac{1}{\eta_t},$$

where the markup ϱ_t is the price markup over marginal cost and the markdown μ_t is the ratio of the marginal product of labor to the wage. With this notation, equations (47) and (48) become

$$\begin{aligned} F_N(K_{j,t}, A_t N_{j,t}) A_t &= \varrho_t \mu_t w_{j,t}, \\ F_K(K_{j,t}, A_t N_{j,t}) &= \varrho_t R_t. \end{aligned} \quad (49)$$

These are exactly the reduced-form factor-pricing conditions assumed in Section 3. Note in particular that the markdown μ_t appears only in the labor-side equation, while the markup ϱ_t appears in both. This is the source of the asymmetry between the two distortions.

By symmetry across varieties, we have $K_{j,t} = K_t$, $N_{j,t} = N_t$, $y_{j,t} = y_t$, $p_{j,t} = 1$, and $w_{j,t} = w_t$ for all j . Aggregation yields $Y_t = y_t = F(K_t, A_t N_t)$. Define the intensive variables

$$x_t = \frac{K_t}{A_t N_t}, \quad \tilde{y}_t = \frac{Y_t}{A_t N_t} = f(x_t), \quad \tilde{c}_t = \frac{C_t}{A_t N_t}.$$

With F constant returns to scale, Euler's theorem implies $F = F_K K + F_N(A N)$, so the aggregate labor share is

$$\alpha_L \equiv \frac{w_t N_t}{Y_t} = \frac{1}{\varrho_t \mu_t} \cdot \frac{F_N(K_t, A_t N_t) A_t N_t}{Y_t} = \frac{1}{\varrho_t \mu_t} \cdot \frac{f(x_t) - x_t f'(x_t)}{f(x_t)}. \quad (50)$$

Equation (50) recovers both equation (1) (set $\mu_t = 1$) and equation (3) (set $\varrho_t = 1$) of Section 3.

Households own all firms and receive aggregate profits, which are rebated lump-sum. The household budget constraint is

$$C_t + \dot{A}_t = w_t N_t + r_t A_t + \Pi_t,$$

where A_t is total household wealth (equal to K_t in equilibrium) and $r_t = R_t - \delta$ is the net rate of return. The household's intertemporal first-order condition yields the standard Euler equation

$$\gamma \frac{\dot{C}_t}{C_t} = r_t - \rho.$$

Using the firm's capital FOC (49), we have $R_t = F_K(K_t, A_t N_t)/\varrho_t = f'(x_t)/\varrho_t$, so

$$\gamma \frac{\dot{C}_t}{C_t} = \frac{f'(x_t)}{\varrho_t} - \delta - \rho.$$

Converting to intensive form using $\dot{\tilde{c}}_t/\tilde{c}_t = \dot{C}_t/C_t - g - \dot{N}_t/N_t$, we obtain

$$\gamma \left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g + \frac{\dot{N}_t}{N_t} \right) = \frac{f'(x_t)}{\varrho_t} - \delta - \rho. \quad (51)$$

Equation (51) recovers both equation (2) and equation (4) of Section 3, under a BGP with constant hours worked. Crucially, the markdown μ_t does not appear in the Euler equation, while the markup ϱ_t does.

We can now make formal the BGP non-existence result stated in Section 3. Suppose first that $\mu_t = 1$ for all t and that the markup grows at the constant rate $\dot{\varrho}_t/\varrho_t = g_\varrho > 0$. A balanced growth path requires a constant capital-output ratio, which in turn requires $x_t = \bar{x}$ constant. From the intensive Euler equation (51), in BGP with $\dot{\tilde{c}}_t/\tilde{c}_t = 0$, we have

$$\gamma g = \frac{f'(\bar{x})}{\varrho_t} - \delta - \rho.$$

The left-hand side is constant and positive; the right-hand side declines monotonically as $\varrho_t \rightarrow \infty$ and eventually becomes negative. Hence, so no BGP exists.

Suppose instead that $\varrho_t = 1$ is constant and the markdown grows at rate $\dot{\mu}_t/\mu_t = g_\mu > 0$. From equation (51), the Euler equation reverts to the standard form

$$\gamma g = f'(\bar{x}) - \delta - \rho,$$

which has a unique solution for $\bar{x} > 0$. The capital-output ratio is then constant along the BGP, and from equation (50), the labor share declines at the rate $-g_\mu$ as required.

To establish Result 2 of Section 3 formally, consider the household's intratemporal first-order condition for labor supply, which under preferences (45) reads

$$\psi N_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t.$$

Substituting for the wage using the firm's labor FOC we obtain

$$N_t^{\frac{1}{\theta} + \gamma} = \frac{1}{\varrho_t \mu_t} \tilde{c}_t^{-\gamma} A_t^{1-\gamma} [f(x_t) - x_t f'(x_t)]. \quad (52)$$

Along any BGP with x_t constant, increasing markups and markdowns will generate a decline in labor supply. If ϱ_t and μ_t grow at constant rate and x_t and $\tilde{c}_t$ are constant, equation (52) implies

$$\left(\frac{1}{\theta} + \gamma \right) g_N = (1 - \gamma)g - g_\varrho - g_\mu.$$

This establishes Result 2.

B Model solution

To solve the model, we first write the Hamiltonian of the household, defining investments $q_t^s = \dot{A}_t^s$ and $q_t^i = \dot{A}_t^i$,

$$\begin{aligned} \mathcal{H} = & \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \lambda_t [-C_t - q_t^s - q_t^i + w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i] + \\ & + \mu_t^s q_t^s + \mu_t^i q_t^i + \phi_t [-h_t + n_t^s + n_t^i] \end{aligned}$$

The FOCs of the household, described in the text, together with firm optimality and market clearing, give rise to the following system of nine equations, that govern the evolution of the SSMPE.

$$\begin{aligned} \psi h_t^{\frac{1}{\theta}} &= C_t^{-\gamma} p_t^i (1-\alpha) (K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \\ n_t^s + n_t^i &= h_t \\ \gamma \frac{\dot{C}_t}{C_t} &= \frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s - \rho \\ C_t + \dot{K}_t^s + \dot{K}_t^i &= Y_t - \delta^s K_t^s - \delta^i K_t^i \\ Y_t &= \left[\zeta^{\frac{1}{\sigma}} ((K_t^s)^\alpha (A_t^s n_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} ((K_t^i)^\alpha (A_t^i n_t^i)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ (K_t^s)^\alpha (A_t^s n_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} Y_t, \\ (K_t^i)^\alpha (A_t^i n_t^i)^\alpha &= (1-\zeta) (p_t^i)^{-\sigma} Y_t \\ \frac{\sigma m - 1}{\sigma m} p_t^s (1-\alpha) (K_t^s)^\alpha (n_t^s)^{-\alpha} (A_t^s)^{1-\alpha} &= p_t^i (1-\alpha) (K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \left[1 + \alpha \frac{1}{m} \frac{n_t^s}{n_t^i} \right] \\ \frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s &= p_t^i \alpha (K_t^i)^{\alpha-1} (A_t^i n_t^i)^{1-\alpha} - \delta^i \end{aligned}$$

where $K_t^s = m \bar{K}_t^s, y_t^s = m \bar{y}_t^s, n_t^s = \bar{n}_t^s$ are defined in equation (17) with the symmetric equilibrium. We can rewrite this system in intensive form, where we define

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s}, \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i}, \quad c_t = \frac{C_t}{A_t^s n_t^s}, \quad y_t = \frac{Y_t}{A_t^s n_t^s}$$

$$\begin{aligned}
\psi h_t^{\frac{1}{\theta}} &= c_t^{-\gamma} p_t^i (1 - \alpha) \frac{A_t^i}{(A_t^s n_t^s)^\gamma} (x_t^i)^\alpha \\
\frac{\dot{c}_t}{c_t} &= \frac{\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha - 1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s} \\
c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} &= y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s} \right) x_t^s - \left(\delta^i + g^i + \frac{\dot{n}_t^i}{n_t^i} \right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} \\
y_t &= \left[\zeta^{\frac{1}{\sigma}} ((x_t^s)^{1 - \alpha})^{\frac{\sigma - 1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1 - \alpha} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \\
(x_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t , \\
\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^\alpha &= (1 - \zeta) (p_t^i)^{-\sigma} y_t \\
\frac{\sigma m - 1}{\sigma m} p_t^s (1 - \alpha) A_t^s (x_t^s)^\alpha &= p_t^i (1 - \alpha) A_t^i (x_t^i)^\alpha \left[1 + \frac{\alpha}{m} \frac{n_t^s}{n_t^i} \right] \\
\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha - 1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha - 1} - \delta^i
\end{aligned}$$

The system of growth rates for the monopsonistic decentralized economy is (at infinity already zeroing all growth rates of converging values, $g_{p^s} = g_{x^s} = 0$ and imposing $g_C = g_Y$)

$$\begin{aligned}
g^s &= g^i + g_{p^i} + \alpha g_{x^i} + g_{n^s} - g_{n^i} \\
g_{p^i} + (\alpha - 1) g_{x^i} &= 0 \\
g^i + g_{n^i} + \alpha g_{x^i} &= -\sigma g_{p^i} + g^s + g_{n^s} \\
\frac{1}{\theta} g_{n^s} &= -\gamma (g^s + g_{n^s}) + g_{p^i} + g^i + \alpha g_{x^i} \\
g_h &= g_{n^s} \\
g_C &= g^s + g_h
\end{aligned}$$

As already expressed in the text, the decentralized growth rates are

$$\begin{aligned}
g_C &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1 - \alpha)(\sigma - 1)}{1 + \alpha + \sigma(1 - \alpha)} (g^s - g^i) \\
g_h &= g_{n^s} = \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1 - \alpha)(\sigma - 1)}{1 + \alpha + \sigma(1 - \alpha)} (g^s - g^i) \\
g_{n^s} - g_{n^i} &= \frac{(1 - \alpha)(\sigma - 1)}{1 + \alpha + \sigma(1 - \alpha)} (g^s - g^i)
\end{aligned}$$

Growth rates of additional variables of interest are

$$g_\mu = g_{n^s} - g_{n^i} , \quad g_w = g_{p^i} + g^i + \alpha g_{x^i} .$$

B.1 Optimal-competitive economy

The competitive economy is very similar to the one above, without markup and markdown.

$$\begin{aligned}
\psi h_t^{\frac{1}{\theta}} &= c_t^{-\gamma} p_t^i (1 - \alpha) \frac{A_t^i}{(A_t^s n_t^s)^\gamma} (x_t^i)^\alpha \\
\frac{\dot{c}_t}{c_t} &= \frac{p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s} \\
c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} &= y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s} \right) x_t^s - \left(\delta^i + g^i \frac{\dot{n}_t^i}{n_t^i} \right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} \\
y_t &= \left[\zeta^{\frac{1}{\sigma}} ((x_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
(x_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t, \\
\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^\alpha &= (1 - \zeta) (p_t^i)^{-\sigma} y_t \\
p_t^s (1 - \alpha) A_t^s (x_t^s)^\alpha &= p_t^i (1 - \alpha) A_t^i (x_t^i)^\alpha \\
p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i
\end{aligned}$$

The corresponding system of growth rates for the competitive economy (equivalent to the optimal growth problem) is

$$\begin{aligned}
g^s &= g^i + g_{p^i}^* + \alpha g_{x^i}^* \\
g_{p^i}^* + (\alpha - 1) g_{x^i}^* &= 0 \\
g^i + g_{n^i}^* + \alpha g_{x^i}^* &= -\sigma g_{p^i}^* + g^s + g_{n^s}^* \\
\frac{1}{\theta} g_{n^s}^* &= -\gamma (g^s + g_{n^s}^*) + g_{p^i}^* + g^i + g_{n^i}^* + \alpha g_{x^i}^* \\
g_h^* &= g_{n^s}^* \\
g_C^* &= g^s + g_h^*
\end{aligned}$$

The optimal growth rates are

$$\begin{aligned}
g_C^* &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s \\
g_h^* &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s \\
g_{n^s}^* - g_{n^i}^* &= (1 - \alpha)(\sigma - 1)(g^s - g^i) .
\end{aligned}$$

Notice that if we let the substitutability between goods increase, as we approach the perfect substitutes limit the growth rate of labor in the inferior sector goes to minus infinity, which is coherent with the intuition (that can be verified directly) that whenever goods are perfect substitutes, the inferior sector closes right away in the optimal solution, since there is no reason to use an inefficient inferior technology when there are constant returns to scale in factors of production.

B.2 Worker-capitalist economy

Define

$$c_t^c = \frac{C_t^c}{A_t^s h_t}, \quad c_t^w = \frac{C_t^w}{A_t^i h_t}, \quad x_t^s = \frac{K_t^s}{A_t^s h_t}, \quad x_t^i = \frac{K_t^i}{A_t^i h_t}, \quad \kappa_t = \frac{K_t^i}{K_t}, \quad \nu_t = \frac{n_t^i}{h_t}, \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$\begin{aligned} c_t^w &= p_t^i (1 - \alpha) (x_t^i)^\alpha \\ \psi h_t^{\frac{1}{\theta} + \gamma} &= (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma} (1 - \alpha) (x_t^i)^\alpha \\ n_t^s + n_t^i &= h_t \\ \frac{\dot{c}_t^c}{c_t^c} &= \frac{\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\ c_t^c + \frac{A_t^i}{A_t^s} c_t^w + \dot{x}_t &= y_t - \delta^s x_t^s + \delta^i x_t^i \\ y_t &= \left[\zeta^{\frac{1}{\sigma}} ((1 - \nu_t) (x_t^s)^\alpha)^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ (1 - \nu_t) (x_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t, \quad \frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha = (1 - \zeta) (p_t^i)^{-\sigma} y_t \\ \frac{\sigma m - 1}{\sigma m} p_t^s (1 - \alpha) A_t^s (x_t^s)^\alpha &= p_t^i (1 - \alpha) A_t^i (x_t^i)^\alpha \left[1 + \frac{\alpha}{m} \frac{1 - \nu_t}{\nu_t} \right] \\ \frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i \end{aligned}$$

Alternatively, define

$$K_t = K_t^s + K_t^i, \quad c_t^c = \frac{C_t^c}{A_t^s h_t}, \quad c_t^w = \frac{C_t^w}{A_t^i h_t}, \quad x_t = \frac{K_t}{A_t^s h_t}, \quad \kappa_t = \frac{K_t^i}{K_t}, \quad \nu_t = \frac{n_t^i}{h_t}, \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$\begin{aligned}
c_t^w &= p_t^i (1 - \alpha) \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^\alpha \\
\psi h_t^{\frac{1}{\theta} + \gamma} &= (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma} (1 - \alpha) \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^\alpha \\
n_t^s + n_t^i &= h_t \\
\frac{\dot{c}_t^c}{c_t^c} &= \frac{\frac{\sigma m - 1}{\sigma m} p_t^s \alpha \left(\frac{1 - \kappa_t}{1 - \nu_t} x \right)^{\alpha - 1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
c_t^c + \frac{A_t^i}{A_t^s} c_t^w + \dot{x}_t &= y_t - ((1 - \kappa_t) \delta^s + \kappa_t \delta^i) x_t \\
y_t &= \left[\zeta^{\frac{1}{\sigma}} \left((1 - \nu_t) \left(\frac{1 - \kappa_t}{1 - \nu_t} x_t \right)^\alpha \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i}{A_t^s} \nu_t \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^\alpha \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \\
(1 - \nu_t) \left(\frac{1 - \kappa_t}{1 - \nu_t} x_t \right)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t, \quad \frac{A_t^i}{A_t^s} \nu_t \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^\alpha = (1 - \zeta) (p_t^i)^{-\sigma} y_t \\
\frac{\sigma m - 1}{\sigma m} p_t^s (1 - \alpha) A_t^s \left(\frac{1 - \kappa_t}{1 - \nu_t} x_t \right)^\alpha &= p_t^i (1 - \alpha) A_t^i \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^\alpha \left[1 + \frac{\alpha}{m} \frac{1 - \nu_t}{\nu_t} \right] \\
\frac{\sigma m - 1}{\sigma m} p_t^s \alpha \left(\frac{1 - \kappa_t}{1 - \nu_t} x_t \right)^{\alpha - 1} - \delta^s &= p_t^i \alpha \left(\frac{\kappa_t A_t^s}{\nu_t A_t^i} x_t \right)^{\alpha - 1} - \delta^i
\end{aligned}$$

The study of this system transformed in growth rates at infinity delivers the growth rates in the main text and is consistent with the notation used to prove stability of the main model.

The superstar firm problem under a minimum wage. We report here the details of the firm problem with an exogenous minimum wage $\bar{w}_t$ that we omitted from the main text in Section 5. The inferior sector and the household problem are unchanged. The problem of a representative superstar firm f in the superior sector becomes

$$\begin{aligned}
\max_{N_t^{f,s}, K_t^{f,s}, W_t^{f,s}} \quad & \Pi_t^{f,s} = p_t^s (y_t^{f,s}; y_t^{-f,s}) y_t^s (K_t^{f,s}, N_t^{f,s}, A_t^s) - W_t^{f,s} N_t^{f,s} - R_t^s K_t^{f,s} \\
\text{s.t.} \quad & W_t^{f,s} \geq \bar{w}_t.
\end{aligned}$$

Notice that the minimum wage need not be binding at every date. Since the superstar firms take the capital allocated to the inferior sector as given, it may happen that the marginal product of labor in the inferior sector remains above the minimum wage even when all labor is supplied to it—that is, when $p_t^i (1 - \alpha) (H_t)^{-\alpha} (K_t^i)^\alpha (A_t^i)^{1-\alpha} > \bar{w}_t$. Whenever the constraint $W_t^{f,s} \geq \bar{w}_t$ does bind, there exists a unique $\hat{N}_t^{f,s} \in (0, H_t)$ that characterizes the optimal labor demand of the inferior sector at the minimum wage. The effective wage schedule faced by the superstar firms then takes the piecewise form

$$w_t^s = \begin{cases} p_t^i (K_t^i)^\alpha (H_t - N_t^{f,s})^{-\alpha} (A_t^i)^{1-\alpha} (1 - \alpha) & N_t^{f,s} \in [H_t - \hat{N}_t^{f,s}, H_t] \\ \bar{w}_t & N_t^{f,s} \in [0, H_t - \hat{N}_t^{f,s}] \end{cases}$$

together with the output and technology constraints

$$y_t^{f,s} = \zeta(p_t^s)^{-\sigma} Y_t - y_t^{-f,s}, \quad y_t^{f,s}(K_t^{f,s}, N_t^{f,s}, A_t^s) = (K_t^{f,s})^\alpha (A_t^s N_t^{f,s})^{1-\alpha}.$$

When each superstar firm optimally chooses $w_t^{f,s} = \bar{w}_t$ for all f , the minimum wage is binding and the superstar firms effectively become pure oligopolists: the wage is exogenously set by the policy, so there is no scope for the firm to exercise monopsonistic markdown. The intuition for this is straightforward. With $\bar{w}_t$ given, the firm's labor cost is linear in $N_t^{f,s}$ (marginal cost equals average wage = $\bar{w}_t$), and the markdown factor in the marginal cost of labor disappears.

For the minimum wage to remain binding indefinitely along the equilibrium path, it must grow at least as fast as the shadow monopsonistic wage that would prevail in the absence of the policy. Since the shadow monopsonistic wage in the worker-capitalist economy grows at rate $g_w = g^s - g_\mu$, while the consumption-leisure condition with a binding minimum wage requires $g_{\bar{w}}$ to be linked to the growth rates of consumption and hours, the growth rate $g_{\bar{w}}^* = g^s$ derived in the main text is the unique rate consistent with both binding and efficient outcomes.

C The System with general technology

With general constant returns to scale production function for the superior and inferior sectors indicated respectively with $F(K^s, N^s, A^s)$ and $G(K^i, N^i, A^i)$, the general system for the SMPE

$$\begin{aligned} \frac{\sigma m - 1}{\sigma m} p_t^s F_N(K_t^s, n_t^s, A_t^s) &= p_t^i \left[G_N(K_t^i, n_t^i, A_t^i) - \frac{1}{m} \frac{n_t^s}{n_t^i} G_{NN}(K_t^i, n_t^i, A_t^i) \right] \\ \frac{\sigma m - 1}{\sigma m} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s &= p_t^i G_K(K_t^i, n_t^i, A_t^i) - \delta^i \\ G(K_t^i, n_t^i, A_t^i) &= (1 - \zeta)(p_t^i)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ F(x_t^s, n_t^s, A_t^s) &= \zeta(p_t^s)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ \psi h_t^{\frac{1}{\theta}} &= C_t^{-\gamma} p_t^i G_N(K_t^i, n_t^i, A_t^i) \\ h_t &= n_t^s + n_t^i \\ \gamma \frac{\dot{C}_t}{C_t} &= \frac{\sigma m - 1}{\sigma m} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho \\ C_t + \dot{K}_t^s + \dot{K}_t^i &= \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - \delta^s K_t^s - \delta^i K_t^i \end{aligned}$$

where from the first equation we can see that we can find a very nice expression for the markdown, i.e.

$$\mu_t = 1 - \frac{1}{m} \frac{n_t^s}{n_t^i} \frac{G_{NN}(K_t^i, n_t^i, A_t^i)}{G_N(K_t^i, n_t^i, A_t^i)}.$$

It is simple to show that with general CRS technologies one can't solve for the balanced growth path since for p_t^s and x_t^s converging to a finite number and for $p_t^i \rightarrow \infty$ it must be that $x_t^i = \frac{K_t^i}{A_t^i n_t^i}$

tend to ∞ . To study this, one needs to know the shape of G (while in principle, we could keep F a neoclassical production function that satisfies Uzawa's theorem).

There is one special case which is tractable though: if the two intermediaries are perfect substitutes, the complications arising from prices disappear since the two goods must trade at the same price. With no diverging price for the inferior good, x_t^i will converge to a constant as well, and we can solve for the balanced growth path with a general inferior technology G .

D Stability

The system we use to prove stability is the following.

$$\begin{aligned}
\frac{\dot{c}}{c} &= \frac{\frac{\sigma m - 1}{\sigma m} p^s F_K \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right) - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
\psi h^{\frac{1}{\sigma} + \gamma} &= c^{-\gamma} p^i \frac{A^i}{(A^s)^\gamma} G_N \left(\frac{\kappa}{\nu} \frac{A^s}{A^i} x, 1, 1 \right) \\
\frac{\sigma m - 1}{\sigma m} p_s F_K \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right) - \delta^s &= p_i G_K \left(\frac{\kappa}{\nu} \frac{A^s}{A^i} x, 1, 1 \right) - \delta^i \\
\frac{\sigma m - 1}{\sigma m} p^s A^s F_N \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right) &= p^i A^i \left[G_N \left(\frac{\kappa}{\nu} \frac{A^s}{A^i} x, 1, 1 \right) - \frac{1}{m} \frac{1 - \nu}{\nu} G_{NN} \left(\frac{\kappa}{\nu} \frac{A^s}{A^i} x, 1, 1 \right) \right] \\
c + \dot{x} &= \left[\zeta^{\frac{1}{\sigma}} \left\{ (1 - \nu) F \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right) \right\}^{\frac{\sigma - 1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left\{ \nu \frac{A^i}{A^s} G \left(\frac{\kappa}{\nu} \frac{A^s}{A^i} x, 1, 1 \right) \right\}^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} - \left((1 - \kappa) \delta^s + \kappa \delta^i + g^s + \frac{\dot{h}}{h} \right) \\
1 &= \zeta p_s^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left\{ \frac{A_i}{A_i} \frac{\nu}{1 - \nu} \frac{G \left(\frac{\kappa}{\nu} \frac{A_s}{A_s} x, 1, 1 \right)}{F \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right)} \right\}^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \\
1 &= (1 - \zeta) p_i^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} \left\{ \frac{A_s}{A_i} \frac{1 - \nu}{\nu} \frac{F \left(\frac{1 - \kappa}{1 - \nu} x, 1, 1 \right)}{G \left(\frac{\kappa}{\nu} \frac{A_s}{A_i} x, 1, 1 \right)} \right\}^{\frac{\sigma - 1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}
\end{aligned}$$

Theorem 2. Assume that $\delta^s = \delta^i$, then the model is locally saddle path stable.

We will prove stability in five steps (all values are assumed at the steady state $(c^*, 0, 0, x^*)$).

1. Show that $\frac{\partial \dot{h}}{\partial h} = g_h < 0$ and that for all other variables, the derivative with respect to h is zero.
2. Show that $\frac{\partial \dot{\nu}}{\partial \nu} = g_{n^i} - g_{n^s} < 0$ and that with respect to all other variables, the derivative of $\dot{\nu}$ is zero.
3. Show that $\frac{\partial \dot{x}}{\partial c} = -\frac{1}{1 + \theta \alpha_i}$ (under $\alpha_s = \alpha_i$, in general one should get $\frac{\partial \dot{x}}{\partial c} < 0$).
4. Show that $\frac{\partial \dot{c}}{\partial c} = -c^* \frac{\partial(\dot{h}/h)}{\partial c}$ and that $\frac{\partial \dot{c}}{\partial x} = \frac{c^*}{\gamma} \frac{\sigma - 1}{\sigma} \zeta^{\frac{1}{\sigma - 1}} \alpha_s (\alpha_s - 1) (x^*)^{\alpha_s - 2} - c^* \frac{\partial(\dot{h}/h)}{\partial x}$.

These steps together will generate a Jacobian of the type

$$J_\Phi = \begin{pmatrix} -c^* \frac{\partial(\dot{h}/h)}{\partial c} & 0 & * & \frac{c^*}{\gamma} \frac{\sigma m - 1}{\sigma m} \zeta^{\frac{1}{\sigma - 1}} \alpha_s (\alpha_s - 1) (x^*)^{\alpha_s - 2} - c^* \frac{\partial(\dot{h}/h)}{\partial x} \\ * & g_h & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} & 0 \\ -\frac{1}{1 + \theta \alpha_i} & 0 & * & * \end{pmatrix}$$

The determinant of this matrix is equal to that of the matrix (we are basically subtracting the third row from the first, this leaves the determinant unchanged.)

$$\begin{pmatrix} 0 & 0 & * & \underbrace{\frac{c^* \sigma m - 1}{\gamma \sigma m} \zeta^{\frac{1}{\sigma-1}} \alpha_s (\alpha_s - 1) (x^*)^{\alpha_s - 2}}_{<0} \\ * & \underbrace{g_h}_{<0} & * & * \\ 0 & 0 & \underbrace{g_{n^i} - g_{n^s}}_{<0} & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & * \end{pmatrix}$$

which is strictly negative. Then the matrix is non-singular and the steady state is hyperbolic. Also, either one or three eigenvalues have positive real part. But eigenvalues are roots of the characteristic polynomial which solves

$$\det \begin{pmatrix} k \frac{\partial \dot{x}}{\partial c} - \lambda & 0 & * & (<0) + k \frac{\partial \dot{x}}{\partial x} \\ * & g_h - \lambda & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} - \lambda & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & * - \lambda \end{pmatrix} = (g_h - \lambda)(g_{n^i} - g_{n^s} - \lambda) \begin{pmatrix} k \frac{\partial \dot{x}}{\partial c} - \lambda & (<0) + k \frac{\partial \dot{x}}{\partial x} \\ -\frac{1}{1+\theta\alpha_i} & * - \lambda \end{pmatrix}$$

which implies that two of these are negative, and thus three eigenvalues must have negative real part.

Proof. The system of equations with Cobb-Douglas production functions reads as follows (assuming $\delta^s = \delta^i = \delta$, but leaving $\alpha_s \neq \alpha_i$)

$$\frac{\dot{c}}{c} = \frac{1}{\gamma} \frac{\sigma m - 1}{\sigma m} \zeta^{\frac{1}{\sigma}} \left\{ \zeta^{\frac{1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left[\frac{A_i}{A_s} \frac{1 - \nu}{\nu} \frac{\left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_s}}{\left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_i}} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{1}{\sigma-1}} \alpha_s \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_s - 1} - \frac{\rho + \delta}{\gamma} - g^s - \frac{\dot{h}}{h} \quad (55)$$

$$\psi h^{\frac{1}{\sigma} + \gamma} = c^{-\gamma} \frac{A_i}{A_s^\gamma} (1 - \alpha_i) \left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_i} (1 - \zeta)^{\frac{1}{\sigma}} \left\{ \zeta^{\frac{1}{\sigma}} \left[\frac{A_s}{A_i} \frac{\nu}{1 - \nu} \frac{\left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_i}}{\left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_s}} \right]^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \right\}^{\frac{1}{\sigma-1}} \quad (56)$$

$$\begin{aligned} \frac{\sigma m - 1}{\sigma m} \left(\frac{A_i}{A_s} \frac{\nu}{1 - \nu} \frac{\left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_s}}{\left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_i}} \right)^{\frac{1}{\sigma}} \alpha_s \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_s - 1} &= \alpha_i \left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_i - 1} \\ A_s \frac{\sigma m - 1}{\sigma m} \left(\frac{A_i}{A_s} \frac{\nu}{1 - \nu} \frac{\left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_s}}{\left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_i}} \right)^{\frac{1}{\sigma}} (1 - \alpha_s) \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_s} &= (1 - \alpha_i) A_i \left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_i} \left[1 + \frac{1}{m} \frac{1 - \nu}{\nu} \alpha_i \right] \end{aligned}$$

$$c + \dot{x} = \left[\zeta^{\frac{1}{\sigma}} \left\{ (1 - \nu) \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_s} \right\}^{\frac{\sigma-1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left\{ \nu \frac{A_i}{A_s} \left(\frac{\kappa A_s}{\nu A_i} x \right)^{\alpha_i} \right\}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - \left(\delta + g^s + \frac{\dot{h}}{h} \right) \quad (57)$$

Combining (taking the ratio of) the third and the fourth equations delivers

$$\frac{1 - \alpha_s}{\alpha_s} \frac{1 - \kappa}{1 - \nu} = \frac{1 - \alpha_i}{\alpha_i} \frac{\kappa}{\nu} \left[1 + \frac{1 - \nu}{\nu} \alpha_i \right]$$

which allows us to express κ as a function ν . In growth rates this implies

$$\frac{\dot{\kappa}}{\kappa} \frac{1}{1 - \kappa} = \frac{\dot{\nu}}{\nu} \left[1 + \frac{\nu}{1 - \nu} + \frac{(1 - \alpha_i)\nu}{\alpha_i + (1 - \alpha_i)\nu} \right].$$

We can also rewrite the third equation as

$$\frac{\sigma m - 1}{\sigma m} \left(\frac{A_i}{A_s} \frac{\nu}{1 - \nu} \right)^{\frac{1}{\sigma}} \left(\frac{\zeta}{1 - \zeta} \right)^{\frac{1}{\sigma}} \left(\frac{\kappa}{\nu} \frac{A_s}{A_i} x \right)^{1 - \alpha_i \frac{\sigma - 1}{\sigma}} \frac{\alpha_s}{\alpha_i} \left(\frac{1 - \kappa}{1 - \nu} x \right)^{-1 + \alpha_s \frac{\sigma - 1}{\sigma}} = 1. \quad (58)$$

Differentiating with respect to time and imposing $\alpha_s = \alpha_i$ gives in a straightforward manner an equation that depends on ν and κ only (thus ultimately on ν only) and that when evaluated at $\nu = \kappa = 0$ returns

$$\frac{\dot{\nu}}{\nu} = - \frac{(\sigma - 1)(1 - \alpha_i)}{1 + \alpha_i + \sigma(1 - \alpha_i)} (g^s - g^i) = g_{n^i} - g_{n^s}$$

We are now ready to proceed with our four steps of proof. Notice that whenever the ratio A_s/A_i shows up in equations (55) and (57) (which we are not going to differentiate), to make the equation time-invariant we need to substitute it out using (58).

1. Notice that the only equations in which h appears are (55), (56) and (57). We can transform (56) in growth rates. Once one substitutes $\frac{\dot{c}}{c}$ out using (55), the expression reads

$$\begin{aligned} \frac{1}{\theta} \frac{\dot{h}}{h} = & -\zeta^{\frac{1}{\sigma}} \left\{ \zeta^{\frac{1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}} \left[\frac{A_i}{A_s} \frac{1 - \nu}{\nu} \left(\frac{\kappa}{\nu} \frac{A_s}{A_i} x \right)^{\alpha_s} \right]^{\frac{\sigma - 1}{\sigma}} \right\}^{\frac{1}{\sigma - 1}} \alpha_s \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_s - 1} + \rho + \delta + g^i + \alpha_i \left[\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\nu}}{\nu} + g^s - g^i + \frac{\dot{x}}{x} \right] + \\ & + \frac{\zeta^{\frac{1}{\sigma}} \left[\frac{A_s}{A_i} \frac{\nu}{1 - \nu} \left(\frac{1 - \kappa}{1 - \nu} x \right)^{\alpha_i} \right]^{\frac{\sigma - 1}{\sigma}}}{\zeta^{\frac{1}{\sigma}} \left[\frac{A_s}{A_i} \frac{\nu}{1 - \nu} \left(\frac{\kappa}{\nu} \frac{A_s}{A_i} x \right)^{\alpha_s} \right]^{\frac{\sigma - 1}{\sigma}} + (1 - \zeta)^{\frac{1}{\sigma}}} \frac{1}{\sigma} \left[g^i - g^s - \frac{1}{1 - \nu} \frac{\dot{\nu}}{\nu} + (\alpha_s - \alpha_i) \frac{\dot{x}}{x} - \alpha_i (g^s - g^i) + \alpha_s \left(-\frac{\kappa}{1 - \kappa} \frac{\dot{\kappa}}{\kappa} + \frac{\nu}{1 - \nu} \frac{\dot{\nu}}{\nu} \right) - \alpha_i \left(\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\nu}}{\nu} \right) \right] \end{aligned}$$

We then notice that only the growth rate of h appear, not h itself. We can then substitute $\frac{\dot{h}}{h}$ out of (55) and (57). This establishes that no time derivative except $\dot{h}$ depends on h . Next, consider that we are left with an equation arising from differentiating (56) which is of the type

$$\frac{\dot{h}}{h} = \mathcal{H}(\nu, x, \kappa(\nu), \dot{x}(c, \nu, x, \kappa(\nu)))$$

which must be such that

$$\mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

but then clearly

$$\frac{\partial \dot{h}}{\partial h} = \mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

at the steady state. This establishes the first point.

2. For the second point, by differentiating (58) we find

$$\begin{aligned} & \left(1 - \alpha_i \frac{\sigma - 1}{\sigma}\right) \left(\frac{\dot{\kappa}}{\kappa} - \frac{\dot{\nu}}{\nu}\right) - \left(1 - \alpha_s \frac{\sigma - 1}{\sigma}\right) \left(-\frac{\kappa}{1 - \kappa} \frac{\dot{\kappa}}{\kappa} + \frac{\nu}{1 - \nu} \frac{\dot{\nu}}{\nu}\right) + \frac{1}{\sigma}(g^i - g^s) + \\ & + \frac{1}{\sigma} \left(\frac{\dot{\nu}}{\nu} + \frac{\nu}{1 - \nu} \frac{\dot{\nu}}{\nu}\right) + \left(1 - \alpha_i \frac{\sigma - 1}{\sigma}\right) (g^s - g^i) + \frac{\sigma - 1}{\sigma} (\alpha_s - \alpha_i) \frac{\dot{x}}{x} = 0 \end{aligned}$$

Clearly for $\alpha_s = \alpha_i$ this gives an equation for $\frac{\dot{\nu}}{\nu}$ that depend on ν only. Also, again it is an expression for the growth rate of ν , and $\nu^* = 0$, as above

$$\frac{\dot{\nu}}{\nu} = \mathcal{N}(\nu)$$

where

$$\mathcal{N}(0) = -\frac{(\sigma - 1)(1 - \alpha_i)}{1 + \alpha_i + \sigma(1 - \alpha_i)}(g^s - g^i) = g_{n^s} - g_{n^i}$$

so that

$$\frac{\partial \dot{\nu}}{\partial \nu} = \mathcal{N}'(0) = -\frac{(\sigma - 1)(1 - \alpha_i)}{1 + \alpha_i + \sigma(1 - \alpha_i)}(g^s - g^i) < 0 .$$

This completes the second step.

3. For the third point, notice that once we substitute out $\frac{\dot{h}}{h}$ for (57) using (1), we find

$$(1 + \alpha_i \theta) \dot{x} = -c + \text{other terms independent of } c$$

This directly implies

$$\frac{\partial \dot{x}}{\partial c} = -\frac{1}{1 + \alpha_i \theta} < 0 .$$

4. Since (55) is in the form

$$\frac{\dot{c}}{c} = \mathcal{C}(\nu, x, \kappa(\nu), \dot{h}/h) ,$$

we will have

$$\frac{\partial \dot{c}}{\partial c} = \mathcal{C}(0, x^*, 0, g_h) - c^* \frac{\partial(\dot{h}/h)}{\partial c}$$

but since c converges to a constant, we must have $\mathcal{C}(0, x^*, 0, g_h) = 0$. Notice than that $\dot{h}/h$ depends on c only through $\dot{x}$, since

$$\frac{\partial(\dot{h}/h)}{\partial c} = \theta \alpha_i \frac{\partial(\dot{x}/x)}{\partial c} = -\frac{\theta \alpha_i}{1 + \theta \alpha_i} \frac{1}{x^*}$$

On the other hand we have

$$\frac{\partial \dot{c}}{\partial x} = \frac{c^*}{\gamma} \frac{\sigma m - 1}{\sigma m} \zeta^{\frac{1}{\sigma-1}} \alpha_s (\alpha_s - 1) (x^*)^{\alpha_s - 2} - c^* \frac{\partial(\dot{h}/h)}{\partial x}$$

□

E The model with fixed materials

The only difference in the model is the absence of capital and the fact that the intermediaries produce with a fixed material T_t^s and T_t^i so that

$$y_t^s = A_t^s(T_t^s)^\alpha; \quad y_t^i = A_t^i(T_t^i)^\alpha$$

The choice of materials is only influenced by the monopoly power in the labor market and its problem solves.

$$\frac{\partial y_t^s}{\partial T_t^s} p_t^s \frac{\sigma m - 1}{\sigma m} = p^T$$

The fundamental factor allocation for the superstar firms implies

$$\left[1 + \frac{\alpha}{m} \frac{N_t^s}{N_t^i} \right] \frac{N_t^s}{T^s} = \frac{N_t^i}{T^i}$$

where it is clear that as long as $\alpha > 0$, the factor allocation of the superstar firm is distorted vis-a-vis a pure neoclassical factor ratio equilibrium that implies $\frac{N_t^s}{T_t^s} = \frac{N_t^i}{T_t^i}$ that characterizes the optimal growth problem that will be discussed later. The superstar firm makes strictly positive profits that are fully distributed to the consumers who owns the firm, to which we turn next. Using the key conditions of the superstar firms for both labor and land,

The representative consumer problem maximize utility period by period and takes as given the wages w_t^j ($j = s, i$) in the two sectors, and needs to choose the total hours h_t as well as the hours worked in every sector. She is also endowed with the materials T that that she rents to the firm at price P_t^T . Finally, the consumers obtains per capita profits/dividends from the superstar firm that we indicate with π_t^s . The consumer problem is

$$\begin{aligned} \max_{C_t, h_t, n_t^s, n_t^i, T_t^s, T_t^i} & \quad \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} \\ \text{s.t.} & \quad P_t C_t = w_t^s n_t^s + w_t^i n_t^i + P_t^T (T_t^s + T_t^i) + \pi_t^s \\ & \quad (n_t^s + n_t^i) \leq h_t; \quad T_t^s + T_t^i = \bar{T}_t \end{aligned}$$

where P_t is the natural price index of equation 4.1 normalized to one and $\bar{T}_t$ are the fixed material at time t . The key first order conditions for the consumers for the labor and material allocation are

$$\begin{aligned} C_t^{-\gamma} w_t^j &= \psi (n_t^s + n_t^i)^{\frac{1}{\theta}} \quad j = i, s \\ C_t^{-\gamma} \frac{\partial C_t}{\partial T_t^j} &= p_t^T \quad j = i, s \end{aligned}$$

and imply the standard neoclassical labor supply problem for which the marginal rate of substitution between labor and consumption is equal to the wage rate, and the marginal utility of consumption is equal to price of material. Since these factor conditions holds for each sector, the labor supply of the consumer implies a fundamental arbitrage condition so that the wage in the two sectors has to be identical for he worker to be in equilibrium, and $w_t^s = w_t^i$. Similarly, the material allocation is such that the marginal utility of consumption in each of the two sector is identical.

In equilibrium all markets clear, and the labor and material demand quantities coincide with labor supply choice of the consumer. so that

$$C_t = Y_t; \quad n_t^s = N_t^s; \quad n_t^i = N_t^i; \quad T^t = T_t^s + T_t^i,$$

where the condition $C_t = Y_t$ implies that all final output is consumed and there are no savings.

The SPME is very similar but it only features a price for the fixed materials p_t^T in place of the cost of capital R_t . The limit system at time t for obtaining the BGP is made up by the following four equations. The growth rates are

$$\begin{aligned} g_C &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \alpha + \gamma(1 - \alpha)} g^s - (1 - \alpha)\Omega(g^s - g^i) \\ g_{ns} &= \frac{1 - \gamma}{\frac{1}{\theta} + \alpha + \gamma(1 - \alpha)} g^s - \Omega(g^s - g^i) \\ g_\mu &= \frac{\sigma - 1}{1 + \alpha + \sigma(1 - \alpha)} (g^s - g^i) \\ g_{T_i} &= -2g_\mu \end{aligned}$$

where the constant Ω depends on the structural parameters of the model $\Omega = \frac{1}{\frac{1}{\theta} + \alpha + \gamma(1 - \alpha)} \frac{\sigma - 1}{1 + \alpha + \sigma(1 - \alpha)}$. The equilibrium growth rate depends on the exogenous productivity growth rates g^s and g^i in interaction with the structural parameters of the model; TFP growth is exogenous by design, and market power affects growth rates through its presence, not its intensity (see the discussion in the main text). For further characterizing its properties, it is necessary to solve for the corresponding optimal growth problem, to which we next turn.

The optimal growth problem yields the following equations and the growth rates are

$$\begin{aligned} g_C^* &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma + \alpha(1 - \gamma)} g^s \\ g_{ns}^* &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma + \alpha(1 - \gamma)} g^s \\ g_{ni}^* &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma + \alpha(1 - \gamma)} g^s + (\sigma - 1)(g^i - g^s) \\ g_{T_i}^* &= -(\sigma - 1)(g^s - g^i) \end{aligned}$$

We are thus now in a position to study the misallocation and growth effect of the SMPGE.

F Accounting for a Rising Markdown from Concentration Data

This appendix reports the accounting exercise referenced in Section 6.2; its role in the calibration is limited to an independent cross-check on the growth rate of the markdown.

The dynamics of hours and of the labor share ultimately depend on the dynamics of μ_t , which in turn depend on the ratio of employment in the superior sector to employment in the inferior sector, as in equation (14). This accounting exercise is separate from the calibration of the effective-number index m . Let $q \in \{4, 20\}$ denote the number of firms included in CR4 or CR20, and denote by $\hat{n}_t^{s,j,q}$ their aggregate employment share in industry j , with $\hat{n}_t^{i,j,q} = 1 - \hat{n}_t^{s,j,q}$ the employment share of the remaining firms. Under the symmetric-firm approximation used for this exercise, the implied markdown is

$$\hat{\mu}_t^{j,q} = 1 + \alpha \frac{\hat{n}_t^{s,j,q}}{q \hat{n}_t^{i,j,q}} , \quad (63)$$

We use the recent evidence on superstar firms to compute the dynamics of $\hat{\mu}_t^{j,q}$ across industries. Autor et al. (2020) estimate time trends for CR4 and CR20 based on employment data. We use these ratios as $\hat{n}_t^{s,j,q}$ and apply equation (63) separately at $q = 4$ and $q = 20$. In Figure 6 and Table 10 we use $\alpha = 0.2$, consistent with the calibration of the transitional dynamics in Section 6.2. The exercise is an accounting translation of the two census concentration series, not an estimate of the calibrated conduct index m .

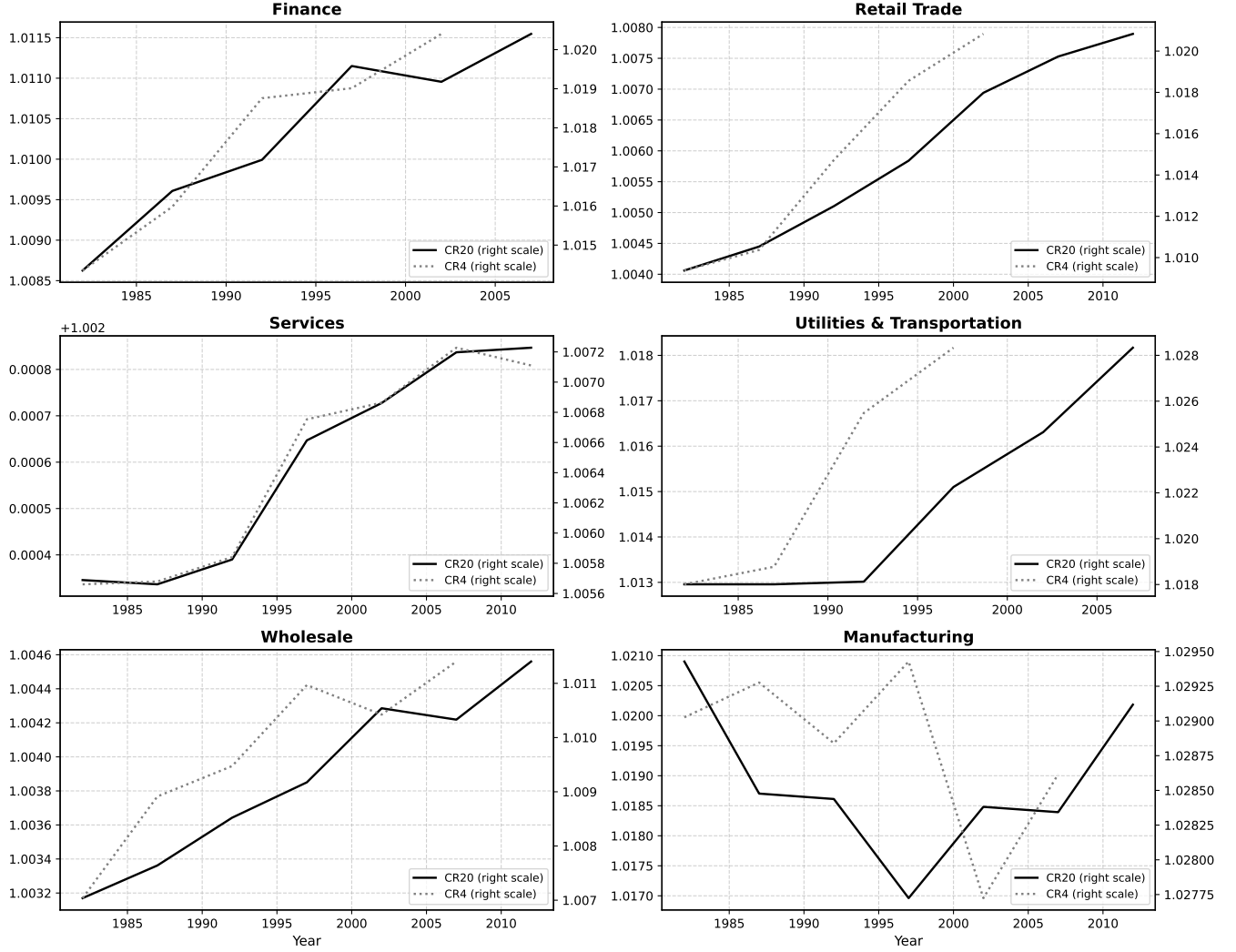
The results of the accounting exercise are reported in Table 10 and Figure 6 for six major industries: Retail Trade, Services, Utilities and Transportation, Wholesale Trade, Finance, and Manufacturing. The data are from Autor et al. (2020) and cover the period from 1982 to roughly 2010, with the exact time span varying by sector. In Figure 6, the left y-axis refers to estimates based on CR20 and the right y-axis to CR4. While the levels differ across industries, the figure reveals an unambiguous upward trend in the estimated markdown in all major industries, with the partial exception of manufacturing. For manufacturing, the estimate is V-shaped, but concentration has been clearly rising since the beginning of the century, in line with the trend estimates of Yeh et al. (2022). Although the exercise is purely accounting-based, the estimates in Figure 6 are clearly consistent with the central driver of the SSMPE.

Table 10 reports $\hat{\mu}_{2002}^{j,q}$ for the six industries together with the average value. The columns labeled “Growth CR4” and “Growth CR20” report the average growth rate of the markdown implied by the accounting exercise. Notably, the order of magnitude is comparable to that of the growth rate of the labor share of the US reported in Table 1, particularly when using the CR20 measure; the post-2000 CR20 growth rate (+0.121 percent per year over 2000–2012, against +0.077 before 2000) is the cross-check used in Section 6.2.

Table 10: Markdown Estimates and Average Growth by Industry

Industry	$CR4_{2002}^a$	$CR20_{2002}^a$	Growth CR4 (%)	Growth CR20 (%)	Period ^b
Finance	1.020	1.011	0.030	0.014	1982-2002
Retail Trade	1.021	1.007	0.056	0.019	1982-2002
Services	1.007	1.003	0.005	0.002	1982-2012
Utilities & Transportation	n.a.	1.016	0.067	0.034	1982-1997
Wholesale	1.010	1.004	0.017	0.006	1982-2007
Manufacturing	1.028	1.018	-0.002	-0.003	1982-2007
Average	1.017	1.010	0.029	0.012	
^a , $CR4_{2002}$ and $CR20_{2002}$ refer to the employment concentration ratio for C4 and C20 respectively.					
^b , Each industry has a different sample period as indicated in the last column.					
Source: Authors' calculations and Autor et al. (2020).					

Figure 6: Markdown Estimates from Concentration Ratio Across Six Main US Sectors



The panels report our estimates of the markdown by sector from the concentration data in Autor et al. (2020).

CR20 refers to the employment of the top 20 firms, CR4 to the employment of the top 4 firms.

The markdown is computed according to $\mu_t^{j,q} = 1 + \alpha n_t^{s,j,q} / (q n_t^{i,j,q})$, for $q \in \{4, 20\}$.