Monopsony in Growth Theory[*]

Pietro Garibaldi[†] Enrico D. Turri[‡]

June 11, 2026

Abstract

The secular decline in the labor share and the long-run reduction in labor supply suggest that imperfect labor markets can play a role in long-run economic growth. Unlike rising markups, rising wage markdowns are compatible with a balanced growth path featuring declining labor share and constant capital-output ratio. We introduce oligopsony and oligopoly power in a neoclassical growth model with superstar firms and an inferior sector which represents workers' outside option. Faster TFP growth in the superstar sector with respect to the inferior sector generates an endogenously increasing markdown, the driver of growth misallocation. The model can be calibrated to simultaneously match the joint trends of GDP growth, declining labor share, and hours worked. For the US, the consumption-equivalent loss with respect to the optimal growth path is around 7.5 percent. An extension of the model with hand-to-mouth workers and capitalists delivers balanced growth with increasing inequality. While—in this context—proportional taxation distorts equilibrium labor supply, a rising minimum wage can restore efficient growth.

Keywords: Monopsony, Growth, Hours Growth, Labor Share, Misallocation.
JEL codes: O40, O41, J23, J30, J42 :

---

[*] We thank Chad Jones, Rachel Ngai, Victor Ríos-Rull and Gianluca Violante for their insightful feedback, and seminar participants at Collegio Carlo Alberto, Birkbeck College, Norwegian Business School, LUISS University, University of Pennsylvania, IMF, Princeton University, Bocconi University, Cyrpus conference in honor of Chris Pissarides, CEPR-RPN conference on Growth, Innovation and the Social Model in Europe at Collége de France, for comments and suggestions.

[†] Collegio Carlo Alberto, University of Torino, CEPR, IZA, pietro.garibaldi@unito.it

[‡] London School of Economics, e.turri@lse.ac.uk

1 Introduction

The neoclassical growth model (NGM) assumes competitive labor markets (Cass, 1965; Koopmans, 1965; Acemoglu, 2009). However, two long-run macroeconomic facts suggest that imperfect labor markets can potentially also play a role in long-run economic growth. First, the secular decline in the labor share implies a long-run reduction in the share of total income accruing to labor (Bergholt et al., 2022; Karabarbounis, 2024). Second, long-run statistics on hours worked by the average individual point to a secular decline in labor supplied to the market (Huberman and Chris, 2007; Ramey and Francis, 2009; Boppart and Krusell, 2020).

Since the seminal contributions of Romer (1990) and Aghion and Howitt (1992), market power in the product market has been the focus of growth theory. Empirically, market power by so-called "superstar firms" (Autor et al., 2020) is associated with stagnant wages (Deb et al., 2022) and declining labor share. In standard growth models, a persistent increase in product-market power can drive a continuous fall in the labor share. Nevertheless—with standard neoclassical production functions—a permanent increase in market power is hardly compatible with a constant capital-output ratio, a key long-run property of any Balanced Growth Path (BGP). Growth models devote less attention to market power in the hiring of labor, even though labor-market imperfections too imply a wedge between the firm's marginal product and the wage paid to individuals; in recent years, the existence of monopsony power in the labor market has attracted considerable attention in both theoretical and empirical work (Manning, 2021; Yeh et al., 2022; Berger et al., 2022). The starting point of our analysis is to show that—unlike monopoly power—a long-lasting increase in monopsony power is consistent with both a constant capital-output ratio and a persistent decline in the labor share.

The paper proposes and solves a multisector growth model with sector-specific TFP growth and a superior sector in which "superstar" firms enjoy both oligopsonistic and oligopolistic power. Superstar firms derive oligopoly power from barriers to entry and from knowledge of the imperfect substitutability between different goods, and oligopsonistic power from knowledge of other firms' labor demand conditional on wages. As in traditional growth theory, the production functions are neoclassical and the demand for goods is isoelastic. When superstar firms enjoy both oligopoly and oligopsony power, labor-market imperfections induce growth effects and the decentralized growth process is inefficient. With standard isoelastic product demand, there are no growth effects from oligopoly power alone, but only level effects, including lower labor share and lower hours worked. Instead, oligopsony power by superstar firms induces a drag on the growth rates of the labor share, hours worked, and total production. The asymmetry reflects a compatibility requirement of the balanced growth path. A rising markup is inconsistent with a constant capital-output ratio, so it cannot sustain a declining labor share in line with the Kaldor facts; a rising markdown faces no such obstacle. Monopsony power is thus the form of market power that can rationalize an ongoing decline in the labor share along a BGP. Furthermore, the decline in hours per capita is a feature of the model also when income and substitution effects from wage changes cancel each other out and preferences are consistent with the King et al. (1988) restriction. The negative effect of oligopsonistic power on growth in hours represents the growth equivalent of the classic static result of the negative effects on hours worked of a generic labor-market wedge (Prescott, 2004).

In the solution of the model, the labor wedge increases over time because of oligopsony power, and is the key driver of the growth results. A simple accounting exercise that exploits the equilibrium equations of the model suggests that rising markdowns are fully consistent with rising sectoral labor concentration by superstar firms, as reported by Autor et al. (2020). In this perspective, the model takes as given the phenomenon of superstar firms and investigates the growth consequences of labor-market power in those markets. Yeh et al. (2022) also report evidence of rising markdowns and monopsony power in US manufacturing.

We solve our model under standard capital accumulation and labor-augmenting technological progress, and require TFP growth to be unbalanced in favor of the superstar sector. Yet, in a version of the model with Hicks-neutral technological progress, TFP growth differentials across sectors are not necessary to generate inefficient growth and declining labor share. Instead, a more capital-intensive superstar sector is sufficient to generate all the implications of the model.

The paper has several quantitative implications and results. Three exercises study the BGP of the representative economy; the fourth studies the transitional dynamics of the US economy.

In the first three quantitative exercises, we assume that a calibrated "representative advanced economy" moves along the asymptotic superstar equilibrium defined by the model. The section first shows that to quantitatively match the three long-run facts concerning (i) growth in GDP/consumption, (ii) (negative) growth in hours worked, and (iii) (negative) growth of the labor share (all reported empirically in Section 2), it is not enough to work with an NGM with preferences that imply an income effect of wage growth larger than the substitution effect, or preferences that belong to the Boppart and Krusell (2020) class. Indeed, we show that only the model with oligopsonistic labor markets can match all three growth moments at the same time. Second, the section quantifies the size of the income effect due to market imperfections and positive profits, and decomposes the part of the income effect that can be attributed to pure preferences (such as in Boppart and Krusell, 2020) from the part attributable to the non-wage transfer in a growth setting (Prescott, 2004). In general, the size of the income effect is much larger if the model is used to match the (larger) negative trend in hours per worker than the trend in hours per person. Quantitatively, market imperfections implied by the model account for 16 percent of the income effect in the case of hours per worker and as much as 80 percent in the case of hours per person. Third, we quantify the size of the asymptotic "growth drag" due to market imperfections vis-à-vis the competitive economy, and find that oligopsony implies an asymptotic loss of 5 percent of GDP growth. The proportional loss of the growth rate of hours is 7 percent lower than in the competitive economy in the case of hours per worker, and falls from -0.02 percent to -0.1 percent in the case of hours per person, which corresponds to an increase by a factor of 5 in proportional terms.

In the fourth quantitative exercise, we focus on the US economy and solve quantitatively the transition of the economy along the asymptotic path. We calibrate a version of the model with a fixed factor of production such as "materials". We match the level of the labor share, hours worked, and other key macro moments to the "time 0" of the calibration to the US economy in the first ten years of the 2000s. Before calibrating the transitional dynamics, we perform a simple accounting exercise based on the employment dynamics of the superstar firms. The accounting exercise implies an upward trend in the monopsonistic markdown that can be used as an empirical benchmark. We then estimate the transition path of the US economy toward the asymptotic balanced growth path, and show that a version of the calibrated economy perfectly matches the growth rate and can account for 60 to 80 percent of the (negative) growth observed in hours per worker and the labor share. We quantitatively assess the consumption-equivalent loss of the calibrated economy. With respect to a competitive economy, the consumption loss is approximately 7.5 percent. Lastly, the paper shows that in the transitional dynamics, an increase in market concentration induces level effects on hours worked and labor share, without influencing their long-run growth properties.

The theory developed in this paper has implications for long-run inequality. A version of the model with capitalists and hand-to-mouth workers shows that the divergence between the capital share and the labor share is accompanied by increasing and long-lasting income and consumption inequality. In addition, the model with heterogeneous agents has implications for the long-run dynamics of labor supply, since the hand-to-mouth individuals work harder than would be optimal for a central planner, and harder than the representative decentralized agent subject to oligopsony and oligopoly. We also discuss the role of policy in this setting. We first study the effects of a proportional tax on capitalists' income rebated to workers, showing that while it addresses the most severe effects of inequality, it depresses labor supply and GDP growth vis-à-vis the efficient levels. We also address the possibility that a growing minimum wage increases both growth of hours and growth of wages, in line with the classical intuition due to Robinson (1969).

Related Literature. This paper sits at the intersection of five strands of literature. First, the literature on *monopsony in labor markets* has expanded rapidly in recent years, both theoretically and empirically.[1] Yet, despite the prominent role of product-market power in the endogenous growth revolution,[2] the growth effects of labor-market imperfections have received remarkably little attention. The early literature on growth and labor-market frictions focused on the relationship between growth and unemployment,[3] while the only prior attempt to study the growth consequences of monopsony, to our knowledge, is Barr and Roy (2008), where spatial mobility costs depress wages and slow human capital accumulation in an endogenous growth setting. Barr and Roy (2008) do not link monopsony to the labor share, to long-run labor supply, or to superstar firms—the three connections that organize our analysis. Second, our paper relates to recent *growth-theoretic explanations of the labor share decline*. Existing models generate level effects on the labor share through endogenous markups, automation, or capital-embodied technological change.[4] Relative to this work, we propose a different mechanism—monopsony rather than monopoly or capital deepening—and, more importantly, generate a balanced growth path along which the labor share declines at a *constant rate*, rather than transitions between BGPs of different levels. Third, the paper refers to the *empirical literature on the secular rise in market concentration* documented for the US and other advanced economies by Autor et al. (2020), Hsieh and Rossi-Hansberg (2023) and Kwon et al. (2024). A growing share of sales and employment in each major industry is accounted for by a small number of large "superstar"

---

[1] See Manning (2021) for a survey, and Berger et al. (2022) and Yeh et al. (2022) for evidence of monopsony in the US labor market. Deb et al. (2022) study superstar firms with labor-market power in a static setting.

[2] Romer (1990); Aghion and Howitt (1992).

[3] Aghion and Howitt (1994); Mortensen and Pissarides (1998); more recently, Martellini and Menzio (2020) embed declining search frictions in a balanced growth model with constant unemployment.

[4] Akcigit and Ates (2021, 2023) build models with endogenous markups; Acemoglu (2025) studies endogenous automation; Jones and Liu (2024) characterize a BGP with constant labor share under capital-embodied technological change.

firms, which exhibit faster productivity growth and higher markups than the non-superstar fringe. Autor et al. (2020) show that industries experiencing larger increases in concentration also experience steeper declines in the labor share, and emphasize rising product-market power as the principal mechanism. More recent evidence indicates that superstar firms exert market power not only in the product market but also in the labor market: Yeh et al. (2022) document rising markdowns in US manufacturing, and Berger et al. (2022) provide quantitative estimates of labor-market power at the firm level. Our paper takes the existence of superstar firms as given, and asks what their oligopsony power implies for long-run growth, the labor share, and hours worked. Fourth, the model features sectoral labor reallocation driven by unbalanced TFP growth, placing it in the tradition of the *structural change* literature.[5] Relative to this strand, we solve a fully decentralized economy with imperfections in both product and labor markets, multiple firms in the superior sector, and endogenous labor supply. Finally, we contribute to the *misallocation* literature, which Jones (2022) identifies as a central research avenue in growth economics.[6] Whereas this literature typically traces TFP differentials to micro-level distortions, in our framework the source of misallocation is monopsony power by superstar firms, which slows the reallocation of factors toward the more productive sector relative to the competitive benchmark.

The paper proceeds as follows. Section 2 reviews the two key stylized facts. Section 3 presents the challenge posed by a rising markup, but not a rising markdown, in obtaining a declining labor share alongside a constant capital-output ratio on a BGP. Section 4 presents the basic structure of the economy, the superstar sector, production technologies, and preferences. It then solves for the balanced growth path with superstar oligopolists and oligopsonists, proving its stability, as well as for the competitive equilibrium, and compares the two solutions, highlighting the growth effects of oligopsony power. Section 5 solves the model with hand-to-mouth workers and capitalists and derives the implications for inequality. Within this framework, we also analyze two types of policy intervention: taxation of capitalists' income and the minimum wage. Section 6 presents the calibration and quantifies the misallocation loss due to monopsony in consumption-equivalent terms. It also presents the accounting exercise on the dynamics of the markdown. Section 7 summarizes and concludes.

2 The Decline in Labor Share and Hours Worked

We document two facts that are well known in the literature, even though they are not typically considered jointly and part of a unified theory. The first fact concerns the long run dynamics of the labor share. The second fact is the long-run dynamic of labor supply. To organize the evidence we use the latest available data from World Penn Table (Feenstra et al., 2015). We focus on nine advanced economies for which data in the Penn World Table cover the period from 1950s to 2023 for labor supply, and from 1980 to 2023 for the labor share.[7] For three of these countries (United States, France and Australia) there are labor share statistics dating back to the 1950s.

---

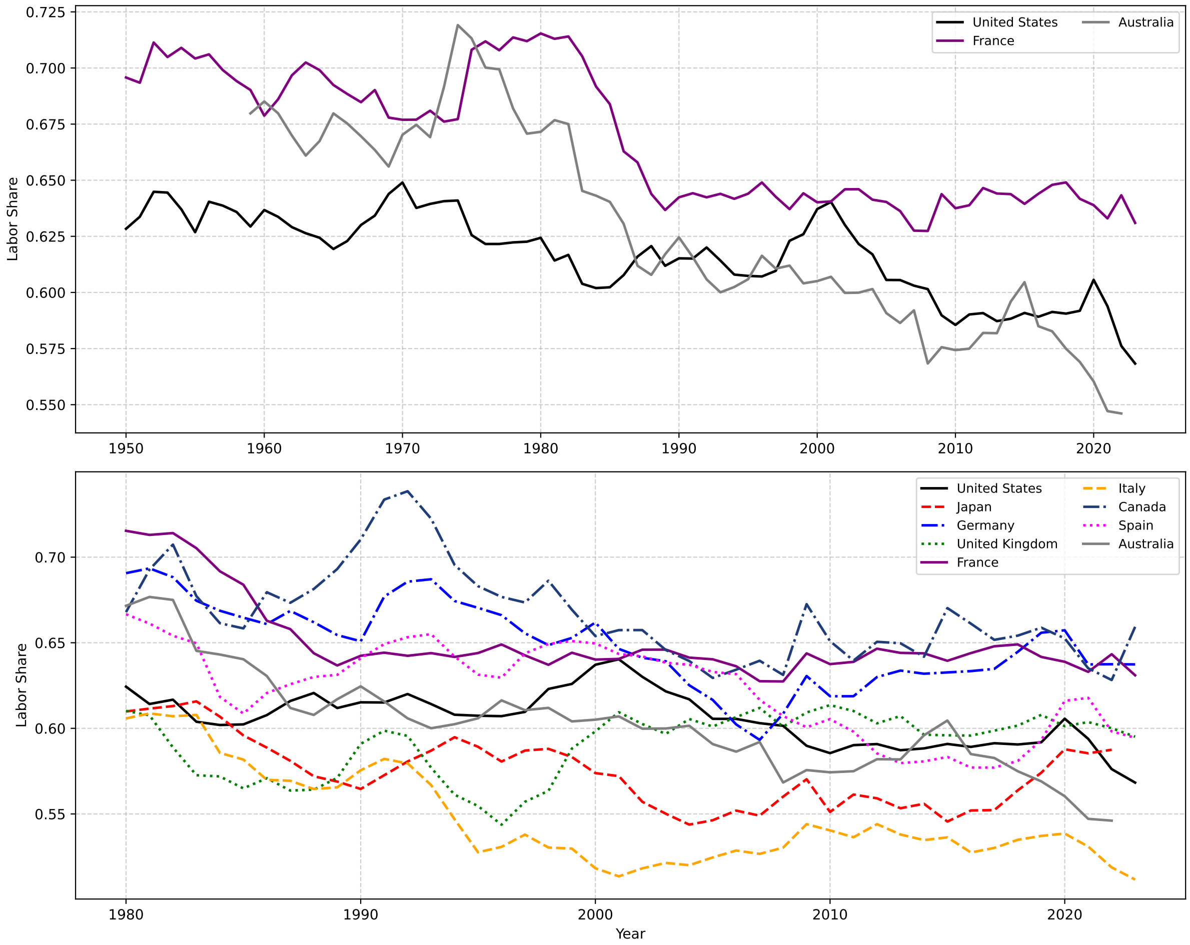
[5] Kongsamut et al. (2001); Ngai and Pissarides (2007); Acemoglu and Guerrieri (2008). The asymptotic BGP of our model mirrors the long-run equilibrium of Acemoglu and Guerrieri (2008).

[6] Seminal contributions include Restuccia and Rogerson (2008) and Hsieh and Klenow (2009); Hsieh et al. (2022) quantify the role of reduced talent misallocation in US TFP growth.

[7] Following Karabarbounis (2024) for the labor share and Boppart and Krusell (2020) for hours worked, it is possible to focus on slightly different advanced countries and on a longer period. In summarizing the evidence, the paper focuses

Using the same dataset, Table 1 reports also the average annual growth rates in GDP per capita and in GDP per worker for the nine countries. In the remainder of our analysis, we will consider GDP growth for the "representative advanced economy" as the growth rates reported in Table 1 for 1980 to 2023 for the nine-country sample, so as to use the same sample and time frame as for labor share and hours.

Figure 1: Dynamics of Labor Share



Source: World Penn Table (Feenstra et al., 2015)

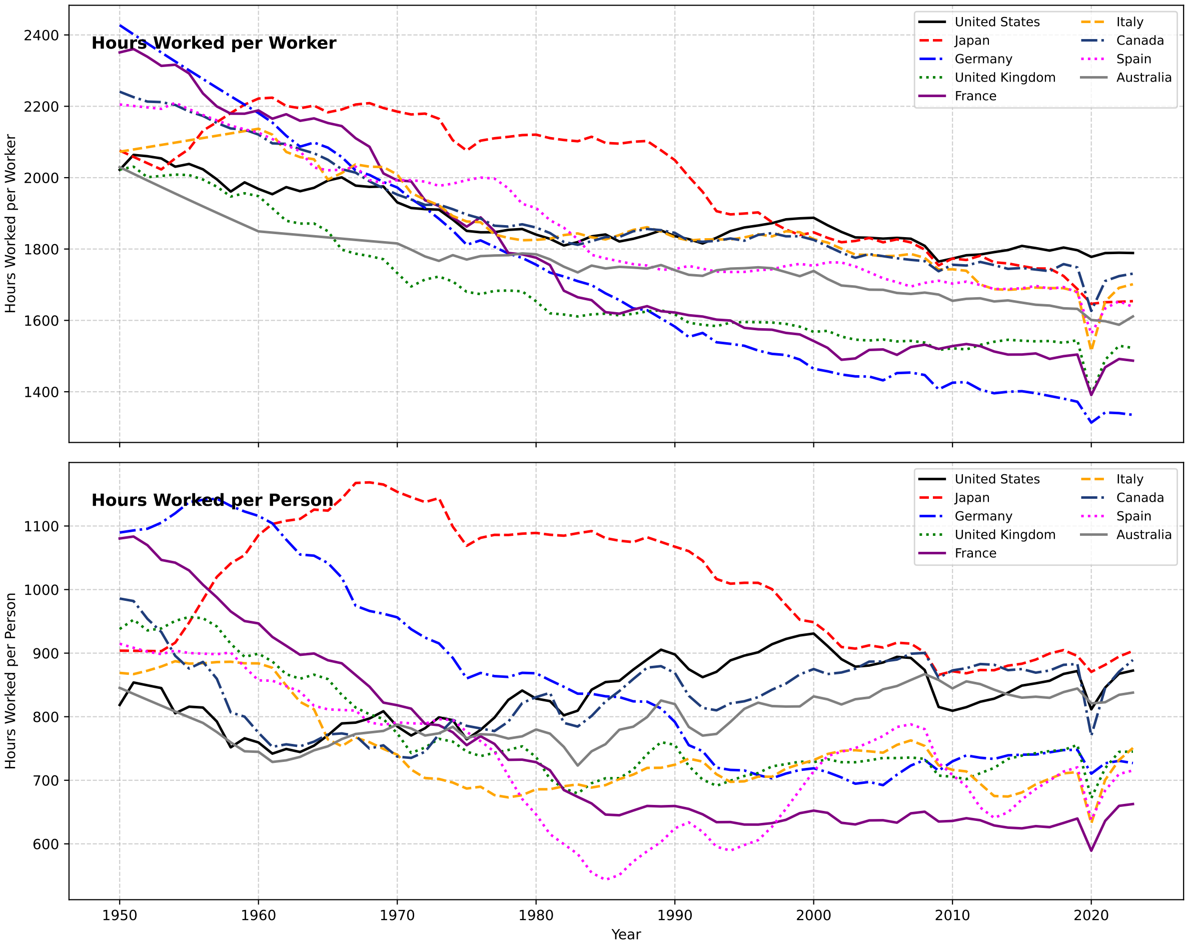
2.1 The Decline of the Labor Share

The secular decline in the labor share has been first recognized in the seminal paper by Karabarbounis and Neiman (2014). Since then, the empirical literature on the measure of the labor share has been very vivid across the two sides of the Atlantic, and in most developing countries (Cette et al.,

---

on consistency and reports statistics on the nine countries listed in Table 1. Adding single advanced economies does not change the nature of the stylized facts.

Figure 2: Dynamics of Hours Worked



Source: World Penn Table (Feenstra et al., 2015)

2019; Brooks et al., 2021). At the empirical methodological level, there is strong debate about measurement issues linked to the role of housing, self employment and proprietary income, and the role of government. Atkenson (2020)- for example- argues that the estimates of the falling labor share are largely due to changes in measurement details by the Bureau of Labor Statistics. More recently, Karabarbounis (2024) reviews the large work on the decline in the labor share and reports unambiguous evidence of its decline. Figure 1 uses data from the Penn World Table and reports the long run trend in the labor share. Quantitatively, Table 1 - in line with Karabarbounis (2024) - shows that since the mid 1980s the labor share has been declining at a rate of -0.11 percent per year for the representative advanced economy, a value that we will consider in the calibration exercise. Several possible explanations for the secular fall in the labor share in the US and in most countries have been proposed, and relate to technology (Acemoglu and Rastrepo, 2022), cost of capital (Kaymak and Schott, 2023), product market power by firms and rising concentration (Autor et al., 2020; Deb et al., 2022), changes in labor market institutions and firms' market power (Yeh et al., 2022), and globalization. While it is difficult to give weights to all the possible concurrent factors, this paper examines the relationship between balanced growth and declining labor share through the lenses of increasing market power by superstar firms, with special emphasis on the hiring of labor.

Table 1: Stylized Facts: Labor Share, Hours, and GDP Growth

Country	**Labor Share** (pp/yr)	**Hours/Worker** (%/yr)	**Hours/Person** (%/yr)	**GDP/Capita** (%/yr)	**GDP/Worker** (%/yr)
Period: 1950 onward					
United States	-0.0708***	-0.3406***	0.1352***	1.9816	2.1534
France	-0.1119***	-1.2975***	-0.5962***	3.1102	3.2451
Australia	-0.2202***	-0.4429***	0.1101***	2.0221	2.0669
Average	-0.1343	-0.6937	-0.1169	2.3713	2.4885
Period: 1980 onward					
United States	-0.0781***	-0.1382***	-0.0246	1.4776	1.6902
France	-0.1152***	-0.5361***	-0.1080***	1.3379	1.4666
Australia	-0.2141***	-0.3909***	0.2036***	1.6874	1.8442
Japan	-0.1007***	-1.1386***	-0.6064***	1.3484	1.4529
Germany	-0.1304***	-0.9044***	-0.2630***	1.3891	1.5230
United Kingdom	0.0750***	-0.3196***	0.0572**	1.9254	2.0556
Italy	-0.1666***	-0.5033***	0.0144	1.0824	1.1962
Canada	-0.1294***	-0.3419***	0.1271***	1.4498	1.5861
Spain	-0.1514***	-0.4160***	0.3392***	1.5892	1.7137
Average	-0.1123	-0.5543	-0.0734	1.4985	1.6920

Notes: Data from the Penn World Table (Feenstra et al., 2015). The Labor Share column reports the OLS slope of the aggregate labor income share on calendar year, expressed in percentage points per year. The Hours columns report OLS slopes of the natural log of hours per worker and hours per person on calendar year, expressed in percent per year. The GDP columns report compound annual growth rates of real GDP per capita and real GDP per worker over each subperiod, expressed in percent per year. Average rows are simple means across countries within each subperiod. Stars denote statistical significance of the underlying slope: $^{***}p \leq 0.01$, $^{**}p \leq 0.05$, $^{*}p \leq 0.10$.

2.2 Labor Supply in The Long Run

Traditional growth theory used to take the long run stability of total labor and leisure as a key stylized fact. Prescott (1986) argued that leisure shows no secular trend while the real wage has grown steadily. In terms of preferences, the stability of leisure and labor supply has been obtained by representative agent models in which the income and substitution effects of wage increases cancel each other out, the well known King et al. (1988) restriction. Recently, however, the stability of hours worked over the long run has been challenged first by Huberman and Chris (2007) and Ramey and Francis (2009), and more recently by Boppart and Krusell (2020).

Indeed, hours worked per worker have been declining in most countries when we consider a time horizon of more than a century, similarly to what is reported in the top panel of Figure 2 for the nine countries we focus on in our analysis. Hours worked per person (bottom panel of Figure 2) have declined too for most countries, even if post World War II data for the US suggest a stable level of hours worked per person. Yet, even the US features a marked secular decline over two centuries (Boppart and Krusell, 2020).[8] Table 1 reports trend estimates for hours worked and shows that for the average country of our sample- in a way fully coherent with the evidence reported in Boppart and Krusell (2020) - hours worked per worker decline at rate approximately equal to -0.55 percentage points per year. With respect to hours per person, the sample used suggests a more modest decline, and the estimates we use for the average country is approximately -0.1 percentage point per year.

3 Markup, Markdown, and the Kaldor Facts: An Asymmetry

This section establishes a preliminary theoretical result that motivates the modeling choices of the paper. We show that in a standard NGM, a permanent decline in the labor share can in principle be generated by either a permanently rising markup or a permanently rising markdown. The two mechanisms, however, are not qualitatively equivalent. A rising markup is incompatible with a constant capital–output ratio in balanced growth, whereas a rising markdown preserves this Kaldor fact.

To formalize these claims, consider the recent neoclassical growth models with market power in product and labor market, proposed - among others - by Deb et al. (2022). In these models the aggregate production function- obtained by solving for the symmetric equilibrium in output and labor varieties- is an equilibrium object. It mimics a standard neoclassical production function with labor-augmenting technological progress in which production takes place according to $Y_t = F(K_t, N_t, A_t)$, where F is neoclassical (exhibiting constant returns to scale in capital K_t and labor N_t), satisfies Inada's conditions and Uzawa's theorem (so that it can be rewritten as a function of K_t and effective labor $A_t N_t$), and A_t grows at rate g. Appendix A provides the micro structure underlying the aggregate model. We define capital and output per unit of efficiency as $x_t = \frac{K_t}{A_t N_t}$ and $\tilde{y}_t = \frac{Y_t}{A_t N_t}$, with intensive-form production function $\tilde{y}_t = f(x_t) = F(x_t, 1, 1)$. Along a balanced growth path,

---

[8] Ramey and Francis (2009) argues that the decline in hours worked per person are mainly accounted for by an increase in schooling by young workers. Conversely, prime age individuals between the ages of 25 and 54 are working the same number of hours now as in 1900, as a combination of a rise in female hours worked and reduction in male hours.

a constant and positive capital–output ratio requires a time-invariant $x_t = x > 0$, [9] and hence a constant $\tilde{y}_t = \tilde{y}$. We denote by $\varrho_t = \frac{p_t}{MC_t}$ the markup of the aggregate price p_t over marginal cost MC_t, and by $\mu_t = \frac{MPN_t}{w_t}$ the markdown of the marginal product of labor over the wage w_t. Two natural mechanisms for generating a declining labor share emerge from the literature: a permanently rising markup ($\dot{\varrho}_t/\varrho_t = g_\varrho > 0$) or a permanently rising markdown ($\dot{\mu}_t/\mu_t = g_\mu > 0$). At this level of generality we do not specify the underlying market structure giving rise to either.

Under a rising markup, the labor share in intensive form is

$$\alpha_t^\varrho = \frac{1}{\varrho_t}\frac{f(x_t) - x_t f'(x_t)}{f(x_t)}, \tag{1}$$

where $\varrho = 1$ in perfect competition. Holding $x_t = x > 0$ constant, a rising markup at rate g_ϱ would drive $\alpha_t^\varrho \to 0$ at the constant rate $-g_\varrho$, generating the permanently declining labor share empirical regularity. However, such path is inconsistent with canonical (CRRA) preferences and neoclassical production technology. The Euler equation for the representative consumer with constant elasticity of the marginal utility of consumption γ reads, when hours are constant,

$$\gamma\left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g\right) = \frac{f'(x_t)}{\varrho_t} - \delta - \rho, \tag{2}$$

where $c_t = C_t/N_t$ is per capita consumption, $\tilde{c}_t = c_t/A_t$ is consumption per unit of efficiency, δ is the depreciation rate, and ρ is the discount rate. In a BGP - if it exists - we have that $d\tilde{c}_t/dt = 0$. For a constant markup, equation (2) is satisfied for a unique stationary x, and the capital–output ratio K_t/Y_t in the solution is thus constant. However, the same mechanism required to deliver a declining labor share in (1) – a permanently rising ϱ_t – enters the right-hand side of (2) and forces x_t to decline and converge to zero. This - in turn- implies that the capital-output ratio converges to zero, in contradiction with empirical evidence documenting its stability (Jones, 2016), which is accepted as one of the Kaldor's facts.[10]

The same tension does not arise when the decline in the labor share is driven by a rising markdown. In that case the analogous equations (when $\varrho_t = 1$) are

$$\alpha_t^\mu = \frac{1}{\mu_t}\frac{f(x_t) - x_t f'(x_t)}{f(x_t)}, \tag{3}$$

$$\gamma\left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g\right) = f'(x_t) - \delta - \rho. \tag{4}$$

A rising μ_t drives $\alpha_t^\mu \to 0$ along a BGP with constant x. Crucially, μ_t does not enter the Euler equation (4): the markdown affects only the labor market, leaving the equilibrium condition for capital – and hence the capital–output ratio – intact. We summarize these observations as our first result.

---

[9] The fact that x_t must be constant follows from rewriting

$$\frac{K}{Y} = \frac{K}{F(K,N,A)} = \frac{K}{ANF(K/AN,1,1)} = \frac{x}{f(x)} \ .$$

If $K/Y > 0$, then $x > 0$ too because of Inada's conditions, by de L'Hôpital rule $\lim_{x\to0} x/f(x) = \lim_{x\to0} 1/f'(x) = 0$.

[10] Ball and Mankiw (2023) study dynamic inefficiency in a similar model with time invariant markups.

Result 1 (Markup–Markdown Asymmetry)**.** *In a neoclassical growth model with labor-augmenting technological progress and canonical preferences:*

(i) A permanently rising markup generates a permanent decline in the labor share but is incompatible with a constant capital–output ratio in balanced growth.

(ii) A permanently rising markdown generates a permanent decline in the labor share and is compatible with a constant capital–output ratio in balanced growth.

The intuition for the asymmetry is straightforward. A markup over the marginal cost of production distorts the return to all factors, including capital, and therefore alters the steady-state condition for capital accumulation. A markdown of the wage over the marginal product of labor distorts only the labor margin, leaving the capital margin – and the Kaldor fact on the capital–output ratio – undisturbed.

Consider now the second stylized fact, the permanent decline in hours worked. In addition to the Euler equation, a model of declining hours implies an extra condition on labor supply along the BGP (see Appendix A). In the influential framework of Boppart and Krusell (2020), a permanent decline in hours is delivered through preferences in which the income effect of rising wages dominates the substitution effect. Raising market power markdown offers an alternative mechanism. In a static setting, Prescott (2004) shows that any wedge between marginal product and wages depresses the level of hours. A positive growth in markup and markdown (g_μ), and (g_ϱ) represents the dynamic extension of such classic result, and represents our second result.

Result 2 (Market Power and Hours Growth)**.** *Permanently rising markups and markdowns generate a permanent decline in hours worked along the BGP, even under preferences satisfying the King et al. (1988) restriction (income and substitution effects offsetting each other).*

The joint combination of Results 1 and 2 makes a methodological case for organizing the rest of the paper around monopsony power rather than monopoly power. A model built on a rising markdown can in principle deliver the two facts facts documented in Section 2 – a declining labor share and declining hours – and also be consistent with a constant capital output ratio along a balanced growth path without requiring auxiliary restrictions on preferences. The remainder of the paper develops such a model from explicit microfoundations and quantifies its implications.

4 A Model of Superstar Monopsonistic Growth

This section develops the model. We first describe the technology, market structure, and household preferences (Sections 4.1–4.2). We then characterize the problems of the two types of firms (Sections 4.3–4.4) and of the representative household (Section 4.5). Section 4.6 defines the symmetric monopsonistic equilibrium, establishes the existence of a unique asymptotic balanced growth path, and derives the key growth rates. Section 4.7 contrasts these results with the competitive benchmark.

4.1 Technology and Market Structure

The economy produces a single final consumption good by competitively combining two intermediate goods, indexed $j \in \{s, i\}$, with superscripts denoting a *superior* and an *inferior* sector. The final good is produced according to a constant-elasticity-of-substitution technology

$$Y_t = \left[\zeta^{\frac{1}{\sigma}}(y_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}(y_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution between the two intermediaries and y_t^j is the output of sector j. We denote by p_t^j the price of intermediary j and normalize the price of the final good to one at all dates, so that

$$1 \equiv P_t = \left[\zeta(p_t^s)^{1-\sigma} + (1-\zeta)(p_t^i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

The two intermediaries are produced with Cobb–Douglas technologies that combine capital K_t^j and labor N_t^j with the same elasticity of output to capital $\alpha \in (0,1)$, but with different productivity levels A_t^j:

$$y_t^j = (K_t^j)^\alpha (A_t^j N_t^j)^{1-\alpha}, \quad A_t^j = A^j e^{g^j t}, \quad j \in \{s, i\},$$

with $A^s > A^i$ and $g^s > g^i$, so that the superior sector has a higher initial level and grows faster.[11]

The two intermediaries differ not only in technology but also in market structure. The inferior good i is produced by a competitive fringe. The superior good s is produced by m identical *superstar* firms indexed $f \in \{1, \ldots, m\}$, all sharing the same productivity A_t^s, so that total sectoral output is $y_t^s = \sum_{f=1}^m y_t^{f,s}$ with $y_t^{f,s} = (K_t^{f,s})^\alpha (A_t^s N_t^{f,s})^{1-\alpha}$. The m superstar firms hold property rights over the superior technology and consequently enjoy both oligopoly power in the product market and oligopsony power in the labor market, in the spirit of Berger et al. (2022) and Robinson (1969). We take this market power as given and study its growth consequences.[12] The capital market is competitive.

4.2 Preferences

The economy is populated by a continuum of measure one of identical, infinitely-lived agents; we abstract from population growth. Each agent is endowed with one unit of time and supplies $h_t \leq 1$ hours to labor. Preferences are time-separable and additively separable in consumption and hours, belonging to the MaCurdy (1981) class within the Boppart and Krusell (2020) family:

$$\int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right] dt, \quad \text{if } \gamma \neq 1,$$

with $u(C_t) = \log C_t$ when $\gamma = 1$. The parameter $\gamma > 0$ is the inverse elasticity of marginal utility of consumption; to be consistent with the Boppart and Krusell (2020) class, we will typically have $\gamma > 1$. The parameter θ is the Frisch elasticity of labor supply. With these preferences, $\epsilon_{w_t,n_t} = \theta$,

---

[11] The assumption $A^s > A^i$ is inessential; the higher growth rate of sector s implies that A_t^s eventually overtakes A_t^i regardless of initial conditions. With Hicks-neutral technological progress, $g^s = g^i$ with a higher capital intensity in the superior sector would deliver the same qualitative results (see Section 4.10).

[12] Autor et al. (2020) show that superstar firms display higher patenting activity and larger R&D investment than ordinary firms. The oligopoly problem extends naturally to the case in which $A_t^{f,s}$ differs across firms, provided they share the same growth rate g^s.

so the firm chooses the wage along a λ-constant labor supply schedule. Agents allocate their hours between the two sectors,

$$n_t^i + n_t^s = h_t, \qquad h_t \leq 1,$$

where $n_t^s = \sum_{f=1}^m n_t^{f,s}$. They own all firms in the economy and save in safe assets that yield rights to the capital stocks in production.

4.3 Final Good Sector and the Problem of the Inferior Intermediary

The final-good firm produces competitively, taking the prices p_t^s and p_t^i as given and solving

$$\Pi_t^f = \max_{y_t^s, y_t^i} Y_t - p_t^s y_t^s - p_t^i y_t^i. \tag{5}$$

The first-order conditions yield the demand functions for the two intermediaries:

$$y_t^s = \zeta (p_t^s)^{-\sigma} Y_t, \tag{6}$$
$$y_t^i = (1-\zeta)(p_t^i)^{-\sigma} Y_t. \tag{7}$$

The inferior intermediary is produced by a competitive fringe that takes prices and wages as given, solving

$$\Pi_t^i = \max_{K_t^i, N_t^i} p_t^i y_t^i - w_t^i N_t^i - R_t^i K_t^i, \tag{8}$$

where w_t^i is the wage in the inferior sector and R_t^i the rental rate of capital. Standard Cobb–Douglas factor demands give

$$w_t^i = p_t^i \frac{(1-\alpha) y_t^i}{N_t^i}, \qquad R_t^i = p_t^i \frac{\alpha y_t^i}{K_t^i}. \tag{9}$$

4.4 The Superstar Oligopolist and Oligopsonist

The core of the model is the problem of the superstar firms in the superior sector. These firms enjoy market power on two margins simultaneously. On the product side, the m superstar firms compete à-la-Cournot facing a downward-sloping demand from the final good sector with constant elasticity σ. On the factor side, each superstar firm is a price taker in the capital market but an oligopsonist in the labor market: each firm internalizes the fact that hiring an additional worker forces it to pay a higher wage, both to that worker and to all its existing workforce. This subsection sets up the problem of a representative superstar firm, derives the markdown of wages over the marginal product of labor that this two-sided market power generates, and characterizes the symmetric Cournot–Nash equilibrium that defines the rest of the model.

We index superstar firms by $f \in \{1, \ldots, m\}$. The choices of firm f at time t are denoted $N_t^{f,s}$ (labor), $K_t^{f,s}$ (capital), $y_t^{f,s}$ (output), and $W_t^{f,s}$ (the wage it pays). To express how firm f's decisions interact with those of its rivals, we collect the analogous variables for the other $m-1$ firms in the superior sector:

$$N_t^{-f,s} \equiv \sum_{l \neq f} N_t^{l,s}, \quad y_t^{-f,s} \equiv \sum_{l \neq f} y_t^{l,s}, \quad K_t^{-f,s} \equiv \sum_{l \neq f} K_t^{l,s}.$$

Total sectoral output and labor are then $y_t^s = y_t^{f,s} + y_t^{-f,s}$ and $N_t^s = N_t^{f,s} + N_t^{-f,s}$. In Cournot fashion, firm f takes the decisions of its rivals $\{N_t^{-f,s}, y_t^{-f,s}, K_t^{-f,s}\}$ as given when choosing its own. The symmetric equilibrium, in which all firms make identical choices, will be characterized below.

The market power of firm f derives from its full knowledge of two equilibrium relationships in the rest of the economy.

The first is the *labor-supply schedule* faced by firm f. Workers in the economy can either work for the superstar firms in sector s or for the competitive fringe in sector i. The wage in the inferior sector is the marginal product of labor there, $w_t^i = p_t^i(1-\alpha)(K_t^i)^\alpha(N_t^i)^{-\alpha}(A_t^i)^{1-\alpha}$. By the household's wage-arbitrage condition (derived in Section 4.5), any wage paid by a superstar firm must equal the wage available in the inferior sector. Using $N_t^i = H_t - N_t^{f,s} - N_t^{-f,s}$, where H_t is total hours supplied in the economy, the labor-supply schedule faced by firm f is

$$w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}) = p_t^i(1-\alpha)(K_t^i)^\alpha \left(H_t - N_t^{f,s} - N_t^{-f,s}\right)^{-\alpha} (A_t^i)^{1-\alpha}. \tag{10}$$

Equation (10) is upward-sloping in $N_t^{f,s}$: as firm f hires more labor, fewer workers remain available for the inferior sector, the marginal product of labor in that sector rises, and so does the outside option of firm f's workforce. This is the classic monopsonistic mechanism of Robinson (1969), generalized here to the case of m firms in the spirit of Berger et al. (2022).

The second fundamental condition is the inverse demand for the superior good s. Substituting $y_t^s = y_t^{f,s} + y_t^{-f,s}$ into the demand function (6) from the final-good sector yields

$$p_t^s = Y_t^{\frac{1}{\sigma}} \left(y_t^{f,s} + y_t^{-f,s}\right)^{-\frac{1}{\sigma}} \zeta^{\frac{1}{\sigma}}, \tag{11}$$

which is downward-sloping in $y_t^{f,s}$ with constant elasticity σ.

While superstar firms have knowledge of the labor supply schedule of its workforce (Eq. 10) and their product demand (Eq. 11), they maximize profits holding fixed three sets of variables:

1. factors inputs of other superstar firms: capital $K_t^{-f,s}$ and labor $N_t^{-f,s}$, coherently with traditional Cournot competition as $y_t^{-f,s}$ is function of capital and labor;
2. variables other than labor input in the inferior sector: output y_t^i, price p_t^i and capital K_t^i; superstar firms ignore additional equilibrium responses from the competitive inferior sector.
3. total aggregate hours H_t; superstar firms do not internalize that a change in the wage level will affect the household's labor supply decision.

Such assumptions are coherent with the discussion in Section 3. To obtain a BGP with constant capital output ratio, it is necessary to build a microfounded model with constant markups and endogenously increasing markdowns.

Taking $\{H_t, N_t^{-f,s}, y_t^{-f,s}, K_t^{-f,s}, R_t^s, p_t^i, K_t^i\}$ as given, firm f chooses labor, capital, and the wage to maximize period-by-period profits subject to the labor-supply constraint (10):

$$\begin{aligned} \max_{N_t^{f,s}, K_t^{f,s}, W_t^{f,s}} \quad & \Pi_t^{f,s} = p_t^s\left(y_t^{f,s}; y_t^{-f,s}\right) y_t^{f,s}\left(K_t^{f,s}, N_t^{f,s}, A_t^s\right) - W_t^{f,s} N_t^{f,s} - R_t^s K_t^{f,s} \\ \text{s.t.} \quad & W_t^{f,s} \geq w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}). \end{aligned} \tag{12}$$

With respect to the wage, the firm optimally sets a *limit wage*: paying any more than necessary would simply transfer rents to workers, while paying any less would leave the firm short of labor. The labor-supply constraint thus binds with equality, and the firm's effective wage choice collapses to its labor choice.

Because the wage paid to all workers depends on the firm's labor input, the marginal cost of hiring an additional worker exceeds the wage that worker is paid. Differentiating total labor costs $w_t^{f,s}(N_t^{f,s};N_t^{-f,s})N_t^{f,s}$ with respect to $N_t^{f,s}$, and using the Cobb–Douglas form of the inferior-sector marginal product, the marginal cost of labor to firm f is

$$\frac{\partial\left[w_t^{f,s}(N_t^{f,s};N_t^{-f,s})N_t^{f,s}\right]}{\partial N_t^{f,s}} = w_t^{f,s}\left[1+\alpha\frac{N_t^{f,s}}{N_t^i}\right] = w_t^{f,s}\left[1+\frac{1}{\epsilon_{N^{f,s},w^{f,s}}}\right]. \tag{13}$$

The expression $\epsilon_{N^{f,s},w^{f,s}} = (dN^{f,s}/dw^{f,s})(w^{f,s}/N^{f,s}) = N_t^i/(\alpha N_t^{f,s})$ is the firm-level elasticity of labor supply, in line with Manning (2021). The bracketed term in equation (13) is the *markdown* of the wage with respect to the marginal cost of labor,

$$\mu_t^{f,s} = 1+\frac{1}{\epsilon_{N^{f,s},w^{f,s}}}, \tag{14}$$

and plays the central role in what follows.[13] The intuition behind the markdown is straightforward: a firm with monopsony power restricts hiring below the competitive level in order to keep its wage bill low, so that in equilibrium the marginal product of labor exceeds the wage. The static analogue of this mechanism is depicted in the top panel of Figure 3. The bottom panel illustrates how an increase in superior-sector productivity A^s shifts the marginal revenue product of labor outward, raising both the firm's employment and the markdown.

On the product side, the firm internalizes that selling an additional unit of output requires lowering the price. From the inverse demand (11), the marginal revenue from increasing $y_t^{f,s}$ is

$$\frac{\partial\left[y_t^{f,s}\cdot p_t^s(y_t^{f,s}+y_t^{-f,s})\right]}{\partial y_t^{f,s}} = \left(1-\frac{1}{\sigma}\frac{y_t^{f,s}}{y_t^{f,s}+y_t^{-f,s}}\right)p_t^s.$$

The bracketed term is below one whenever the firm's market share is positive and shrinks toward one as $\sigma\to\infty$, in which case the firm becomes a price taker. The restriction $\sigma>1$ is standard in monopoly theory.

Combining the marginal revenue of output with the marginal cost of labor, the firm's optimal labor choice equates marginal revenue product to marginal cost:

$$\underbrace{\underbrace{p_t^s\left(1-\frac{1}{\sigma}\frac{y_t^{f,s}}{y_t^{f,s}+y_t^{-f,s}}\right)}_{\text{MR}}\frac{\partial y_t^s}{\partial N_t^{f,s}}}_{\text{MRP}} = \underbrace{\underbrace{p_t^i\frac{\partial y_t^i}{\partial N_t^i}}_{w_t^i}\underbrace{\left[1+\alpha\frac{N_t^{f,s}}{N_t^i}\right]}_{\mu_t}}_{\text{MC}}. \tag{15}$$

---

[13] For a general inferior-sector technology $G(K_t^i,N_t^i,A_t^i)$, the markdown takes the form $\mu_t^{f,s} = 1+(n_t^s/mn_t^i)\cdot[-G_{NN}/G_N]$; see Appendix C.

Equation (15) is the central first-order condition of the model. The left-hand side combines two ingredients: the marginal revenue from selling an additional unit of y_t^s (the markup-adjusted price) and the marginal product of labor in producing it. The right-hand side combines the wage paid (which equals the marginal product of labor in the inferior sector by the wage-arbitrage condition) and the markdown that reflects the firm's awareness of how its hiring affects the wage it must pay to all its workers. The two distortions are conceptually distinct: the markup on the left is an oligopoly distortion of the product market, while the markdown on the right is an oligopsony distortion of the labor market.

The choice of capital is influenced only by product-market power, since the capital market is competitive:

$$p_t^s \frac{\partial y_t^{f,s}}{\partial K_t^{f,s}} = \left(1 - \frac{1}{\sigma} \frac{y_t^{f,s}}{y_t^{f,s} + y_t^{-f,s}}\right) R_t^s. \tag{16}$$

For equations (15) and (16) to admit an interior solution, we need labor to enter production with strictly diminishing returns.

Assumption 1. *The elasticity of output to capital is strictly positive,* $\epsilon_{y_t^s, K_t^s} = \alpha \in (0,1)$.

Assumption 1 is more than a regularity condition: it is essential for the existence of monopsony power. Indeed, the key component of the assumption is that the inferior sector operates a technology with diminishing returns to labor, which gives rise to an upward sloping labor supply, giving bite to oligopsony power.

We focus on the symmetric Cournot–Nash equilibrium in which all m superstar firms make identical choices:

$$N_t^{f,s} = \overline{N}_t^s, \quad K_t^{f,s} = \overline{K}_t^s, \quad y_t^{f,s} = \overline{y}_t^s \quad \forall f, \tag{17}$$

with sectoral totals $N_t^s = m\overline{N}_t^s$, $K_t^s = m\overline{K}_t^s$, and $y_t^s = m\overline{y}_t^s$. Under symmetry, the rivals' aggregates simplify to $N_t^{-f,s} = (m-1)\overline{N}_t^s$ and $y_t^{-f,s} = (m-1)\overline{y}_t^s$, and the firm's market share in its sector is $y_t^{f,s}/y_t^s = 1/m$. Substituting into equations (15) and (16) yields the symmetric first-order conditions:

$$p_t^s \left(\frac{\sigma m - 1}{\sigma m}\right) \frac{\partial y_t^s}{\partial N_t^s} = p_t^i \frac{\partial y_t^i}{\partial N_t^i} \left[1 + \alpha \frac{N_t^s}{m N_t^i}\right], \tag{18}$$

$$p_t^s \left(\frac{\sigma m - 1}{\sigma m}\right) \frac{\partial y_t^s}{\partial K_t^s} = p_t^i \frac{\partial y_t^i}{\partial K_t^i}. \tag{19}$$

The term $\sigma m/(\sigma m - 1)$ is the constant oligopoly markup, which converges to one as either σ or m goes to infinity — recovering the competitive limit. Equation (18) can be written compactly as the product of markup and markdown,

$$\frac{p_t^s \, \partial y_t^s / \partial N_t^s}{w_t^s} = \underbrace{\frac{m\sigma}{m\sigma - 1}}_{\text{markup}} \underbrace{\left(1 + \frac{1}{\epsilon_{N_t^s, w_t^s}}\right)}_{\text{markdown}}, \tag{20}$$

making it transparent that the marginal product of labor in the superior sector exceeds the wage by a factor that combines both forms of market power. While our model with CES implies a constant markup of the superstar firms, one can potentially build models with oligopolistic markups that grow over time, as in Autor et al. (2020). Yet, as shown in Section 3, such models are inconsistent with a capital output ratio along the BGP, at the same time, because of the compositional shift, the aggregate markup of our economy does increase over time. While the markup depends on exogenous parameters of the model, the markdown depends on the endogenous ratio N_t^s/N_t^i, a mechanism that drives most of the results in the paper. The markdown also depends negatively on the exogenously set number of superstar firms m.

The inferior-sector firms equate the marginal revenue products of labor and capital to factor prices in standard fashion: $p_t^i(\partial y_t^i/\partial N_t^i) = w_t^i$ and $p_t^i(\partial y_t^i/\partial K_t^i) = R_t^i$. When both sectors face the same wage and rate of return (which holds in equilibrium when capital depreciation is symmetric across sectors), taking the ratio of the labor and capital first-order conditions across sectors yields the *fundamental factor allocation*:

$$\left[1 + \alpha \frac{N_t^s}{mN_t^i}\right] \frac{N_t^s}{K_t^s} = \frac{N_t^i}{K_t^i}. \tag{21}$$

The pure neoclassical benchmark, which would obtain in a fully competitive economy, would set $N_t^s/K_t^s = N_t^i/K_t^i$: the labor–capital ratio would be equalized across sectors. Equation (21) shows that whenever $\alpha > 0$, the superstar firms operate with a distorted factor mix — specifically, they use *too little* labor relative to capital when compared to the inferior sector. This intersectoral misallocation of factors is the central inefficiency generated by monopsony power, and it is what generates the consumption-equivalent loss we quantify in Section 6. The superstar firms earn strictly positive profits along the equilibrium path, which the household receives as a rebate (see Section 4.5).

4.5 Household Problem

The representative household owns all firms and supplies labor to both sectors. Given the symmetric equilibrium across superstar firms, it is without loss of generality to write the household problem in terms of a single asset position $\mathbb{A}_t^s$ in the superior sector (proportional to total capital $K_t^s = m\overline{K}_t^s$) and a single labor supply $n_t^s = \sum_{f=1}^m n_t^{f,s}$. The household takes as given wages w_t^s, w_t^i, rates of return r_t^s, r_t^i, and aggregate superstar profits $\Pi_t^s = \sum_{f=1}^m \Pi_t^{f,s}$, and solves

$$\begin{aligned}
\max_{\{C_t, \mathbb{A}_t^s, \mathbb{A}_t^i, n_t^s, n_t^i\}_{t\geq 0}} \quad & \int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right] \mathrm{d}t \\
\text{s.t.} \quad & C_t + \dot{\mathbb{A}}_t^s + \dot{\mathbb{A}}_t^i = w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \Pi_t^s \\
& n_t^s + n_t^i = h_t, \quad n_t^s, n_t^i \geq 0, \quad \mathbb{A}_t^s, \mathbb{A}_t^i \geq 0.
\end{aligned} \tag{22}$$

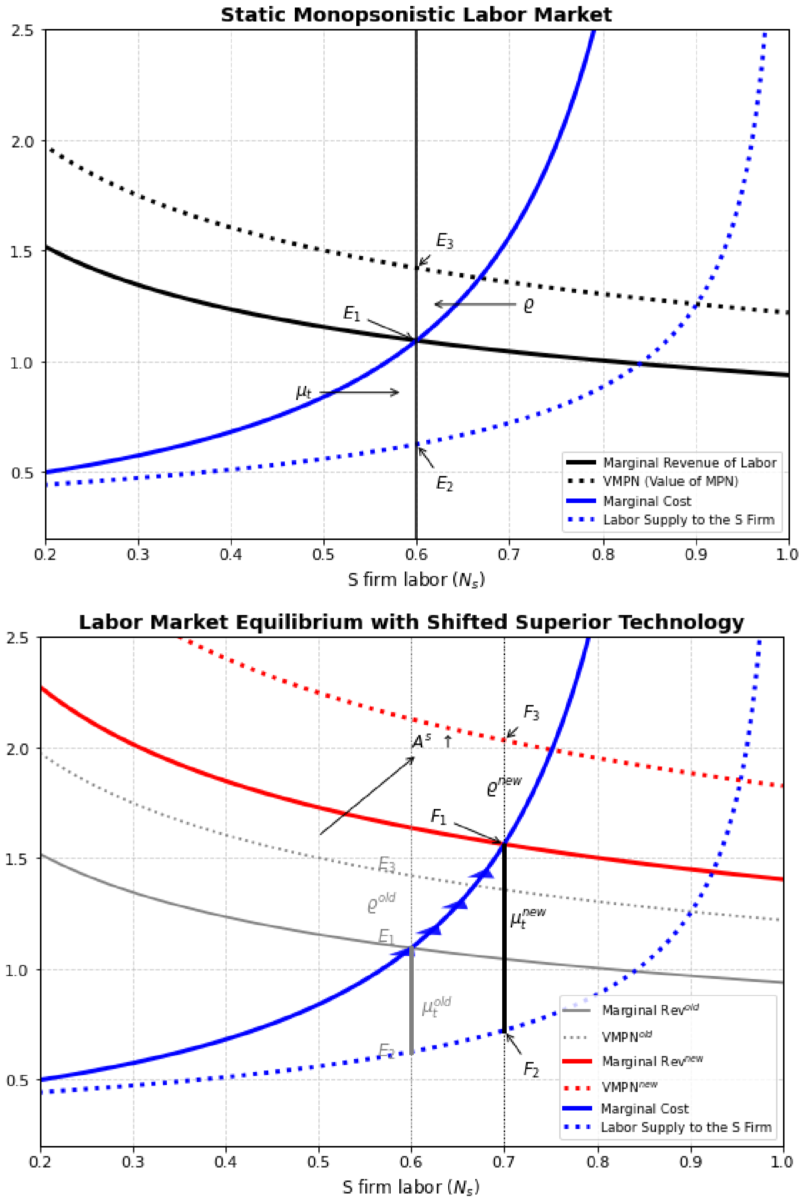
To ensure finite utility along the equilibrium path, we impose

Assumption 2. $-\rho + (1-\gamma)g^s < 0$.

The Hamiltonian is standard and is reported in Appendix B. The first-order conditions deliver an Euler equation, arbitrage conditions for labor and capital, and a transversality condition. The Euler equation reads

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma}, \tag{23}$$

Figure 3: Monopsonistic Labor Market with the Superior Technology



where the capital-arbitrage condition implies

$$r_t^s = r_t^i.$$

and we denote $r_t = r_t^s = r_t^i$. The intratemporal labor-supply conditions for each sector,

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^s, \tag{24}$$

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^i, \tag{25}$$

imply the *fundamental wage-arbitrage condition* $w_t = w_t^s = w_t^i$: along any equilibrium path, the wage paid by the oligopsonists must equal the wage in the inferior sector. The flow budget constraint

$$C_t + \dot{\mathbb{A}}_t^s + \dot{\mathbb{A}}_t^i = w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \Pi_t^s$$

and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} u'(C_t)\mathbb{A}_t^s = \lim_{t\to\infty} e^{-\rho t} u'(C_t)\mathbb{A}_t^i = 0$$

close the system.

4.6 Superstar Monopsonistic Equilibrium

Market clearing requires that asset holdings equal capital stocks net of depreciation, that labor supplied to each sector equal labor demanded, and that rates of return be equalized net of depreciation, so that

$$r_t = R_t^s - \delta^s = R_t^i - \delta^i, \tag{26}$$

$$\mathbb{A}_t^s = K_t^s, \quad \mathbb{A}_t^i = K_t^i, \quad n_t^s = N_t^s, \quad n_t^i = N_t^i, \quad h_t = H_t. \tag{27}$$

Definition 1. *Given a path for productivity* $\{A_t^s, A_t^i\}_{t=0}^{\infty}$ *and a number of superstar firms* m*, a* symmetric superstar monopsonistic equilibrium *(SSMPE) is a set of sequences for allocations, prices, and consumption such that: (i) the symmetric superstar firm's choices* $\overline{n}_t^s, \overline{K}_t^s, \overline{y}_t^s$ *solve the oligopolistic–oligopsonistic problem* (18)–(19)*; (ii) final-good prices and quantities satisfy* (5)–(7)*; (iii) the inferior firm's choices satisfy* (8)–(9)*; (iv) the household's allocations satisfy* (22), (23), *and* (24)–(25)*; (v) wages and rates of return satisfy the arbitrage conditions* (20), (16), (26), (12)*; (vi) all markets clear,* (27)*.*

The equilibrium can be reduced to a system of nine equations in nine unknowns; the full system is reported in Appendix B. The economic content of the model is captured by the equation that determines the intersectoral allocation of labor:

$$\frac{\sigma m - 1}{\sigma m} p_t^s (1-\alpha)(K_t^s)^{\alpha}(n_t^s)^{-\alpha}(A_t^s)^{1-\alpha} = p_t^i (1-\alpha)(K_t^i)^{\alpha}(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha}\left[1 + \frac{\alpha}{m}\frac{n_t^s}{n_t^i}\right]. \tag{28}$$

The left-hand side is the marginal revenue product of labor in the superior sector net of the constant oligopoly markup; the right-hand side is the marginal product of labor in the inferior sector scaled by the markdown. The Euler equation (23) and the standard CES, Cobb–Douglas, and market-clearing conditions close the system.

Since the two sectors feature unbalanced productivity growth, in the limit the superior sector absorbs all factors, and the model admits no finite balanced growth path. We therefore characterize the *asymptotic* BGP in which all variables grow at constant rates. To study the dynamics, we transform the system into intensive form by defining

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s}, \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i}, \quad c_t = \frac{C_t}{A_t^s n_t^s},$$

with details in Appendix B.

Definition 2. *An* (asymptotic) balanced growth path (BGP) *is an SSMPE along which aggregate consumption and GDP grow at constant rates.*

Proposition 1. *The SSMPE admits a unique asymptotic balanced growth path with the following features:*

1. *consumption and output per capita grow at the constant rate $g_C = g_Y = g_{y^s} > g_{y^i}$, with the superior sector absorbing the entire final-good production ($y_t^s/Y_t \to 1/\delta^{1-\sigma}$, $y_t^i/Y_t \to 0$);*
2. *the relative labor allocation in the superior sector grows indefinitely, $\frac{1}{m}\frac{n_t^s}{n_t^i} \to \infty$, but total hours grow at the constant rate $g_h = g_{n^s} > g_{n^i}$;*
3. *the growth rate of wages is strictly below the growth rate of output, $g_w < g_Y$;*
4. *the interest rate r_t and capital per efficiency unit in the superior sector converge to constants ($r_t^s \to r$, $x_t^s \to x^s$), while $x_t^i \to \infty$.*

The asymptotic price of the superior intermediary converges to a constant ($p_t^s \to \delta^{\frac{1}{\sigma-1}}$, $g_{p^s} \to 0$), while the inferior price grows at a constant rate.

The key analytical result is that the markdown grows without bound along the BGP.

Proposition 2. *The markdown of the superstar firms has a constant asymptotic growth rate*

$$g_\mu = g_{n^s} - g_{n^i}.$$

The proof follows directly from differentiating equation (14).[14] Reducing the asymptotic system to a linear system in growth rates (zeroing $g_{p^s} = g_{x^s} = 0$ and imposing $g_C = g_Y$, $g_h = g_{n^s}$; see

---

[14] Specifically, $\frac{\dot{\mu}_t}{\mu_t} = \frac{\alpha(g_{n^s}(t) - g_{n^i}(t))}{(1/m)(n_t^i/n_t^s) + \alpha}$, which converges to $g_{n^s} - g_{n^i}$ as $n_t^i/n_t^s \to 0$. The bottom panel of Figure 3 provides graphical intuition.

Appendix B.1) yields the central analytical results of the paper:

$$g_h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s - \frac{1}{\frac{1}{\theta}+\gamma} \cdot \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i), \tag{29}$$

$$g_C = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma} g^s - \frac{1}{\frac{1}{\theta}+\gamma} \cdot \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i), \tag{30}$$

$$g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i). \tag{31}$$

The SSMPE is a semi-endogenous growth model: equilibrium growth depends on the exogenous productivity growth differential $g^s - g^i$, but interacts with all structural parameters of the model. To gauge the effect of monopsony power, we next compare these results with those of the corresponding competitive economy.

4.7 The Competitive Economy

In the competitive benchmark, all firms are price takers in product markets and wage takers in labor markets. The household problem and the inferior sector are unchanged. The superior sector now operates competitively, so superstar profits vanish ($\Pi_t^s = 0$). We denote competitive growth rates with an asterisk.

The competitive economy is a decentralized version of the unbalanced growth equilibrium of Acemoglu and Guerrieri (2008). The system of equilibrium conditions coincides with that of the SSMPE except for the intersectoral allocation equation, which loses both the markup and the markdown:

$$p_t^s(1-\alpha)(K_t^s)^\alpha(n_t^s)^{-\alpha}(A_t^s)^{1-\alpha} = p_t^i(1-\alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha}. \tag{32}$$

The full system in growth rates is reported in Appendix B.1, and yields the optimal growth rates

$$g_C^* = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma} g^s, \quad g_h^* = g_{n^s}^* = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s, \quad g_{n^s}^* - g_{n^i}^* = (1-\alpha)(\sigma-1)(g^s - g^i). \tag{33}$$

Proposition 3. *The competitive economy admits a unique asymptotic BGP with the same qualitative features as Proposition 1, with one notable exception: the growth rate of wages coincides with the growth rate of output,* $g_w = g_Y$*.*

Comparing equations (29)–(31) with (33), the two distortions in equation (28) relative to (32)—the markup and the markdown—play asymmetric roles. The constant oligopoly markup affects only the level of the steady-state capital–efficiency ratio x^s and has no impact on growth rates, while the rising markdown μ_t depresses the growth rates of both hours and consumption. We formalize and quantify this misallocation in the next subsection.

4.8 Growth Misallocation and Consumption Equivalent Loss

We can formally show that imperfect labor markets induce negative growth effects in the SSMPE. The following proposition summarizes the main result.

Proposition 4. *The balanced growth path of the SSMPE is inefficient: both output per capita and total hours grow more slowly than in the corresponding optimal growth problem.*

The proof is immediate from comparing the balanced growth path for hours and consumption in the decentralized monopsonistic equilibrium of equations (29) and (30) with the optimal ones in equation (33). The growth expressions can be rewritten as

$$g_C = g_C^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu \ , \quad g_h = g_h^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu \ , \tag{34}$$

where $g_\mu = g_{n^s} - g_{n^i} > 0$ as reported in equation (31). Equation (34) immediately shows that, since $g_\mu > 0$ along the decentralized equilibrium, monopsony power distorts both hours and consumption growth, generating a "growth drag" ($g_h < g_h^*$ and $g_C < g_C^*$). The growth rates of both consumption and total hours are thus negatively affected by the growth of the markdown, and imperfect labor markets introduce an additional negative channel in the long-run dynamics of labor supply.

The total growth misallocation depends on the preference parameters γ and θ as well as on the technological parameters α and σ. We first discuss the role of the technological parameters. Assumption 1 highlighted that diminishing returns to capital—and thus $0 < \alpha < 1$—is an essential prerequisite for the existence of monopsony. In addition, equation (34) shows that misallocative effects on output are present as long as $\alpha < 1$, that is, as long as labor has a productive role in the superior technology. The role of σ operates through its effect on the growth of the markdown g_μ: a higher value of σ implies higher markdown growth ($\partial g_\mu / \partial \sigma > 0$). The parameter σ in the model has a dual role, since it reflects both the elasticity of substitution between the two intermediaries and the oligopoly markup of the superstar firms. The model requires $\sigma > 1/m$, since the markup is $\varrho = \frac{\sigma m}{\sigma m - 1}$. A higher σ implies a lower markup and lower oligopoly power. Yet one can easily establish that there is no *direct* growth effect of monopoly power in the model: a simple monopoly model without monopsony yields $g_C^m = g_C^*$, where g_C^m is the growth rate of consumption in the version of the model with only monopoly power in the product market and no monopsony power in the labor market. In the SSMPE, however, a higher σ implies lower growth through an *indirect* channel that operates via the markdown μ_t. The mechanism is the following. A higher σ implies higher substitutability between the goods produced in the superior and inferior sectors. Higher substitutability translates into faster growth of the markdown g_μ and thus larger monopsony power, which in turn leads to higher overall distortion and lower growth.

An Alternative Explanation for Declining Hours

The preference parameters θ and γ also play an important role in the growth process, and in particular in the associated long-run decline in hours per person and hours per worker. In the recent literature surveyed in Section 2, much of the attention has been devoted to the income effect of long-run wage increases. Boppart and Krusell (2020) propose a class of preferences coherent with the observed long-run decline in hours worked along a balanced growth path. With respect to the MaCurdy (1981) preferences specified in our model, the Boppart and Krusell (2020) class requires the elasticity of marginal utility of consumption to be strictly larger than one (i.e. $\gamma > 1$). In other words, $\gamma > 1$ is a necessary condition for obtaining a neoclassical growth model with declining labor supply, under the assumption of competitive labor markets. This is apparent from equation (33), in which the growth rate of hours is zero when $\gamma = 1$.

In the theory of labor supply, the MaCurdy (1981) utility function with $\gamma = 1$ implies a set of preferences in which the income and substitution effects of wage changes cancel each other out, as discussed by King et al. (1988). The SSMPE delivers an important result in this respect, summarized in the following proposition.

Proposition 5. *In the asymptotic balanced growth path of the SSMPE, the growth rate of hours is negative even when the income and substitution effects of wage changes cancel each other out (i.e.* $\gamma = 1$*).*

To see this, set $\gamma = 1$ in the monopsonistic growth rate of hours to obtain

$$g_h = -\frac{1}{\frac{1}{\theta}+1}\frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) < 0 \ .$$

The result is driven by the additional income effect generated by returns to capital and the profit rebate Π_t^s. Prescott (2004) used a closely related static mechanism to explain steady-state differences in hours worked across the two sides of the Atlantic. Our result can be seen as the dynamic, growth-theoretic counterpart of that classic labor-supply result.

Labor Share Dynamics

The model also has qualitative implications for the second macro fact outlined in Section 2, namely the dynamics of the aggregate labor share. In the competitive equilibrium, the labor share is time-invariant and equals

$$\alpha_{L,t}^* = \frac{w_t^{*s} n_t^{*s} + w_t^{*i} n_t^{*i}}{Y_t^*} = 1 - \alpha \ ,$$

where the optimal decentralized wages w_t^{*s} and w_t^{*i} coincide with the marginal product of labor in each sector. There is thus no labor share dynamics in the optimal growth problem. By contrast, in the SSMPE the labor share is time-varying and its level depends on the state of the economy. The decentralized labor share is

$$\alpha_{L,t} = \frac{w_t^s n_t^s + w_t^i n_t^i}{Y_t} \ ,$$

and Proposition 1 delivers the following result for its dynamics.

Proposition 6. *The growth rate of the labor share is negative in the SSMPE, and the capital share (which includes profits) tends to one.*

The result follows directly from the fact that, along the asymptotic balanced growth path, the growth rate of the labor share equals the negative of the growth rate of the markdown:

$$\frac{\dot{\alpha}_L}{\alpha_L} = -g_\mu < 0 \ .$$

The Role of Market Concentration

Market concentration in the model has a dual interpretation: it can describe either the market share of the top firms within the superior sector or the share of the superior sector in the total economy. The superior sector contains m symmetric firms that grow faster than the inferior firms and eventually absorb the entire production of the economy. With respect to the asymptotic growth rates, changing the number of firms m has no *growth* effect, as is evident from equation (30). However, the number of firms m does have a key *level* effect on the economy.

In this context, an increase in market concentration within the superior sector can be described as a reduction in m. Formally, the SSMPE can be defined for a given path $\{m_t\}_{t=0}^{\infty}$, and one way to model rising concentration within sector s is through an exogenous reduction over time in the number of firms. Such an increase in concentration implies various level effects along the transitional dynamics, as highlighted in the quantitative section.

The alternative interpretation of market concentration relates to the size of the superior sector within the economy. Since the firms in sector s are superstars, the competitive fringe represents ordinary firms. Under this interpretation, market concentration can be measured as the share of sales (or employment) of the top z firms in the economy. The time-t concentration of the first z firms (with $z \leq m$) reads

$$\text{conc}_t^z = \frac{p_t^s z \overline{y}_t^s}{Y_t^s} = \frac{z}{m}\frac{p_t^s y_t^s}{Y_t^s} = \frac{z}{m}\left(1 - \frac{p_t^i y_t^i}{Y_t}\right) \ , \qquad \lim_{t\to\infty} \text{conc}_t^z = \frac{z}{m} \ .$$

Concentration rises endogenously along the transitional dynamics as the superior sector grows to dominate the economy and the value of production from the inferior sector becomes negligible. In Section 6.2 we exploit this alternative interpretation to derive accounting implications of our theory.

Consumption Equivalent Loss

To estimate the misallocation effects of the superstar model, we perform a Lucas-style consumption-equivalent-loss (CEL) exercise. The exercise rests on the numerical simulation of the model and is carried out in Section 6; here we describe the logic. We simulate a path of length T of the competitive economy and denote the resulting sequences by $\{c_t^*, h_t^*\}_{t=0}^T$. Similarly, the path of the monopsonistic equilibrium is $\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T$. The corresponding utility level is

$$\tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T) = \sum_{t=0}^{T} \beta^t \left[\frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} - \psi \frac{\tilde{h}_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1}\right] \ ,$$

where $\beta = \frac{1}{1+\rho dt}$ is the discount factor and dt is the length of the period. We approximate TFP growth over a period dt as $A_{t+1}^s - A_t^s \approx dt\, g^s A_t^s$. The consumption equivalent loss asks what fraction λ of the competitive consumption path the representative agent would be willing to give up in order to obtain the same utility as the misallocated economy. Formally, we seek a λ such that

$$\sum_{t=0}^{T} \beta^t \left[\frac{[(1-\lambda)c_t^*]^{(1-\gamma)}}{1-\gamma} - \psi \frac{(h_t^*)^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1}\right] = \tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^T) < U(\{c_t^*, h_t^*\}_{t=0}^T) \ .$$

The results are reported in the numerical Section 6.

4.9 Stability of the SSMPE

We now investigate whether the SSMPE approaches the BGP. We focus on allocations in the neighborhood of the BGP, thus investigating only local (saddle-path) stability. To prove stability, we cannot use the formulation of the system above with x_t^s and x_t^i: since capital per unit of efficiency is unbounded ($x_t^i \to \infty$), we cannot write a Jacobian for the dynamical system evaluated at the BGP. We thus introduce variables capturing the share of capital and labor in the two sectors, in line with Acemoglu and Guerrieri (2008),

$$K_t = K_t^s + K_t^i \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad x_t = \frac{K_t}{A_t^s h_t} \ , \quad c_t = \frac{C_t}{A_t^s h_t} \ .$$

Along the BGP, κ_t, ν_t, and h_t all converge to zero, while x_t and c_t converge to positive finite values $\bar{x}$ and $\bar{c}$. We can study the behavior of the system in a neighborhood of these values. For the purposes of the proof, the production functions of the intermediaries are Cobb–Douglas and the depreciation rates are symmetric ($\delta^s = \delta^i = \delta$). The full system of equations is presented in Appendix D.

Theorem 1. *If capital depreciates at the same rate in both sectors ($\delta^s = \delta^i$), the SSMPE exhibits local saddle-path stability.*

The details of the proof are in Appendix D. The key idea is to rewrite the system of equations as

$$\dot{z} = \Phi(z),$$

where $z = (c, h, \nu, x)$ and Φ is the transition of the dynamical system. Four equations is the minimum number of equations we can reduce to: the Euler equation, the aggregate resource constraint, and the consumption–leisure tradeoff together with the monopsonistic labor supply, both differentiated with respect to time. We can substitute out κ since the equation stemming from equality of the rates of return to capital is time-invariant; for the opposite reason, we cannot dispense with the consumption–leisure and labor-demand equations, since they are time-dependent. Because we have endogenous labor supply, we end up with one more equation than Acemoglu and Guerrieri (2008). In our setting, we have three predetermined variables: h_0, κ_0, and ν_0. One of these, κ_0, follows naturally from $K_0 = K_0^s + K_0^i$ being given. The two remaining predetermined values arise because two of our differential equations come from differentiating intertemporal constraints: once C_0 is chosen, these equations determine h_0 and ν_0. We therefore need to show, first, that the BGP is hyperbolic—that is, that all eigenvalues of the Jacobian of the transition evaluated at the BGP have non-zero real part—and, second, that three eigenvalues are negative while one is positive.

We show that the determinant of the Jacobian of the transition evaluated at the BGP equals the determinant of the following matrix (where $*$ denotes a finite, non-zero constant):

$$\hat{J}_\Phi = \begin{pmatrix} 0 & 0 & * & \frac{\bar{c}}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha(\alpha-1)\bar{x}^{\alpha-2} \\ * & g_h & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * \end{pmatrix},$$

where asterisks denote finite, possibly non-zero values. We then have

$$\det J_\Phi = \det \hat{J}_\Phi = -\left(-\frac{1}{1+\theta\alpha}\right) g_h (g_{n^i} - g_{n^s}) \frac{c}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha(\alpha-1)x^{\alpha-2} < 0 \ ,$$

so the BGP is hyperbolic. Since the determinant is negative, either one or three eigenvalues are negative. We then compute the characteristic polynomial of the Jacobian, whose roots are the eigenvalues. Letting λ denote the variable of the polynomial and $*$ again denote a finite, non-zero constant, we find

$$P(\lambda) = \det(J_\Phi - \lambda I) = \begin{pmatrix} * - \lambda & 0 & * & * \\ * & g_h - \lambda & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} - \lambda & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * - \lambda \end{pmatrix} .$$

It is then straightforward to verify that

$$P(\lambda) = (g_h - \lambda)(g_{n^i} - g_{n^s} - \lambda) \det \begin{pmatrix} * - \lambda & * \\ -\frac{1}{1+\theta\alpha} & * - \lambda \end{pmatrix} .$$

Since both g_h and $g_{n^i} - g_{n^s}$ are negative, two of the eigenvalues must be negative. The only configuration consistent with a negative determinant of the Jacobian is then that three eigenvalues are negative and one positive, which establishes stability.

4.10 Implications of Different Capital Intensities

In our baseline model, the results are driven by unbalanced TFP growth between the superior superstar sector and the inferior competitive sector. Autor et al. (2020) provide evidence on the productivity growth differential between superstar and ordinary firms.

Our analysis, however, suggests that an alternative mechanism may also be at work, namely different capital intensities between superstar and fringe firms. Indeed, Autor et al. (2020) also document higher capital intensity in superstar firms. While the model and the SSMPE were solved under the assumption of symmetric capital intensity across sectors ($\alpha^i = \alpha^s$) with asymmetric labor-augmenting technological progress, the model can equally be solved under the alternative case of $\alpha^s > \alpha^i$ with Hicks-neutral TFP growth, as in Acemoglu and Guerrieri (2008). Under Hicks-neutral technological progress, all the growth results presented above hold as long as

$$\frac{g^s}{1-\alpha^s} > \frac{g^i}{1-\alpha^i} .$$

Hence, to obtain a model with declining labor share, declining hours worked, and a growth drag, it is sufficient to assume $g^s = g^i$ together with $\alpha^s > \alpha^i$.

5 A Worker-Capitalist Model and Policy

One concern related to the decline of the labor share is the rise in inequality across individuals and the shifting distribution of income across segments of the population. In a representative agent (RA) framework such mechanisms are muted: while payments shift from labor to profits, the RA owns both labor and capital, and receives all capital income in the form of interest and dividends. As discussed in Section 4.8, from the labor-supply perspective the rebate of profits to the RA generates an additional income effect beyond that of a pure wage increase. This is one of the primary sources of growth misallocation in the SSMPE, closely related to the classic labor-supply effect of public spending highlighted by Prescott (2004).

This section explores how an economy populated by separate *workers* and *capitalists* changes the predictions of our model. Workers are the only suppliers of labor; they cannot save and consume their labor income period by period. Capitalists do not work but save in a safe asset that gives them full claims to capital and profits, so that they earn both the marginal product of capital and oligopoly rents. The setup is in the spirit of Straub and Werning (2020), who build on the classic paper by Judd (1985) on optimal taxation.

The decentralized worker-capitalist equilibrium. There is a unit measure of workers and a unit measure of capitalists. Workers solve a static problem period by period:

$$\max_{C_t^w, h_t} \frac{(C_t^w)^{1-\gamma}-1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} , \qquad \text{s.t.} \quad C_t^w = w_t^s n_t^s + w_t^i n_t^i \ , \quad n_t^s + n_t^i = h_t \ ,$$

where the budget constraint shows that workers finance period-t consumption C_t^w exclusively out of labor income from the two sectors. The first-order conditions deliver a standard consumption-leisure tradeoff and the wage-arbitrage condition,

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} w_t \ , \qquad w_t^{f,s} = w_t^i = w_t \quad \text{for all } f \in \{1, \ldots, m\}.$$

Capitalists derive utility from consumption only and solve a standard forward-looking problem

$$\max_{\{C_t^c\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{(C_t^c)^{1-\gamma}-1}{1-\gamma} \, dt \ , \qquad \text{s.t.} \quad C_t^c + \dot{\mathbb{A}}_t^s + \dot{\mathbb{A}}_t^i = r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \Pi_t^s,$$

which delivers a standard Euler equation and the return-equalization condition,

$$\gamma \frac{\dot{C}_t^c}{C_t^c} = r_t - \rho \ , \qquad r_t^{f,s} = r_t^i = r_t \quad \text{for all } f.$$

The production side of the economy is identical to the SSMPE of Section 4, with the same symmetric Cournot equilibrium $K_t^s = m\overline{K}_t^s$, $n_t^s = m\overline{n}_t^s$, $y_t^s = m\overline{y}_t^s$. The equilibrium system of nine ODEs is therefore essentially identical to that of the representative-agent SSMPE, with one substitution: the household Euler equation (23) is replaced by the workers' consumption-leisure condition and budget constraint,

$$C_t^w = w_t h_t, \qquad \psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} w_t,$$

and the capitalists' Euler equation now governs capital accumulation. The full intensive-form system is presented in Appendix B.2.

Because the production side of the economy is unchanged, the growth rate of the markdown, the growth of the relative labor allocation, and the growth of the inferior intermediate price all coincide with the RA solution of Section 4.6:

$$g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \ , \qquad g_{p^i} = 2\frac{1-\alpha}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \ .$$

The growth rate of wages is likewise unchanged from the RA monopsonistic case, since it depends only on the inferior price and parameters: $g_w = g^s - g_\mu < g^s$. What differs from the SSMPE is the equilibrium growth rate of labor supply. Since workers finance consumption entirely out of labor income and do not receive a profit rebate, the growth of C_t^w is proportional to the growth of wages, and the equilibrium growth rate of hours reads

$$g_h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g_w = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}\left[g^s - \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i)\right] . \tag{35}$$

Capitalists, who earn both the rate of return on capital and oligopoly profits, see their consumption grow at $g_{C^c} = g^s + g_h$, while workers' consumption grows only at $g_{C^w} = g_w + g_h$. Since $g_w < g^s$, the divergence is permanent.

Proposition 7. *In the balanced growth equilibrium of the worker-capitalist economy, consumption growth is unbalanced between the two types of agents and, asymptotically, capitalists absorb the entire share of aggregate consumption.*

The result follows immediately from $g_{C^c} - g_{C^w} = g^s - g_w = g_\mu > 0$, so that $C_t^c/(C_t^w + C_t^c) \to 1$.

The utilitarian efficient benchmark. To evaluate the welfare properties of the decentralized worker-capitalist equilibrium, we introduce a utilitarian planner who assigns a welfare weight $\omega > 0$ to capitalists.[15] The planner solves

$$\max_{\{C_t^c, C_t^w, h_t\}_{t=0}^\infty} \int_0^\infty e^{-\rho t}\left[\frac{(C_t^w)^{1-\gamma}-1}{1-\gamma} - \psi\frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \omega\frac{(C_t^c)^{1-\gamma}-1}{1-\gamma}\right] dt$$

subject to the aggregate resource constraint, the CES aggregator, and the time constraint $n_t^s + n_t^i = h_t$. The planner uses the two CRS technologies directly and abstracts from the number of firms in the superior sector. The first-order conditions imply that the two agents consume a constant fraction of output, $\omega^{1/\gamma} C_t^{*w} = C_t^{*c}$, and that the consumption-leisure tradeoff takes the standard neoclassical form $\psi(h_t^*)^{1/\theta} = (C_t^{*w})^{-\gamma}\partial Y_t/\partial h_t$. The optimal growth rates therefore read

$$g_h^* = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s , \qquad g_{C^w}^* = g_{C^c}^* = g^s + g_h^* = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma} g^s , \tag{36}$$

which coincide exactly with the optimal growth rates of the representative-agent model. Two conclusions follow. First, the inequality result is dramatic: while the decentralized monopsonistic market implies ever-increasing consumption inequality between the two groups, the utilitarian planner delivers balanced consumption growth with a constant relative weight determined by ω. Second, comparing the labor-supply rates in equations (35) and (36) reveals a striking and counterintuitive result.

Proposition 8. *In the decentralized worker-capitalist monopsonistic economy, the growth rate of labor supply differs from the utilitarian optimum as follows:*

---

[15] We assume the planner cannot make capitalists work. This affects the levels but not the growth rates of the solution.

1. *If preferences are in the Boppart and Krusell (2020) class ($\gamma > 1$), workers supply* more *hours than is optimal: $g_h > g_h^*$.*
2. *If preferences satisfy the King et al. (1988) restriction ($\gamma = 1$), hours are constant in both the efficient and decentralized monopsonistic equilibrium.*

Algebraically, the deviations from the optimum can be expressed compactly as

$$g_h = g_h^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu \ , \quad g_{C^c} = g_{C^c}^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu \ , \quad g_{C^w} = g_{C^w}^* - \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g_\mu \ . \tag{37}$$

Proposition 8 highlights the role of profits in shaping labor supply. In the SSMPE with representative agents, the rebate of profits to the household generated an additional income effect that depressed hours growth *below* the optimal level. In the worker-capitalist economy this channel is shut down: workers decide their labor supply on the basis of labor income alone and never see the profits. The result is that workers now supply *more* hours than is socially optimal — the labor-supply distortion has flipped sign. This experiment confirms that the labor-supply channel emphasized in Section 4.8 is indeed driven by the profit-rebate income effect, and constitutes the long-run growth counterpart of Prescott (2004).

Redistributive taxation. The analysis of the worker-capitalist economy raises a natural policy question: can a redistributive tax restore the efficient allocation? Consider a proportional tax τ on capitalists' total income Ω_t^c, whose proceeds are rebated to workers. Capitalists' total income equals output net of labor payments and capital depreciation,

$$\Omega_t^c = Y_t - w_t h_t - \delta^s K_t^s - \delta^i K_t^i,$$

which combines returns to capital and oligopoly profits. The only modification to the model is the workers' budget constraint, which becomes

$$C_t^w = (1 - \tau) w_t h_t + \tau \big(Y_t - \delta^s K_t^s - \delta^i K_t^i \big). \tag{38}$$

Equation (38) implies that, asymptotically, workers consume a constant fraction τ of output net of depreciation.[16] Since the production side is unchanged, the growth rates of the markdown, the inferior price, and the wage are identical to the no-tax case. The transfer of profits to workers, however, reintroduces the additional income effect that was active in the representative agent SSMPE. The resulting growth rates with the tax read

$$g_h^\tau = g_h^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu, \qquad g_{C^w}^\tau = g_{C^c}^\tau = g_C^* - \frac{1}{\frac{1}{\theta} + \gamma} g_\mu. \tag{39}$$

Comparing equation (39) with the no-tax outcome of equation (37) yields a clear picture: the tax pushes the growth rate of hours from *above* the optimum (Proposition 8) to *below* the optimum, mirroring the SSMPE distortion. The tax thus eliminates the growth inequality between workers and capitalists at the cost of overshooting the efficient labor supply.

---

[16] By Walras's law, capitalists' consumption is pinned down by the aggregate resource constraint and does not enter the system of equations explicitly.

Proposition 9. *A proportional tax on capitalists' income, rebated to workers:*

1. *eliminates the growth inequality between workers' and capitalists' consumption;*
2. *depresses the growth rate of hours and total output below the utilitarian optimum, reintroducing the labor-supply distortion of the representative-agent SSMPE.*

A tax levied on profits only would have the same qualitative effects, since asymptotically the marginal product of labor accrues to profits.

Minimum wage. In the presence of monopsony, a binding minimum wage can in principle restore efficient outcomes, in the spirit of the classic Robinson (1969) argument. We show that along the balanced growth path of the worker-capitalist economy, a minimum wage that grows at the rate g^s achieves the efficient growth rates of hours and consumption.

To formalize the argument, consider an exogenously specified minimum wage $\bar{w}_t$. The inferior sector and the household problem are unchanged. Each superstar firm chooses labor, capital, and the wage to maximize profits subject to $W_t^{f,s} \geq \bar{w}_t$; the firm's problem and the conditions under which the minimum wage is binding are reported in Appendix B.2.[17] When the minimum wage binds, each superstar firm sets $w_t^{f,s} = \bar{w}_t$, so that the firm becomes effectively a pure oligopolist: the wage is exogenously given and the firm has no scope for monopsonistic markdown.

The consumption-leisure condition of workers under a binding minimum wage reads

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} \bar{w}_t.$$

Combining this condition with the utilitarian growth rates from equation (36), the minimum wage growth rate that delivers the efficient allocation is

$$g_{\bar{w}}^* = \frac{1}{\theta} g_h^* + \gamma g_{C^w}^* = g^s.$$

Proposition 10. *A minimum wage growing at the rate of superior-sector productivity, $g_{\bar{w}}^* = g^s$, achieves the utilitarian-efficient growth rates of hours worked and consumption in the worker-capitalist monopsonistic economy.*

Two remarks are in order. First, although the minimum wage achieves the efficient growth rate of hours, it actually *depresses* hours growth relative to the no-policy decentralized equilibrium of Proposition 8. This is a direct consequence of the fact that, in the worker-capitalist economy, monopsony causes workers to *work too much* rather than too little — the conventional intuition of monopsony reducing labor input is inverted here because the profit-rebate channel is shut down. Second, the result for the worker-capitalist economy contrasts with what a minimum wage would do in the representative agent SSMPE of Section 4. In the RA case, where workers also receive profits, a growing minimum wage would deliver the classic Robinson (1969) outcome: it would *raise* both the growth of labor supply and the growth of wages relative to the decentralized monopsonistic equilibrium.

---

[17] The minimum wage need not be binding at every date: when the inferior-sector marginal product of labor exceeds $\bar{w}_t$ even at $N_t^i = H_t$, the constraint is slack. For the analysis below we focus on the case in which $\bar{w}_t$ binds.

6 Quantifying the Growth Misallocation and the Consumption Equivalent Loss

We are now in a position to quantify the misallocative effects of the distortions induced by market power in the superior sector. We perform four quantitative exercises. First, in Section 6.1 we calibrate the asymptotic BGP of a "representative advanced economy" and show that only the SSMPE can simultaneously match the three key growth moments: GDP growth, hours growth, and labor share growth. Second, we estimate the size of the income effect required to match the decline in hours that can be attributed to misallocation. Third, we quantify the asymptotic growth drag between the SSMPE and the competitive benchmark. Fourth, in Section 6.2, we focus on the US economy and solve numerically for the transition along the asymptotic path. While Section 6.1 relies on the version of the model with capital, Section 6.2 confronts the well-known instability problems that affect shooting algorithms in growth models with capital and forward-looking consumption (Judd, 1998; Miranda and Fackler, 2002).[18] In our model these problems are exacerbated by the complexity of the joint labor and capital allocation across the two sectors. To circumvent the instability, we calibrate a version of the model in which capital is replaced by a fixed factor of production, which can be interpreted as "materials" or "land". In this version there are no savings and consumption absorbs period-t output; the model can be solved and simulated for any exogenous productivity path $\{A_t^s, A_t^i\}_{t=0}^T$. The details of the model and the few differences with respect to the full model with capital are reported in Appendix E. The simulations in Section 6.2 refer to the model with the fixed factor.

6.1 Asymptotic Calibration and Growth Drag

In the first quantitative exercise, we assume that the "representative advanced economy" features a long-run growth behavior summarized by the three moments reported in Section 2, and moves along the asymptotic superstar equilibrium defined by the model. The three long-run facts concern growth in GDP per capita, (negative) growth in hours worked, and (negative) growth of the labor share. Table 2, in the column labeled "data", reports the target moments for both GDP per capita and GDP per worker. The values correspond to the rounded averages of the empirical moments for the period 1980–2023.

We denote the empirical estimates of the three per-capita moments by $\hat{g}_C^{pc}$, $\hat{g}_h^{pc}$, and $\hat{g}_{\alpha_L}^{pc}$, and the corresponding per-worker moments by $\hat{g}_C^{pw}$, $\hat{g}_{n^s}^{pw}$, and $\hat{g}_{\alpha_L}^{pw}$. The quantitative goal is to find values of the parameters γ, g^s, θ that match the three theoretical moments derived in Section 4.6. We take θ from the literature (λ-constant elasticity of labor supply from Chetty et al., 2011), so two parameters remain: g^s and γ.

Consider first the monopsonistic SSMPE. For given θ and the three empirical moments, the

---

[18] The shooting algorithm is unstable because small errors in guessing initial conditions are magnified through the integration of the differential equations, leading to slow or failed convergence.

theoretical growth rates derived in Section 4.6 can be inverted to back out $\hat{\gamma}$ and $\hat{g}^s$ in closed form:

$$\hat{\gamma}^j = 1 - \frac{1}{1+\theta}\frac{\hat{g}_h^j}{\hat{g}_C^j} + \frac{\hat{g}_{\alpha_L}^j}{\hat{g}_C^j}, \qquad j = pc, pw \tag{40}$$

$$\hat{g}^{s,j} = \frac{\hat{g}_h^j\left(\frac{1}{\theta} + \hat{\gamma}^j\right)}{1 - \hat{\gamma}^j}, \qquad j = pc, pw \tag{41}$$

Finally, the SSMPE implies that the asymptotic growth rate of the markdown is the negative of the asymptotic growth rate of the labor share,

$$\hat{g}_\mu = -\hat{g}_{\alpha_L}, \tag{42}$$

which is independent of whether one uses per-capita or per-worker data. Given the closed-form expressions (40)–(42), the SSMPE matches the three moments exactly in both cases, as confirmed in Table 2.

We now turn to the column labeled "NGM Model" in Table 2. Equations (40) and (41) can still be used to match GDP and hours growth, but the growth rate of the labor share cannot be matched, because the NGM implies a constant asymptotic labor share equal to $1 - \alpha$ with our calibration $\hat{\alpha} = 0.33$, as outlined in Section 4.8. Although the elasticity of output to capital is in principle a free parameter, the implied growth rate of the labor share is zero regardless. The bottom line of this first exercise is therefore clear: to match the three growth moments of Table 2, it is not sufficient to work with preferences in which the income effect of wage growth dominates the substitution effect, as in Boppart and Krusell (2020). A model with a permanently declining labor share, such as the SSMPE, is necessary.

Decomposing the income effect. Table 2 also performs an additional exercise concerning the income effect and the size of γ in the MaCurdy (1981) preferences. Boppart and Krusell (2020) show that, within an NGM, matching the declining trend of labor supply requires $\gamma > 1$, and indeed the calibrated $\hat{\gamma}^j$ in Table 2 is always greater than one. The larger the downward trend of hours worked, the larger the income effect needed. In the SSMPE, however, profits are rebated to the representative consumer, and this profit channel generates an additional income effect that depresses hours worked — the dynamic counterpart of the Prescott (2004) static result. Since the SSMPE has a built-in profit channel arising from market imperfections, it relies less on the preference parameter γ to match the observed decline in hours. We measure the extra income effect attributable to misallocation as

$$\Delta IE = \left| \frac{\gamma^{NGM,j} - \gamma^{SSMPE,j}}{\gamma^{NGM,j}} \right|, \qquad j = pc, pw,$$

where $\gamma^{NGM,j}$ and $\gamma^{SSMPE,j}$ are the values of γ calibrated in the two models. In the definition of ΔIE, the value $\gamma = 1$ corresponds to the traditional preferences used in the business-cycle literature since King et al. (1988), under which hours are constant. Table 2 shows that the additional income effect attributable to misallocation accounts for 16 percent of the calibrated γ in the case of hours per worker, and as much as 81 percent in the case of hours per person.

The growth drag. The third quantitative exercise linked to the asymptotic equilibrium estimates the size of the "growth drag" due to market imperfections. The SSMPE delivers a growth drag for both GDP and hours, the analytical expressions for which were derived in Section 4.8. Table 3 quantifies this drag using the calibrated values from Table 2. Oligopsony implies an asymptotic loss of 5 percent of GDP growth in the per-capita case and 10 percent in the per-worker case. The growth rate of hours per worker is roughly 8 percent lower than in the competitive economy, while the growth rate of hours per person falls from -0.02 percent to -0.1 percent — a fivefold increase in proportional terms.

Table 2: Calibration of Asymptotic Balanced Growth Path

	Model SSMPE[a]		*NGM Model with MaCurdy preferences*[b]		*Data*	
	per capita	per worker	per capita	per worker	per capita	per worker
Moments						
GDP growth	0.0150	0.0169	0.0150	0.0169	0.0150	0.0169
Growth of Hours	-0.0010	-0.0050	-0.0010	-0.0050	-0.0010	-0.0050
Growth Labor Share	-0.0011	-0.0011	0.0000	0.0000	-0.0011	-0.0011
Parameters[c]						
γ	1.017	1.335	1.090	1.400		
g^s	0.016	0.022	0.016	0.022		
θ	2.84	2.84	2.84	2.84		
Share of Income Effect due to Misallocation (percentage points)[d]						
	81.350	16.270				

[a] , SSMPE is the calibrated asymptotic Superstar Monopsonistic Equilibrium.

[b] , in NGM there are no market imperfections and the labor share is constant, so we calibrate GDP and hours growth only.

[b] , a special case of Boppart and Krusell (2020).

[c] , γ is the elasticity of the marginal utility of consumption in MaCurdy (1981) preferences.

[c] , g^s is the growth rate of the superior technology.

[c] , θ is the elasticity of the marginal disutility of labor in MaCurdy (1981) preferences.

[d] , The relative increase in income effect due to γ between a Boppart and Krusell (2020) economy and the model SSMPE.

Source: Authors' calculations.

6.2 Calibration of the Growth Transition in the US and Consumption Equivalent Loss

In the fourth quantitative exercise we focus on the US economy and solve numerically for the transition along the asymptotic path. The ultimate goal of this section is to quantify the consumption equivalent loss for the US induced by the growth misallocation of the SSMPE. To address the numerical instability discussed at the beginning of the section, we calibrate a version of the model with a fixed factor of production, interpreted as "materials".[19] The logic of the exercise is to calibrate the model to relevant features of the US economy at the beginning of the 21st century, and then simulate the dynamic transition toward the asymptotic SSMPE. Before discussing the details of the calibration, we perform an accounting exercise that delivers a value of the markdown $\hat{\mu}_0$ to be used as a benchmark.

---

[19] The equations of the model with a fixed factor are reported in Appendix E.

Table 3: Asymptotic Growth Drag

	Model SSMPE[a]		*Competitive Economy*[b]		*Data*	
	per capita	Per Person	per capita	Per Person	per capita	Per Person
Moments						
Gdp growth	0.0150	0.0169	0.0159	0.0188	0.0150	0.0169
Growth of Hours	-0.0010	-0.0050	-0.0002	-0.0047	-0.0010	-0.0050
Growth Labor Share	-0.0011	-0.0011	0.0000	0.0000	-0.0011	-0.0011
Parameters[b]						
γ	1.017	1.335	1.017	1.335		
g^s	0.016	0.022	0.016	0.022		
θ	2.84	2.84	2.84	2.84		
Growth Drag[c]						
Gdp Growth	-5.47	-10.09				
Hours	406.96[e]	7.39				

[a], SSMPE Is the asymptotic Superstar Monopsonistic Equilibrium.

[b], Optimal Competitive Growth with the parameters of the SSMPE.

[c] , γ is the elasticity of the marginal utility of consumption in MaCurdy (1981) preferences.

[c] , θ is the elasticity of the marginal disutility of labor in MaCurdy (1981) preferences.

[c] , g^s is the growth rate of the superior technology.

[d] , Growth Drag is the loss in asymptotic growth rate between SSMPE and Optimal Growth.

[e] , The large per-capita value reflects the fact that hours per capita grow at a near-zero rate in the competitive economy (-0.02%); in the SSMPE the same rate falls to -0.10%, a fivefold proportional increase from a near-zero base.

Source: Authors' calculations.

Accounting for Rising Markdown

The key driver of our growth theory is the dynamics of the markdown. The dynamics of hours and of the labor share ultimately depend on the dynamics of μ_t, which in turn depend on the ratio of employment in the superior sector to employment in the inferior sector, as in equation (14). Denoting by $\hat{n}_t^{s,j}$ and $\hat{n}_t^{i,j}$ the employment shares of the two sectors in some industry j, the markdown at date t in that industry can be estimated as

$$\hat{\mu}_t^j = 1 + \alpha \frac{\hat{n}_t^{i,j}}{m\hat{n}_t^{s,j}} \ ,$$

where m is the number of firms in the oligopsonistic market.

We use the recent evidence on superstar firms to calibrate the dynamics of $\hat{\mu}_t^j$ across industries. Autor et al. (2020) estimate time trends for the concentration ratios of the top superstar firms in each industry, both for the top 4 firms (CR4) and for the top 20 firms (CR20), based on employment data. We treat $\hat{n}_t^{s,j}$ as the superstar employment share in industry j and use $\hat{n}^{i,j} = 1 - \hat{n}^{s,j}$ for the competitive fringe. To compute the implied markdown we then need values for α and m. In Figure 4 and Table 4 we use $\alpha = 0.2$, consistent with the calibration of the transitional dynamics in the next subsection.

The results of the accounting exercise are reported in Table 4 and Figure 4 for six major industries: Retail Trade, Services, Utilities and Transportation, Wholesale Trade, Finance, and Manufacturing. The data are from Autor et al. (2020) and cover the period from 1982 to roughly 2010, with the exact time span varying by sector. In Figure 4, the left y-axis refers to estimates based on CR20 and the right y-axis to CR4. While the levels differ across industries, the figure reveals an unambiguous upward trend in the estimated markdown in all major industries, with the partial exception of manufacturing. The average level at the beginning of the period is approximately 1.017. For manufacturing, the estimate is V-shaped, but concentration has been clearly rising since the begin-

Table 4: Markdown Estimates and Average Growth by Industry

Industry	$CR4^a_{2002}$	$CR20^a_{2002}$	Growth CR4 (%)	Growth CR20 (%)	Period^b
Finance	1.020	1.011	0.030	0.014	1982-2002
Retail Trade	1.021	1.007	0.056	0.019	1982-2002
Services	1.007	1.003	0.005	0.002	1982-2012
Utilities & Transportation	nan	1.016	0.067	0.034	1982-1997
Wholesale	1.010	1.004	0.017	0.006	1982-2007
Manufacturing	1.028	1.018	-0.002	-0.003	1982-2007
Average	1.017	1.010	0.029	0.012	

[a] , $CR4_{2002}$ and $CR20_{2002}$ refer to the employment concentration ratio for C4 and C20 respectively.

[b] , Each industry has a different sample period as indicated in the last column.

Source: Authors' calculations and Autor et al. (2020).

ning of the century, in line with the trend estimates of Yeh et al. (2022). Although the exercise is purely accounting-based, the estimates in Figure 4 are clearly consistent with the central driver of the SSMPE.

Table 4 reports $\hat{\mu}^j_{2002}$ for the six industries together with the average value, which we use as the benchmark for the US moments in Table 5. The columns labeled "Growth CR4" and "Growth CR20" report the average growth rate of the markdown implied by the accounting exercise. Notably, the order of magnitude is comparable to that of the growth rate of the labor share of the US reported in Table 1, particularly when using the CR20 measure.

US Calibration and Consumption Equivalent Loss

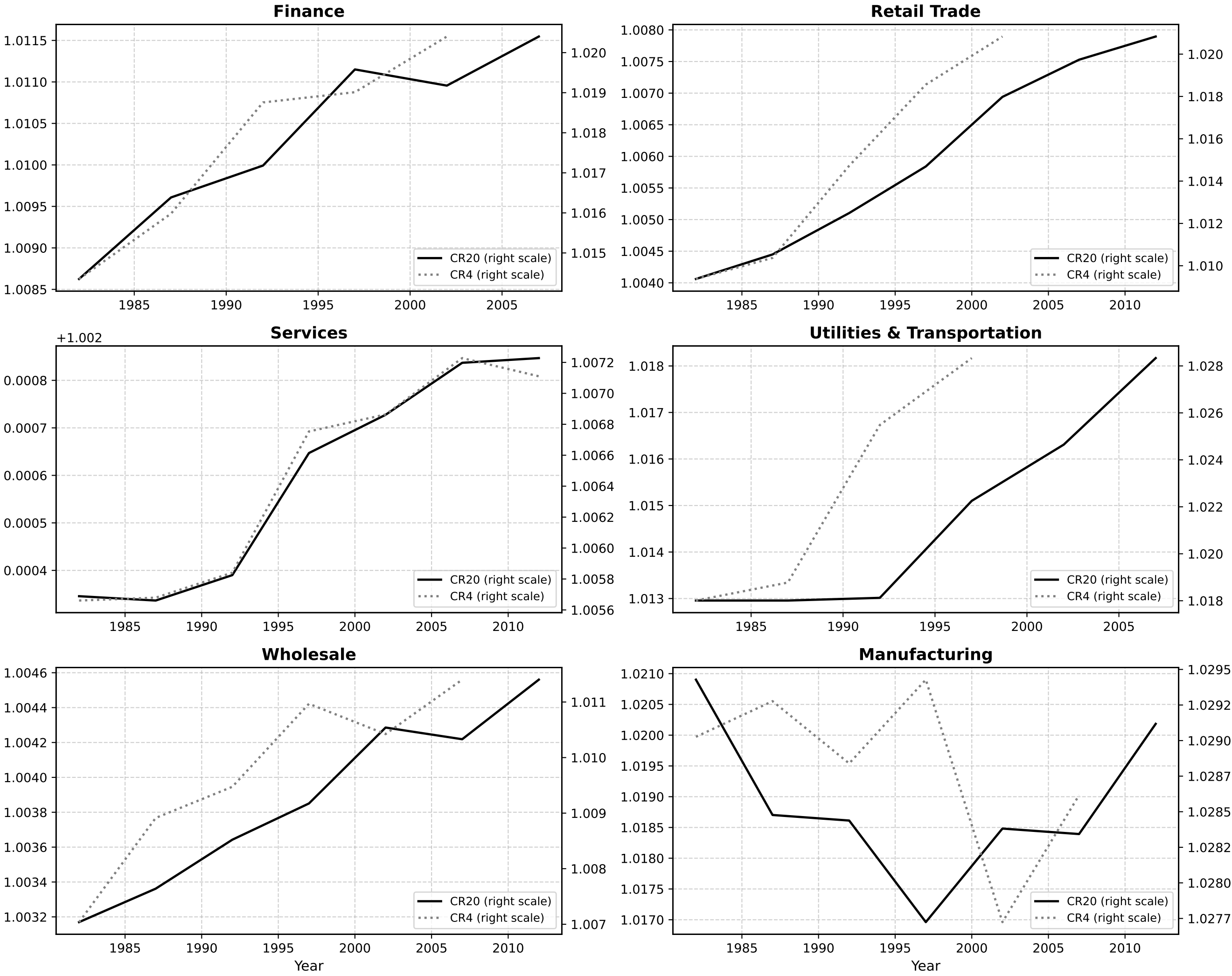
Calibrating the transitional dynamics for the US economy requires four steps.

Step 1: asymptotic moments. We assume that the US economy features asymptotic growth moments consistent with the model. The empirical moments $\hat{g}_C^{j,us}, \hat{g}_h^{j,us}, \hat{g}_{\alpha_L}^{j,us}$ describe the asymptotic behavior of the economy, and the calibrated US economy converges to such a stable asymptotic path for given initial conditions. Following the logic of Table 6, this assumption pins down θ, γ, g^s, and g^i. Table 6 shows that the calibrated $\gamma^{US,pc}$ is below one in absolute value and lower than that of the representative advanced economy. This is unsurprising given the slight upward trend of hours worked per capita in the US. For hours per worker, Table 6 delivers $\gamma^{US,pw} > 1$.

Step 2: parameters from the literature. The remaining parameters in the second panel of Table 6 are set as follows. The combination of σ and m determines the markup $\sigma m/(\sigma m - 1)$. We target an initial markup of 1.30 from Autor et al. (2020). Although m is a natural number in the model, we treat it as a continuous measure of concentration for flexibility, fixing $m = 1.25$ and backing out the corresponding σ. The discount rate ρ is set to a standard value yielding a yearly long-run interest rate, and the initial level of inferior-sector technology is normalized to $A^i = 1$. The parameter α is particularly important for the transitional dynamics. With the standard benchmark $\alpha = 0.33$, the speed of convergence to the asymptotic SSMPE is far too slow. In the model with fixed materials, however, the elasticity of output with respect to the fixed factor need not coincide with the elasticity of output to capital. We set $\alpha = 0.2$, which delivers a reasonable transition speed.

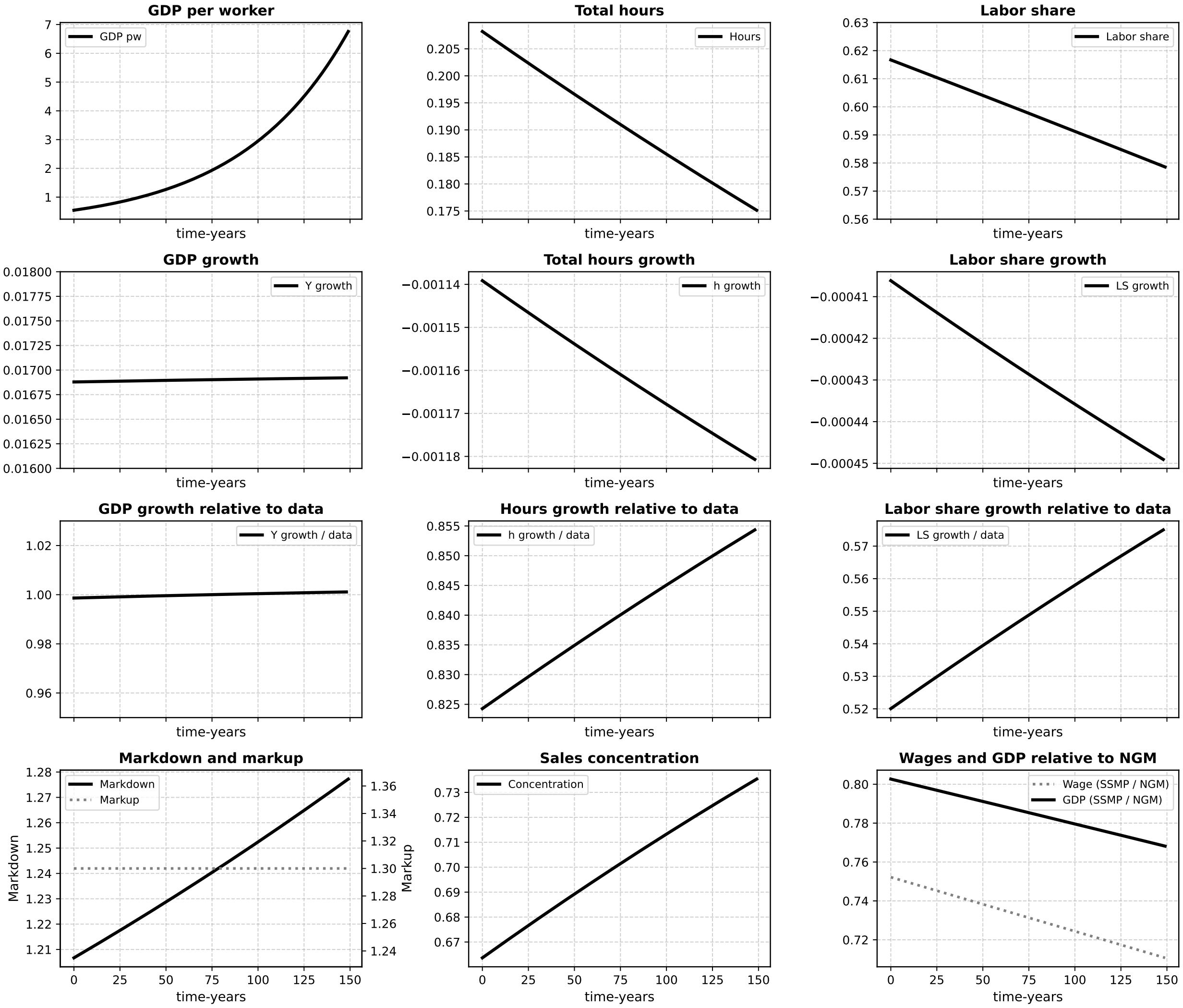
Step 3: identification of the remaining parameters. The core of the calibration concerns the four remaining parameters A^s, ψ, T, ζ. Ideally, we would match four time-zero moments of the US economy: the average markdown $\hat{\mu}_0$, the level of the labor share (Karabarbounis, 2024) $\hat{\alpha}_{L0}$, the level of hours worked $\hat{h}_0$, and the concentration ratio of the superior sector (Autor et al., 2020) $\hat{C}_{m0}$.

Figure 4: Markdown Estimates from Concentration Ratio Across Six Main US Sectors



The panels report our estimates of the markdown by sector from the concentration data in Autor et al. (2020). CR20 refers to the employment of the top 20 firms, C4 to the employment of the top 4 firms. The markdown is computed according to $\mu = 1 + \alpha \frac{n_t^s}{n_t^i}$

Figure 5: Transitional Dynamics of the Calibrated US Economy



However, the analytics of the model imply that these moments are not independent. In particular, the equilibrium of the model delivers

$$\hat{\mu}_0 = 1 + \frac{\alpha}{m}\frac{n_0^s}{n_0^i}$$

$$\frac{\sigma m}{\sigma m - 1}\frac{n^s}{n^i}\left(1 + \frac{\alpha}{m}\frac{n^s}{n^i}\right) = \frac{\hat{C}_{m0}}{1 - \hat{C}_{m0}}$$

$$\frac{\sigma m}{\sigma m - 1}\frac{n^s}{n^i}\left(1 + \frac{\alpha}{m}\frac{n^s}{n^i}\right) + 1 = \frac{(1-\alpha)}{\hat{\alpha}_{L0}}\left(1 + \frac{n^s}{n^i}\right) .$$

The three conditions show that only one of the three moments — markdown, labor share, and concentration — can be matched independently. Consistent with the focus of the paper on the declining labor share (Karabarbounis, 2024), we calibrate $\hat{\alpha}_{L0}$ to the level of the US labor share in 2002 and let the implied levels of $\hat{\mu}_0$ and $\hat{C}_{m0}$ be determined endogenously.

Step 4: identification of (A^s, ψ, T, ζ). The four parameters A^s, ψ, T, ζ are not separately identified: only two nonlinear combinations can be pinned down from data, namely

$$\text{(i)} \quad \frac{\zeta}{1-\zeta}\left(\frac{A^s}{A^i}\right)^{\sigma-1}, \qquad \text{(ii)} \quad \frac{T^{\alpha(1-\gamma)}}{\psi},$$

with details in the appendix. The final step is therefore to choose A^s and ψ for given T and ζ such that, through combinations (i) and (ii) and the equations above, the model exactly matches initial hours worked $\hat{h}_0$ and the initial labor share $\hat{\alpha}_{L0}$.

Once these four steps are completed, the model is fully calibrated. One might worry that since some objects — such as the level of output at time zero — depend on the exact values of A^s, ψ, T, ζ rather than on their identifiable combinations, the consumption equivalent loss might not be well-defined. The next proposition shows that this concern does not arise.

Proposition 11. *Given combinations of* ζ *and* A^s, *and of* T *and* ψ, *that reproduce the moments from the data (labor share and hours worked), the consumption equivalent loss can be computed and does not depend on the individual values of these parameters — only on the combinations that can be identified from the data.*

The proposition follows from the fact that the consumption equivalent loss compares utility in the monopsonistic and competitive economies, so that levels — such as the level of output — cancel out.

Results. The calibration is run over 150 years and the results are plotted in Figure 5. The model replicates the three growth facts during the transition: the calibrated economy exhibits positive GDP growth alongside negative growth in both hours and the labor share. The match is exact in the asymptotic equilibrium and gradually improves along the transition. GDP growth matches the actual historical growth rate already during the transition. The growth rates of total hours and of the labor share are smaller in absolute value at the start of the transition, accounting for approximately 80 percent and 50 percent of the empirical trends, respectively. Figure 5 also reports the dynamics of the implied moments. As anticipated, to deliver reasonable convergence the model requires the value of α used in Table 6, which implies an initial markdown about 20 percent above that obtained from the accounting exercise of Table 4. Similarly, the simulated economy features a concentration ratio higher than the simple CR4 estimate of Autor et al. (2020). The final panel of Figure 5 reports the wage and GDP levels relative to the first-best competitive economy: both gaps widen over time, motivating the consumption equivalent loss calculation that follows.

The consumption equivalent loss (CEL), derived analytically in Section 4.8, is reported in Table 7. Overall, the growth misallocation of the SSMPE implies a CEL of approximately 7.6 percent. The result is essentially identical when calibrating to hours per worker or hours per person, and is robust to the time horizon of the simulation.

Level versus growth effects of concentration. As a final exercise, we simulate the transitional dynamics under an unanticipated shock that reduces the concentration of the superior sector by 20 percent.[20] Figure 6 reports the resulting dynamics. The markdown-and-markup panel displays the one-off increase induced by the fall in m. The main message of Figure 6 concerns the distinction — a central one in growth theory since Lucas (1988) — between level and growth effects: rising concentration produces marked negative *level* effects on the labor share and hours worked, but essentially no *growth* effects, as visible in the growth dynamics panels.

Table 5: Calibrated Moments of Monopsonistic Material-Labor Growth Path - US

Moment	Data		Model	
	per capita	per worker	per capita	per worker
Growth moments				
GDP growth	0.0148	0.0169	0.0148	0.0169
Total hours growth	-0.0002	-0.0014	-0.0002	-0.0014
Labor share growth	-0.0008		-0.0008	
Time 0 moments				
Moments from the literature				
λ-constant elasticity of l.s.	2.8400		2.8400	
Monopolistic markup	1.3000		1.3000	
Matched moments				
Total hours worked	0.0954	0.2145	0.0954	0.2145
Labor share	0.6167		0.6167	
Implied moments				
Concentration[a]	0.2750		0.6478	
Monopolistic markdown[b]	1.0170		1.1902	

[a] , C4 concentration ratio from Autor et al. (2020).

[b] , Authors' calculations from Section 6.2.

Source: Authors' calculations.

7 Conclusion

Long-run dynamics of labor supply and imperfect labor markets are becoming more relevant in macroeconomics and in growth theory. Models of growth should address the long-run decline in hours per worker and the fall in the labor share observed in most advanced economies. This paper takes first steps in this direction, proposing and solving a standard growth model with declining hours worked and declining labor share driven by oligopsonistic power by superstar firms. In a

---

[20] Formally, we re-run the transition of Figure 5 assuming an unexpected mid-path reduction in m.

Figure 6: Dynamic Simulation of Calibrated Economy with Rising Concentration

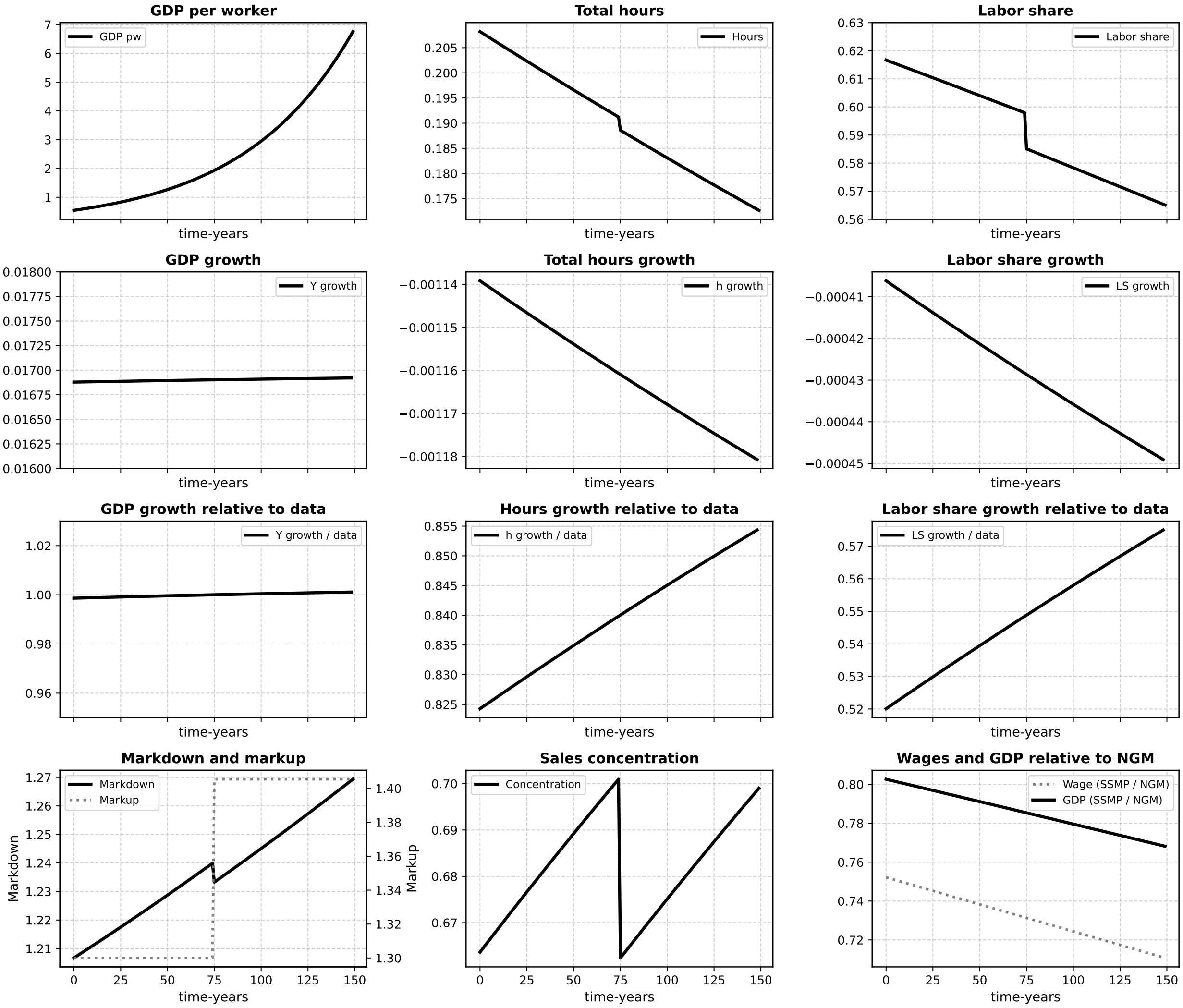


Table 6: Calibrated Parameters of Monopsonistic Material-Labor Growth Path - US

Parameter	Notation	per capita	per worker
Parameters from the literature and normalized			
Discount rate	ρ	0.0500	0.0500
Frisch elasticity	θ	2.8400	2.8400
Cobb-Douglas technology	α	0.2000	0.2000
CES share	ζ	0.1250	0.1250
Fixed materials supply	T	1.0000	1.0000
Time 0 productivity, inferior	A^i	1.0000	1.0000
Number of superstar firms	m	1.2500	1.2500
Calibrated parameters			
Utility curvature	γ	0.9697	1.0643
Productivity growth, superior	g^s	0.0150	0.0180
Productivity growth, inferior	g^i	0.0137	0.0167
CES elasticity	σ	3.4667	3.4667
Time 0 productivity, superior	A^s	4.2106	4.2106
Disutility of work	ψ	14.2265	5.1457

Source: Authors' calculations.

Table 7: Consumption equivalent loss in percentage terms - US

Time periods	per capita	per worker
100	7.69	7.60
500	7.69	7.60
1000	7.69	7.60

Discrete time approximation with $dt = 1$.

Source: Authors' calculations.

broader perspective, the paper calls for expanding the intersection between growth theory and labor economics. Market power by firms, well established in economics of growth, should be extended to factor markets, incorporating empirically relevant features of real life firms. This research avenue is likely to uncover novel and interesting policy implications. The paper studied the growth and distributive effects of proportional taxation and rising minimum wage in economies with declining hours worked and labor share. In such economies, both policies can be welfare improving.

Much remains to be done. Once monopsonistic power by firms is recognized as part of growth theory, it should be studied how its existence shapes incentives for innovation and technology adoption. Imperfect labor markets have implications also for the empirics of growth. Almost seventy years ago, Solow (1957) introduced growth accounting under the assumption of perfect labor and capital markets. In his remarkable and influential analytical derivation, Solow argued that the empirical exercise should be extended to the case of imperfect labor markets. These concerns should now be taken seriously by national and international statistical offices.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth.* Princeton University Press.

Acemoglu, D. (2025). Capital and wages. *International Economic Review 66(1)*, 3–24.

Acemoglu, D. and V. Guerrieri (2008). Capital deepening and nonbalanced economic growth. *Journal of Political Economy 116(3)*, 467–498.

Acemoglu, D. and P. Rastrepo (2022). Tasks, automation and the rise in wage inequality. *Econometrica 90(5)*, 1973–2016.

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60(2)*, 323–351.

Aghion, P. and P. Howitt (1994). Growth and unemployment. *The Review of Economic Studies 61(3)*, 477–494.

Akcigit, U. and S. T. Ates (2021). Ten facts on declining business dynamism and lessons from endogenous growth theory. *American Economic Journal: Macroeconomics 13*(1), 257–298.

Akcigit, U. and S. T. Ates (2023). What happened to US business dynamism? *Journal of Political Economy 131*(8), 2059–2124.

Atkenson, A. (2020). Alternative facts regarding the labor share. *The Quarterly Journal of Economics 37(1)*, 167–180.

Autor, D., D. Dorn, L. F. Katz, and J. Van Reneen (2020). The fall of the labor share and the rise of superstar firms. *Quarterly Journal of Economics 135(2)*, 645–709.

Ball, L. and N. G. Mankiw (2023). Market power in neoclassical growth models. *The Review of Economic Studies 90*(2), 572–596.

Barr, T. and U. Roy (2008). The effect of labor market monopsony on economic growth. *Journal of Macroeconomics 30*(4), 1446–1467.

Berger, D., K. herkenhoff, and S. Mongey (2022). Labor market power. *American Economic Review 112(4)*, 1147–1193.

Bergholt, D., F. Furlanetto, and M.-F. Nicolo (2022). The decline of the labor share: New empirical evidence. *American Economic Journal: Macroeconomics 14(3)*, 163–98.

Boppart, T. and P. Krusell (2020). Labor supply in the past, present, and future: A balanced-growth perspective. *Journal of Political Economy 128(1)*, 118–157.

Brooks, W. J., J. P. Kaboski, and Y. A. Li (2021). Agglomeration, misallocation, and (the lack of) competition. *American Economic Journal: Macroeconomics 13*(4), 483–519.

Cass, D. (1965). Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies 32(3)*, 233–240.

Cette, G., L. Koehl, and T. Philippon (2019). The labor share in the long term: A decline? *Economie et Statistique / Economics and Statistics 510–511*, 35–51.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review: Papers and Proceedings 101*(3), 471–475.

Deb, S., J. Eeckout, A. Pate, and W. Lawrence (2022). What drives wage stagnation: monopsony or monopoly? *Journal of the European Economic Asociation 20(6)*, 2181–2225.

Dixit, A. K. and J. E. Stiglitz (1977, June). Monopolistic competition and optimum product diversity. *American Economic Review 67*(3), 297–308.

Feenstra, R., R. Inklaar, and M. Timmer (2015). The next generation of the penn world table. *American Economic Review 105(10)*, 3150–3182.

Hsieh, C., J. Hurst, C. I. Jones, and P. Klenow (2022). The allocation of talent and U.S. economic growth. *Econometrica 87(5)*, 1439–174.

Hsieh, C. and P. Klenow (2009). Misallocation and manufacturing TFP in China and India. *Quarterly Journal of Economics 87(5)*, 1403–48.

Hsieh, C. and E. Rossi-Hansberg (2023). The industrial revolution in services. *Journal of Political Economy Macroeconomics 28*, 3–40.

Huberman, M. and M. Chris (2007). The times they are not changing': Days and hours work in old and new worlds, 1870-2000. *Explorations Economic History 38(3)*, 538–67.

Jones, B. F. and X. Liu (2024). A framework for economic growth with capital-embodied technical change. *American Economic Review 114*(5), 1448–1487.

Jones, C. I. (2016). The facts of economic growth. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2A, pp. 3–69. Elsevier.

Jones, C. I. (2022). The past and future of economic growth: a semi-endogenous pperspective. *Annual Reviews in Economics 14*, 125–152.

Judd, K. J. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics 28*, 59–83.

Judd, K. L. (1998). *Numerical Methods in Economics.* Cambridge, MA: MIT Press. See Chapter 10, especially Section 10.4 on shooting methods and saddle-path instability.

Karabarbounis, L. (2024). Perspectives on the labor share. *Journal of Econmomic Perspectives 38(2)*, 107–136.

Karabarbounis, L. and B. Neiman (2014). The global decline of the labor share. *The Quarterly Journal of Economics 129(3)*, 61–103.

Kaymak, B. and I. Schott (2023). Corporate tax decline in the manufacturing labor share. *Econometrica 91(6)*, 2371–2408.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988). Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics 21(2/3)*, 195–232.

Kongsamut, P., S. Rebelo, and X. Danyang (2001). Beyond balanced growth. *The Review of Economic Studies 68(4)*, 869–882.

Koopmans, T. C. (1965). On the concept of optimal economic growth. In J. Johansen (Ed.), *The Econometric Approach to Development Planning*, pp. 225–287. Amsterdam: North-Holland. Reprinted in *Scientific Papers of Tjalling C. Koopmans*, Vol. II, 1970, North-Holland.

Kwon, S., Y. Ma, and K. Zimmerman (2024). 100 years of rising corporate concentration. *American Economic Review 114 (4)*, 2111–40.

Lucas, R. J. (1988). On the mechanics of economics development. *Journal of Monetary Economics 22(1)*, 3–42.

MaCurdy, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of Political Economy 89*(6), 1059–1085.

Manning, A. (2021). Monopsony in labor markets: A review. *ILR Review 74(1)*, 3–26.

Martellini, P. and G. Menzio (2020). Declining search frictions, unemployment, and growth. *Journal of Political Economy 128*(12), 4387–4437.

Miranda, M. J. and P. L. Fackler (2002). *Applied Computational Economics and Finance.* Cambridge, MA: MIT Press. Chapters 17–18 discuss instability of boundary value problems and recommend reverse-time or relaxation methods.

Mortensen, D. T. and C. A. Pissarides (1998). Technological progress, job creation, and job destruction. *Review of Economic Dynamics 1*(4), 733–753.

Ngai, R. L. and C. A. Pissarides (2007). Structural change in a multisector model of growth. *American Economic Review 97(1)*, 429–443.

Prescott, E. C. (1986). Theory ahead of business-cycle measurement. *Carnagie-Rochester Conference Series on Public Policy 25*, 11–44.

Prescott, E. C. (2004). Why do Americans work so much more than Europeans? *Quarterly Review 28*(1), 2–13.

Ramey, V. A. and N. Francis (2009). A century of work and leisure. *American Economic Journal: Macroeconomics 1(2)*, 189–224.

Restuccia, D. and R. Rogerson (2008). Policy distortions and aggregate productivity with heterogeneous agents. *Review of Economic Dynamics 87(5)*, 707–20.

Robinson, J. (1969). *The Economics of Imperfect Competition.* London: Macmillan. First published 1933.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy 98*(5, Part 2), S71–S102.

Solow, R. (1957). Technical change and the aggregate production function. *Review of Economics and Statistics 39(3)*, 312–20.

Straub, L. and I. Werning (2020). Positive long-run capital taxation: Chamley-Judd revisited. *American Economic Review 110(1)*, 86–119.

Yeh, C., C. Macaluso, and H. Brad (2022). Monopsony in the US labor market. *American Economic Review 112(7)*, 2099–2138.

A NGM with Markups and Markdowns

This appendix provides a minimal microfounded version of the one-sector neoclassical growth model of Section 3. The reduced-form markup ϱ_t and markdown μ_t that enter the equations of Section 3 are obtained as equilibrium objects of a model featuring monopolistic competition over differentiated intermediate varieties (Dixit and Stiglitz, 1977) and monopsonistic competition in the labor market associated with differentiated labor (Berger et al., 2022; Deb et al., 2022). The derivation grounds the labor share and Euler equation of Section 3 in explicit firm optimization.

Consider a model in which a single homogeneous final good Y_t is produced competitively by aggregating a continuum of intermediate varieties $j \in [0,1]$ according to the CES technology

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y_{j,t}$ is the output of variety j and $\varepsilon > 1$ is the elasticity of substitution across varieties. The price of the final good is normalized to one. Profit maximization of the final-good producer yields the standard isoelastic demand for each variety,

$$y_{j,t} = p_{j,t}^{-\varepsilon} Y_t, \qquad 1 = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{43}$$

Each variety j is produced by a single firm using capital $K_{j,t}$, labor $N_{j,t}$, and labor-augmenting technology A_t, with a neoclassical constant-returns-to-scale production function

$$y_{j,t} = F(K_{j,t}, A_t N_{j,t}), \tag{44}$$

identical across varieties. Productivity A_t grows at the constant rate g. Capital is supplied competitively at the common rental rate R_t, and depreciates at rate δ.

A representative household has preferences

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right] dt. \tag{45}$$

We assume that jobs at different varieties are imperfect substitutes from the worker's perspective. Total hours are aggregated through the CES index

$$N_t = \left(\int_0^1 N_{j,t}^{\frac{\eta+1}{\eta}} \, dj \right)^{\frac{\eta}{\eta+1}},$$

with $\eta > 0$ the firm-level elasticity of labor supply. Cost minimization by the household across varieties then yields the firm-level labor-supply schedule and aggregate wage index

$$N_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{\eta} N_t, \qquad w_t = \left(\int_0^1 w_{j,t}^{1+\eta} \, dj \right)^{\frac{1}{1+\eta}}, \tag{46}$$

and the wage bill aggregates as $\int_0^1 w_{j,t}N_{j,t}\,dj = w_tN_t$.

A typical intermediate firm j chooses capital, labor, and the wage to maximize period-t profits, taking as given the rental rate R_t, the aggregate wage index w_t and the aggregate final-good output Y_t. The firm maximizes

$$\max_{K_{j,t},N_{j,t},w_{j,t},p_{j,t}} p_{j,t}y_{j,t} - w_{j,t}N_{j,t} - R_tK_{j,t}$$

subject to (44), (43), and (46). The firm optimally sets the wage to bind the labor-supply constraint with equality. The first-order conditions for labor and capital then deliver

$$p_{j,t}\left(1-\frac{1}{\varepsilon}\right)F_N(K_{j,t},A_tN_{j,t})A_t = w_{j,t}\left(1+\frac{1}{\eta}\right), \tag{47}$$

$$p_{j,t}\left(1-\frac{1}{\varepsilon}\right)F_K(K_{j,t},A_tN_{j,t}) = R_t. \tag{48}$$

The bracketed term on the left of equation (47) is the monopoly markdown of marginal revenue over price; the bracketed term on the right is the monopsony markdown of marginal cost of labor over the wage. Both ε and η are structural parameters: in order to generate rising markups or markdowns in view of the discussion in Section 3, in the remainder of this appendix the we treat the elasticities as time-varying and denote them by ε_t and η_t.

We define

$$\varrho_t \equiv \frac{\varepsilon_t}{\varepsilon_t - 1}, \qquad \mu_t \equiv 1 + \frac{1}{\eta_t},$$

where the markup ϱ_t is the price markup over marginal cost and the markdown μ_t is the ratio of the marginal product of labor to the wage. With this notation, equations (47) and (48) become

$$F_N(K_{j,t},A_tN_{j,t})A_t = \varrho_t\mu_tw_{j,t},$$

$$F_K(K_{j,t},A_tN_{j,t}) = \varrho_tR_t. \tag{49}$$

These are exactly the reduced-form factor-pricing conditions assumed in Section 3. Note in particular that the markdown μ_t appears only in the labor-side equation, while the markup ϱ_t appears in both. This is the source of the asymmetry between the two distortions.

By symmetry across varieties, we have $K_{j,t} = K_t$, $N_{j,t} = N_t$, $y_{j,t} = y_t$, $p_{j,t} = 1$, and $w_{j,t} = w_t$ for all j. Aggregation yields $Y_t = y_t = F(K_t, A_tN_t)$. Define the intensive variables

$$x_t = \frac{K_t}{A_tN_t}, \qquad \tilde{y}_t = \frac{Y_t}{A_tN_t} = f(x_t), \qquad \tilde{c}_t = \frac{C_t}{A_tN_t}.$$

With F constant returns to scale, Euler's theorem implies $F = F_KK + F_N(AN)$, so the aggregate labor share is

$$\alpha_L \equiv \frac{w_tN_t}{Y_t} = \frac{1}{\varrho_t\mu_t}\cdot\frac{F_N(K_t,A_tN_t)A_tN_t}{Y_t} = \frac{1}{\varrho_t\mu_t}\cdot\frac{f(x_t)-x_tf'(x_t)}{f(x_t)}. \tag{50}$$

Equation (50) recovers both equation (1) (set $\mu_t = 1$) and equation (3) (set $\varrho_t = 1$) of Section 3.

Households own all firms and receive aggregate profits, which are rebated lump-sum. The household budget constraint is

$$C_t + \dot{\mathbb{A}}_t = w_tN_t + r_t\mathbb{A}_t + \Pi_t,$$

where $\mathbb{A}_t$ is total household wealth (equal to K_t in equilibrium) and $r_t = R_t - \delta$ is the net rate of return. The household's intertemporal first-order condition yields the standard Euler equation

$$\gamma \frac{\dot{C}_t}{C_t} = r_t - \rho.$$

Using the firm's capital FOC (49), we have $R_t = F_K(K_t, A_t N_t)/\varrho_t = f'(x_t)/\varrho_t$, so

$$\gamma \frac{\dot{C}_t}{C_t} = \frac{f'(x_t)}{\varrho_t} - \delta - \rho.$$

Converting to intensive form using $\dot{\tilde{c}}_t/\tilde{c}_t = \dot{C}_t/C_t - g - \dot{N}_t/N_t$, we obtain

$$\gamma \left(\frac{d\tilde{c}_t/dt}{\tilde{c}_t} + g + \frac{\dot{N}_t}{N_t} \right) = \frac{f'(x_t)}{\varrho_t} - \delta - \rho. \tag{51}$$

Equation (51) recovers both equation (2) and equation (4) of Section 3, under a BGP with constant hours worked. Crucially, the markdown μ_t does not appear in the Euler equation, while the markup ϱ_t does.

We can now make formal the BGP non-existence result stated in Section 3. Suppose first that $\mu_t = 1$ for all t and that the markup grows at the constant rate $\dot{\varrho}_t/\varrho_t = g_\varrho > 0$. A balanced growth path requires a constant capital-output ratio, which in turn requires $x_t = \bar{x}$ constant. From the intensive Euler equation (51), in BGP with $\dot{\tilde{c}}_t/\tilde{c}_t = 0$, we have

$$\gamma g = \frac{f'(\bar{x})}{\varrho_t} - \delta - \rho.$$

The left-hand side is constant and positive; the right-hand side declines monotonically as $\varrho_t \to \infty$ and eventually becomes negative. Hence, so no BGP exists.

Suppose instead that $\varrho_t = 1$ is constant and the markdown grows at rate $\dot{\mu}_t/\mu_t = g_\mu > 0$. From equation (51), the Euler equation reverts to the standard form

$$\gamma g = f'(\bar{x}) - \delta - \rho,$$

which has a unique solution for $\bar{x} > 0$. The capital-output ratio is then constant along the BGP, and from equation (50), the labor share declines at the rate $-g_\mu$ as required.

To establish Result 2 of Section 3 formally, consider the household's intratemporal first-order condition for labor supply, which under preferences (45) reads

$$\psi N_t^{\frac{1}{\theta}} = C_t^{-\gamma} w_t.$$

Substituting for the wage using the firm's labor FOC we obtain

$$N_t^{\frac{1}{\theta}+\gamma} = \frac{1}{\varrho_t \mu_t} \tilde{c}_t^{-\gamma} A_t^{1-\gamma} \big[f(x_t) - x_t f'(x_t) \big]. \tag{52}$$

Along any BGP with x_t constant, increasing markups and markdowns will generate a decline in labor supply. If ϱ_t and μ_t grow at constant rate and x_t and $\tilde{c}_t$ are constant, equation (52) implies

$$\left(\frac{1}{\theta} + \gamma \right) g_N = (1-\gamma) g - g_\varrho - g_\mu.$$

This establishes Result 2.

B Model solution

To solve the model, we first write the Hamiltonian of the household, defining investments $q_t^s = \dot{A}_t^s$ and $q_t^i = \dot{A}_t^i$,

$$\mathcal{H} = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \lambda_t \left[-C_t - q_t^s - q_t^i + w_t^s n_t^s + w_t^i n_t^i + r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i \right] + \\ + \mu_t^s q_t^s + \mu_t^i q_t^i + \phi_t \left[-h_t + n_t^s + n_t^i \right]$$

The FOCs of the household, described in the text, together with firm optimality and market clearing, give rise to the following system of nine equations, that govern the evolution of the SSMPE.

$$\begin{aligned}
\psi h_t^{\frac{1}{\theta}} &= C_t^{-\gamma} p_t^i (1-\alpha)(K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \\
n_t^s + n_t^i &= h_t \\
\gamma \frac{\dot{C}_t}{C_t} &= \frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s - \rho \\
C_t + \dot{K}_t^s + \dot{K}_t^i &= Y_t - \delta^s K_t^s - \delta^i K_t^i \\
Y_t = \Bigg[\zeta^{\frac{1}{\sigma}} ((K_t^s)^\alpha (A_t^s n_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} &+ (1-\zeta)^{\frac{1}{\sigma}} ((K_t^i)^\alpha (A_t^i n_t^i)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \Bigg]^{\frac{\sigma}{\sigma-1}} \\
(K_t^s)^\alpha (A_t^s n_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} Y_t \ , \\
(K_t^i)^\alpha (A_t^i n_t^i)^\alpha &= (1-\zeta)(p_t^i)^{-\sigma} Y_t \\
\frac{\sigma m - 1}{\sigma m} p_t^s (1-\alpha)(K_t^s)^\alpha (n_t^s)^{-\alpha} (A_t^s)^{1-\alpha} &= p_t^i (1-\alpha)(K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \left[1 + \alpha \frac{1}{m} \frac{n_t^s}{n_t^i} \right] \\
\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s &= p_t^i \alpha (K_t^i)^{\alpha-1} (A_t^i n_t^i)^{1-\alpha} - \delta^i
\end{aligned}$$

where $K_t^s = m\overline{K}_t^s, y_t^s = m\overline{y}_t^s, n_t^s = \overline{n}_t^s$ are defined in equation (17) with the symmetric equilibrium. We can rewrite this system in intensive form, where we define

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s} \ , \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i} \ , \quad c_t = \frac{C_t}{A_t^s n_t^s} \ , \quad y_t = \frac{Y_t}{A_t^s n_t^s}$$

$$\begin{aligned}
\psi h_t^{\frac{1}{\theta}} &= c_t^{-\gamma} p_t^i (1-\alpha) \frac{A_t^i}{(A_t^s n_t^s)^{\gamma}} (x_t^i)^{\alpha} \\
\frac{\dot{c}_t}{c_t} &= \frac{\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s} \\
c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} &= y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s}\right) x_t^s - \left(\delta^i + g^i + \frac{\dot{n}_t^i}{n_t^i}\right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} \\
y_t = \Bigg[\zeta^{\frac{1}{\sigma}} \left((x_t^s)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} &+ (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\Bigg]^{\frac{\sigma}{\sigma-1}} \\
(x_t^s)^{\alpha} &= \zeta (p_t^s)^{-\sigma} y_t \ , \\
\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{\alpha} &= (1-\zeta)(p_t^i)^{-\sigma} y_t \\
\frac{\sigma m-1}{\sigma m} p_t^s (1-\alpha) A_t^s (x_t^s)^{\alpha} &= p_t^i (1-\alpha) A_t^i (x_t^i)^{\alpha} \left[1 + \frac{\alpha}{m} \frac{n_t^s}{n_t^i}\right] \\
\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i
\end{aligned}$$

The system of growth rates for the monopsonistic decentralized economy is (at infinity already zeroing all growth rates of converging values, $g_{p^s} = g_{x^s} = 0$ and imposing $g_C = g_Y$)

$$\begin{aligned}
g^s &= g^i + g_{p^i} + \alpha g_{x^i} + g_{n^s} - g_{n^i} \\
g_{p^i} + (\alpha - 1) g_{x^i} &= 0 \\
g^i + g_{n^i} + \alpha g_{x^i} &= -\sigma g_{p^i} + g^s + g_{n^s} \\
\frac{1}{\theta} g_{n^s} &= -\gamma (g^s + g_{n^s}) + g_{p^i} + g^i + \alpha g_{x^i} \\
g_h &= g_{n^s} \\
g_C &= g^s + g_h
\end{aligned}$$

As already expressed in the text, the decentralized growth rates are

$$\begin{aligned}
g_C &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1-\alpha)(\sigma-1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i) \\
g_h = g_{n^s} &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1-\alpha)(\sigma-1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i) \\
g_{n^s} - g_{n^i} &= \frac{(1-\alpha)(\sigma-1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i)
\end{aligned}$$

Growth rates of additional variables of interest are

$$g_{\mu} = g_{n^s} - g_{n^i} \ , \quad g_w = g_{p^i} + g^i + \alpha g_{x^i} \ .$$

B.1 Optimal-competitive economy

The competitive economy is very similar to the one above, without markup and markdown.

$$
\begin{aligned}
\psi h_t^{\frac{1}{\theta}} &= c_t^{-\gamma} p_t^i (1-\alpha) \frac{A_t^i}{(A_t^s n_t^s)^\gamma} (x_t^i)^\alpha \\
\frac{\dot{c}_t}{c_t} &= \frac{p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s} \\
c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} &= y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s}\right) x_t^s - \left(\delta^i + g^i \frac{\dot{n}_t^i}{n_t^i}\right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} \\
y_t = \left[\zeta^{\frac{1}{\sigma}} ((x_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
(x_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t \ , \\
\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^\alpha &= (1-\zeta)(p_t^i)^{-\sigma} y_t \\
p_t^s (1-\alpha) A_t^s (x_t^s)^\alpha &= p_t^i (1-\alpha) A_t^i (x_t^i)^\alpha \\
p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i
\end{aligned}
$$

The corresponding system of growth rates for the competitive economy (equivalent to the optimal growth problem) is

$$
\begin{aligned}
g^s &= g^i + g_{p^i}^* + \alpha g_{x^i}^* \\
g_{p^i}^* + (\alpha - 1) g_{x^i}^* &= 0 \\
g^i + g_{n^i}^* + \alpha g_{x^i}^* &= -\sigma g_{p^i}^* + g^s + g_{n^s}^* \\
\frac{1}{\theta} g_{n^s}^* &= -\gamma (g^s + g_{n^s}^*) + g_{p^i}^* + g^i + g_{n^i}^* + \alpha g_{x^i}^* \\
g_h^* &= g_{n^s}^* \\
g_C^* &= g^s + g_h^*
\end{aligned}
$$

The optimal growth rates are

$$
\begin{aligned}
g_C^* &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s \\
g_h^* &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s \\
g_{n^s}^* - g_{n^i}^* &= (1-\alpha)(\sigma - 1)(g^s - g^i) \ .
\end{aligned}
$$

Notice that if we let the substitutability between goods increase, as we approach the perfect substitutes limit the growth rate of labor in the inferior sector goes to minus infinity, which is coherent with the intuition (that can be verified directly) that whenever goods are perfect substitutes, the inferior sector closes right away in the optimal solution, since there is no reason to use an inefficient inferior technology when there are constant returns to scale in factors of production.

B.2 Worker-capitalist economy

Define

$$c_t^c = \frac{C_t^c}{A_t^s h_t} \ , \quad c_t^w = \frac{C_t^w}{A_t^i h_t} \quad x_t^s = \frac{K_t^s}{A_t^s h_t} \ , \quad x_t^i = \frac{K_t^i}{A_t^i h_t} \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$\begin{aligned}
c_t^w &= p_t^i(1-\alpha)(x_t^i)^\alpha \\
\psi h_t^{\frac{1}{\theta}+\gamma} &= (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma}(1-\alpha)(x_t^i)^\alpha \\
n_t^s + n_t^i &= h_t \\
\frac{\dot{c}_t^c}{c_t^c} &= \frac{\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
c_t^c + \frac{A_t^i}{A_t^s} c_t^w + \dot{x}_t &= y_t - \delta^s x_t^s + \delta^i x_t^i \\
y_t &= \left[\zeta^{\frac{1}{\sigma}} ((1-\nu_t)(x_t^s)^\alpha)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
(1-\nu_t)(x_t^s)^\alpha &= \zeta (p_t^s)^{-\sigma} y_t \ , \qquad \frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha = (1-\zeta)(p_t^i)^{-\sigma} y_t \\
\frac{\sigma m-1}{\sigma m} p_t^s (1-\alpha) A_t^s (x_t^s)^\alpha &= p_t^i (1-\alpha) A_t^i (x_t^i)^\alpha \left[1 + \frac{\alpha}{m} \frac{1-\nu_t}{\nu_t} \right] \\
\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s &= p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i
\end{aligned}$$

Alternatively, define

$$K_t = K_t^s + K_t^i \ , \quad c_t^c = \frac{C_t^c}{A_t^s h_t} \ , \quad c_t^w = \frac{C_t^w}{A_t^i h_t} \quad x_t = \frac{K_t}{A_t^s h_t} \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$\begin{aligned}
c_t^w &= p_t^i(1-\alpha)\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha} \\
\psi h_t^{\frac{1}{\theta}+\gamma} &= (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma}(1-\alpha)\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha} \\
n_t^s + n_t^i &= h_t \\
\frac{\dot{c}_t^c}{c_t^c} &= \frac{\frac{\sigma m-1}{\sigma m}p_t^s\alpha\left(\frac{1-\kappa_t}{1-\nu_t}x\right)^{\alpha-1}-\delta^s-\rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
c_t^c + \frac{A_t^i}{A_t^s}c_t^w + \dot{x}_t &= y_t - ((1-\kappa_t)\delta^s + \kappa_t\delta^i)x_t \\
y_t &= \left[\zeta^{\frac{1}{\sigma}}\left((1-\nu_t)\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}\left(\frac{A_t^i}{A_t^s}\nu_t\left(\frac{\kappa_t A_t^s}{\nu A_t^i}x_t\right)^{\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
(1-\nu_t)\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^{\alpha} &= \zeta(p_t^s)^{-\sigma}y_t\ , \qquad \frac{A_t^i}{A_t^s}\nu_t\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha} = (1-\zeta)(p_t^i)^{-\sigma}y_t \\
\frac{\sigma m-1}{\sigma m}p_t^s(1-\alpha)A_t^s\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^{\alpha} &= p_t^i(1-\alpha)A_t^i\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha}\left[1+\frac{\alpha}{m}\frac{1-\nu_t}{\nu_t}\right] \\
\frac{\sigma m-1}{\sigma m}p_t^s\alpha\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^{\alpha-1} - \delta^s &= p_t^i\alpha\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha-1} - \delta^i
\end{aligned}$$

The study of this system transformed in growth rates at infinity delivers the growth rates in the main text and is consistent with the notation used to prove stability of the main model.

The superstar firm problem under a minimum wage. We report here the details of the firm problem with an exogenous minimum wage $\bar{w}_t$ that we omitted from the main text in Section 5. The inferior sector and the household problem are unchanged. The problem of a representative superstar firm f in the superior sector becomes

$$\begin{aligned}
\max_{N_t^{f,s},K_t^{f,s},W_t^{f,s}} \Pi_t^{f,s} \quad &= \quad p_t^s(y_t^{f,s};y_t^{-f,s})y_t^s(K_t^{f,s},N_t^{f,s},A_t^s) - W_t^{f,s}N_t^{f,s} - R_t^s K_t^{f,s} \\
\text{s.t.} \qquad &\qquad W_t^{f,s} \geq \bar{w}_t.
\end{aligned}$$

Notice that the minimum wage need not be binding at every date. Since the superstar firms take the capital allocated to the inferior sector as given, it may happen that the marginal product of labor in the inferior sector remains above the minimum wage even when all labor is supplied to it—that is, when $p_t^i(1-\alpha)(H_t)^{-\alpha}(K_t^i)^{\alpha}(A_t^i)^{1-\alpha} > \bar{w}_t$. Whenever the constraint $W_t^{f,s} \geq \bar{w}_t$ does bind, there exists a unique $\hat{N}_t^{f,s} \in (0,H_t)$ that characterizes the optimal labor demand of the inferior sector at the minimum wage. The effective wage schedule faced by the superstar firms then takes the piecewise form

$$w_t^s = \begin{cases} p_t^i(K_t^i)^{\alpha}(H_t - N_t^{f,s})^{-\alpha}(A_t^i)^{1-\alpha}(1-\alpha) & N_t^{f,s} \in [H_t - \hat{N}_t^{f,s}, H_t] \\ \bar{w}_t & N_t^{f,s} \in [0, H_t - \hat{N}_t^{f,s}] \end{cases}$$

together with the output and technology constraints

$$y_t^{f,s} = \zeta(p_t^s)^{-\sigma} Y_t - y_t^{-f,s}, \qquad y_t^{f,s}(K_t^{f,s}, N_t^{f,s}, A_t^s) = (K_t^{f,s})^{\alpha}(A_t^s N_t^{f,s})^{1-\alpha}.$$

When each superstar firm optimally chooses $w_t^{f,s} = \bar{w}_t$ for all f, the minimum wage is binding and the superstar firms effectively become pure oligopolists: the wage is exogenously set by the policy, so there is no scope for the firm to exercise monopsonistic markdown. The intuition for this is straightforward. With $\bar{w}_t$ given, the firm's labor cost is linear in $N_t^{f,s}$ (marginal cost equals average wage $= \bar{w}_t$), and the markdown factor in the marginal cost of labor disappears.

For the minimum wage to remain binding indefinitely along the equilibrium path, it must grow at least as fast as the shadow monopsonistic wage that would prevail in the absence of the policy. Since the shadow monopsonistic wage in the worker-capitalist economy grows at rate $g_w = g^s - g_\mu$, while the consumption-leisure condition with a binding minimum wage requires $g_{\bar{w}}$ to be linked to the growth rates of consumption and hours, the growth rate $g_{\bar{w}}^* = g^s$ derived in the main text is the unique rate consistent with both binding and efficient outcomes.

C The System with general technology

With general constant returns to scale production function for the superior and inferior sectors indicated respectively with $F(K^s, N^s, A^s)$ and $G(K^i, N^i, A^i)$, the general system for the SMPE

$$\begin{aligned}
\frac{\sigma m - 1}{\sigma m} p_t^s F_N(K_t^s, n_t^s, A_t^s) &= p_t^i \left[G_N(K_t^i, n_t^i, A_t^i) - \frac{1}{m}\frac{n_t^s}{n_t^i} G_{NN}(K_t^i, n_t^i, A_t^i) \right] \\
\frac{\sigma m - 1}{\sigma m} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s &= p_t^i G_K(K_t^i, n_t^i, A_t^i) - \delta^i \\
G(K_t^i, n_t^i, A_t^i) &= (1-\zeta)(p_t^i)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
F(x_t^s, n_t^s, A_t^s) &= \zeta(p_t^s)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
\psi h_t^{\frac{1}{\theta}} &= C_t^{-\gamma} p_t^i G_N(K_t^i, n_t^i, A_t^i) \\
h_t &= n_t^s + n_t^i \\
\gamma \frac{\dot{C}_t}{C_t} &= \frac{\sigma m - 1}{\sigma m} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho \\
C_t + \dot{K}_t^s + \dot{K}_t^i &= \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - \delta^s K_t^s - \delta^i K_t^i
\end{aligned}$$

where from the first equation we can see that we can find a very nice expression for the markdown, i.e.

$$\mu_t = 1 - \frac{1}{m}\frac{n_t^s}{n_t^i}\frac{G_{NN}(K_t^i, n_t^i, A_t^i)}{G_N(K_t^i, n_t^i, A_t^i)} \ .$$

It is simple to show that with general CRS technologies one can't solve for the balanced growth path since for p_t^s and x_t^s converging to a finite number and for $p_t^i \to \infty$ it must be that $x_t^i = \frac{K_t^i}{A_t^i n_t^i}$

tend to ∞. To study this, one needs to know the shape of G (while in principle, we could keep F a neoclassical production function that satisfies Uzawa's theorem).

There is one special case which is tractable though: if the two intermediaries are perfect substitutes, the complications arising from prices disappear since the two goods must trade at the same price. With no diverging price for the inferior good, x_t^i will converge to a constant as well, and we can solve for the balanced growth path with a general inferior technology G.

D Stability

The system we use to prove stability is the following.

$$\begin{aligned}
\frac{\dot{c}}{c} &= \frac{\frac{\sigma m-1}{\sigma m} p^s F_K\left(\frac{1-\kappa}{1-\nu}x,1,1\right) - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
\psi h^{\frac{1}{\theta}+\gamma} &= c^{-\gamma} p^i \frac{A^i}{(A^s)^\gamma} G_N\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right) \\
\frac{\sigma m-1}{\sigma m} p_s F_K\left(\frac{1-\kappa}{1-\nu}x,1,1\right) - \delta^s &= p_i G_K\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right) - \delta^i \\
\frac{\sigma m-1}{\sigma m} p^s A^s F_N\left(\frac{1-\kappa}{1-\nu}x,1,1\right) &= p^i A^i \left[G_N\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right) - \frac{1}{m}\frac{1-\nu}{\nu} G_{NN}\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right)\right] \\
c + \dot{x} &= \left[\zeta^{\frac{1}{\sigma}}\left\{(1-\nu)F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)\right\}^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}\left\{\nu\frac{A^i}{A^s}G\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right)\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \left((1-\kappa)\delta^s + \kappa\delta^i + g^s + \frac{\dot{h}}{h}\right)
\end{aligned}$$

$$\begin{aligned}
1 &= \delta p_s^{-\sigma}\left[\delta^{\frac{1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}}\left\{\frac{A_i}{A_i}\frac{\nu}{1-\nu}\frac{G\left(\frac{\kappa}{\nu}\frac{A_s}{A_s}x,1,1\right)}{F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
1 &= (1-\delta)p_i^{-\sigma}\left[\delta^{\frac{1}{\sigma}}\left\{\frac{A_s}{A_i}\frac{1-\nu}{\nu}\frac{F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)}{G\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x,1,1\right)}\right\}^{\frac{\sigma-1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

Theorem 2. *Assume that $\delta^s = \delta^i$, then the model is locally saddle path stable.*

We will prove stability in five steps (all values are assumed at the steady state $(c^*, 0, 0, x^*)$).

1. Show that $\frac{\partial \dot{h}}{\partial h} = g_h < 0$ and that for all other variables, the derivative with respect to h is zero.
2. Show that $\frac{\partial \dot{\nu}}{\partial \nu} = g_{n^i} - g_{n^s} < 0$ and that with respect to all other variables, the derivative of $\dot{\nu}$ is zero.
3. Show that $\frac{\partial \dot{x}}{\partial c} = -\frac{1}{1+\theta\alpha_i}$ (under $\alpha_s = \alpha_i$, in general one should get $\frac{\partial \dot{x}}{\partial c} < 0$).
4. Show that $\frac{\partial \dot{c}}{\partial c} = -c^* \frac{\partial(\dot{h}/h)}{\partial c}$ and that $\frac{\partial \dot{c}}{\partial x} = \frac{c^*}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2} - c^*\frac{\partial(\dot{h}/h)}{\partial x}$.

These steps together will generate a Jacobian of the type

$$J_\Phi = \begin{pmatrix}
-c^*\frac{\partial(\dot{h}/h)}{\partial c} & 0 & * & \frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2} - c^*\frac{\partial(\dot{h}/h)}{\partial x} \\
* & g_h & * & * \\
0 & 0 & g_{n^i} - g_{n^s} & 0 \\
-\frac{1}{1+\theta\alpha_i} & 0 & * & *
\end{pmatrix}$$

The determinant of this matrix is equal to that of the matrix (we are basically subtracting the third row from the first, this leaves the determinant unchanged.)

$$\begin{pmatrix} 0 & 0 & * & \underbrace{\frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2}}_{<0} \\ * & \underbrace{g_h}_{<0} & * & * \\ 0 & 0 & \underbrace{g_{n^i}-g_{n^s}}_{<0} & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & * \end{pmatrix}$$

which is strictly negative. Then the matrix is non-singular and the steady state is hyperbolic. Also, either one or three eigenvalues have positive real part. But eigenvalues are roots of the characteristic polynomial which solves

$$\det\begin{pmatrix} k\frac{\partial\dot{x}}{\partial c}-\lambda & 0 & * & (<0)+k\frac{\partial\dot{x}}{\partial x} \\ * & g_h-\lambda & * & * \\ 0 & 0 & g_{n^i}-g_{n^s}-\lambda & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & *-\lambda \end{pmatrix} = (g_h-\lambda)(g_{n^i}-g_{n^s}-\lambda)\begin{pmatrix} k\frac{\partial\dot{x}}{\partial c}-\lambda & (<0)+k\frac{\partial\dot{x}}{\partial x} \\ -\frac{1}{1+\theta\alpha_i} & *-\lambda \end{pmatrix}$$

which implies that two of these are negative, and thus three eigenvalues must have negative real part.

Proof. The system of equations with Cobb-Douglas production functions reads as follows (assuming $\delta^s=\delta^i=\delta$, but leaving $\alpha_s\neq\alpha_i$)

$$\frac{\dot{c}}{c}=\frac{1}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma}}\left\{\zeta^{\frac{1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\left[\frac{A_i}{A_s}\frac{1-\nu}{\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{1}{\sigma-1}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}-\frac{\rho+\delta}{\gamma}-g^s-\frac{\dot{h}}{h} \tag{55}$$

$$\psi h^{\frac{1}{\theta}+\gamma}=c^{-\gamma}\frac{A_i}{A_s^{\gamma}}(1-\alpha_i)\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}(1-\zeta)^{\frac{1}{\sigma}}\left\{\zeta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\right\}^{\frac{1}{\sigma-1}} \tag{56}$$

$$\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right)^{\frac{1}{\sigma}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}=\alpha_i\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i-1}$$

$$A_s\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right)^{\frac{1}{\sigma}}(1-\alpha_s)\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s}=(1-\alpha_i)A_i\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}\left[1+\frac{1}{m}\frac{1-\nu}{\nu}\alpha_i\right]$$

$$c+\dot{x}=\left[\zeta^{\frac{1}{\sigma}}\left\{(1-\nu)\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s}\right\}^{\frac{\sigma-1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\left\{\nu\frac{A_i}{A_s}\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-\left(\delta+g^s+\frac{\dot{h}}{h}\right) \tag{57}$$

Combining (taking the ratio of) the third and the fourth equations delivers

$$\frac{1-\alpha_s}{\alpha_s}\frac{1-\kappa}{1-\nu} = \frac{1-\alpha_i}{\alpha_i}\frac{\kappa}{\nu}\left[1+\frac{1-\nu}{\nu}\alpha_i\right]$$

which allows us to express κ as a function ν. In growth rates this implies

$$\frac{\dot{\kappa}}{\kappa}\frac{1}{1-\kappa} = \frac{\dot{\nu}}{\nu}\left[1+\frac{\nu}{1-\nu}+\frac{(1-\alpha_i)\nu}{\alpha_i+(1-\alpha_i)\nu}\right] .$$

We can also rewrite the third equation as

$$\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\right)^{\frac{1}{\sigma}}\left(\frac{\delta}{1-\delta}\right)^{\frac{1}{\sigma}}\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{1-\alpha_i\frac{\sigma-1}{\sigma}}\frac{\alpha_s}{\alpha_i}\left(\frac{1-\kappa}{1-\nu}x\right)^{-1+\alpha_s\frac{\sigma-1}{\sigma}} = 1 . \tag{58}$$

Differentiating with respect to time and imposing $\alpha_s = \alpha_i$ gives in a straightforward manner an equation that depends on ν and κ only (thus ultimately on ν only) and that when evaluated at $\nu = \kappa = 0$ returns

$$\frac{\dot{\nu}}{\nu} = -\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i) = g_{n^i} - g_{n^s}$$

We are now ready to proceed with our four steps of proof. Notice that whenever the ratio A_s/A_i shows up in equations (55) and (57) (which we are not going to differentiate), to make the equation time-invariant we need to substitute it out using (58).

1. Notice that the only equations in which h appears are (55), (56) and (57). We can transform (56) in growth rates. Once one substitutes $\frac{\dot{c}}{c}$ out using (55), the expression reads

$$\frac{1}{\theta}\frac{\dot{h}}{h} = -\delta^{\frac{1}{\sigma}}\left\{\delta^{\frac{1}{\sigma}}+(1-\delta)^{\frac{1}{\sigma}}\left[\frac{A_i}{A_s}\frac{1-\nu}{\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{1}{\sigma-1}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}+\rho+\delta+g^i+\alpha_i\left[\frac{\dot{\kappa}}{\kappa}-\frac{\dot{\nu}}{\nu}+g^s-g^i+\frac{\dot{x}}{x}\right]+$$
$$+\frac{\delta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}}{\delta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}+(1-\delta)^{\frac{1}{\sigma}}}\frac{1}{\sigma}\left[g^i-g^s-\frac{1}{1-\nu}\frac{\dot{\nu}}{\nu}+(\alpha_s-\alpha_i)\frac{\dot{x}}{x}-\alpha_i(g^s-g^i)+\alpha_s\left(-\frac{\kappa}{1-\kappa}\frac{\dot{\kappa}}{\kappa}+\frac{\nu}{1-\nu}\frac{\dot{\nu}}{\nu}\right)-\alpha_i\left(\frac{\dot{\kappa}}{\kappa}-\frac{\dot{\nu}}{\nu}\right)\right]$$

   We then notice that only the growth rate of h appear, not h itself. We can then substitute $\frac{\dot{h}}{h}$ out of (55) and (57). This establishes that no time derivative except $\dot{h}$ depends on h. Next, consider that we are left with an equation arising from differentiating (56) which is of the type

$$\frac{\dot{h}}{h} = \mathcal{H}(\nu, x, \kappa(\nu), \dot{x}(c, \nu, x, \kappa(\nu)))$$

   which must be such that

$$\mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

   but then clearly

$$\frac{\partial \dot{h}}{\partial h} = \mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

   at the steady state. This establishes the first point.

2. For the second point, by differentiating (58) we find

$$\left(1-\alpha_i\frac{\sigma-1}{\sigma}\right)\left(\frac{\dot\kappa}{\kappa}-\frac{\dot\nu}{\nu}\right)-\left(1-\alpha_s\frac{\sigma-1}{\sigma}\right)\left(-\frac{\kappa}{1-\kappa}\frac{\dot\kappa}{\kappa}+\frac{\nu}{1-\nu}\frac{\dot\nu}{\nu}\right)+\frac{1}{\sigma}(g^i-g^s)+$$
$$+\frac{1}{\sigma}\left(\frac{\dot\nu}{\nu}+\frac{\nu}{1-\nu}\frac{\dot\nu}{\nu}\right)+\left(1-\alpha_i\frac{\sigma-1}{\sigma}\right)(g^s-g^i)+\frac{\sigma-1}{\sigma}(\alpha_s-\alpha_i)\frac{\dot x}{x}=0$$

Clearly for $\alpha_s=\alpha_i$ this gives an equation for $\frac{\dot\nu}{\nu}$ that depend on ν only. Also, again it is an expression for the growth rate of ν, and $\nu^*=0$, as above

$$\frac{\dot\nu}{\nu}=\mathcal{N}(\nu)$$

where

$$\mathcal{N}(0)=-\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i)=g_{n^s}-g_{n^i}$$

so that

$$\frac{\partial\dot\nu}{\partial\nu}=\mathcal{N}(0)=-\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i)<0\ .$$

This completes the second step.

3. For the third point, notice that once we substitute out $\frac{\dot h}{h}$ for (57) using (1), we find

$$(1+\alpha_i\theta)\dot x=-c+\text{other terms independent of }c$$

This directly implies

$$\frac{\partial\dot x}{\partial c}=-\frac{1}{1+\alpha_i\theta}<0\ .$$

4. Since (55) is in the form

$$\frac{\dot c}{c}=\mathcal{C}(\nu,x,\kappa(\nu),\dot h/h)\ ,$$

we will have

$$\frac{\partial\dot c}{\partial c}=\mathcal{C}(0,x^*,0,g_h)-c^*\frac{\partial(\dot h/h)}{\partial c}$$

but since c converges to a constant, we must have $\mathcal{C}(0,x^*,0,g_h)=0$. Notice than that $\dot h/h$ depends on c only through $\dot x$, since

$$\frac{\partial(\dot h/h)}{\partial c}=\theta\alpha_i\frac{\partial(\dot x/x)}{\partial c}=-\frac{\theta\alpha_i}{1+\theta\alpha_i}\frac{1}{x^*}$$

On the other hand we have

$$\frac{\partial\dot c}{\partial x}=\frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\delta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2}-c^*\frac{\partial(\dot h/h)}{\partial x}$$

□

E The model with fixed materials

The only difference in the model is the absence of capital and the fact that the intermediaries produce with a fixed material T_t^s and T_t^i so that

$$y_t^s = A_t^s (T_t^s)^\alpha; \quad y_t^i = A_t^i (T_t^i)^\alpha$$

The choice of materials is only influenced by the monopoly power in the labor market and its problem solves.

$$\frac{\partial y_t^s}{\partial T_t^s} p_t^s \frac{\sigma m - 1}{\sigma m} = p^T$$

The fundamental factor allocation for the superstar firms implies

$$\left[1 + \frac{\alpha}{m} \frac{N_t^s}{N_t^i}\right] \frac{N_t^s}{T^s} = \frac{N_t^i}{T^i}$$

where it is clear that as long as $\alpha > 0$, the factor allocation of the superstar firm is distorted vis-a-vis a pure neoclassical factor ratio equilibrium that implies $\frac{N_t^s}{T_t^s} = \frac{N_t^i}{T_t^i}$ that characterizes the optimal growth problem that will be discussed later. The superstar firm makes strictly positive profits that are fully distributed to the consumers who owns the firm, to which we turn next. Using the key conditions of the superstar firms for both labor and land,

The representative consumer problem maximize utility period by period and takes as given the wages w_t^j $(j = s, i)$ in the two sectors, and needs to choose the total hours h_t as well as the hours worked in every sector. She is also endowed with the materials T that that she rents to the firm at price P_t^T. Finally, the consumers obtains per capita profits/dividends from the superstar firm that we indicate with π_t^s. The consumer problem is

$$\begin{aligned}
&\max_{C_t, h_t, n_t^s, n_t^i, T_t^s, T_t^i} \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} \\
&\text{s.t.} \quad P_t C_t = w_t^s n_t^s + w_t^i n_t^i + P_t^T (T_t^s + T_t^i) + \pi_t^s \\
&(n_t^s + n_t^i) \leq h_t; \quad T_t^s + T_t^i = \overline{T}_t
\end{aligned}$$

where P_t is the natural price index of equation 4.1 normalized to one and $\overline{T}_t$ are the fixed material at time t. The key first order conditions for the consumers for the labor and material allocation are

$$\begin{aligned}
C_t^{-\gamma} w_t^j &= \psi (n_t^s + n_t^i)^{\frac{1}{\theta}} \quad j = i, s \\
C_t^{-\gamma} \frac{\partial C_t}{\partial T_t^j} &= p_t^T \qquad\quad j = i, s
\end{aligned}$$

and imply the standard neoclassical labor supply problem for which the marginal rate of substitution between labor and consumption is equal to the wage rate, and the marginal utility of consumption is equal to price of material. Since these factor conditions holds for each sector, the labor supply of the consumer implies a fundamental arbitrage condition so that the wage in the two sectors has to be identical for he worker to be in equilibrium, and $w_t^s = w_t^i$. Similarly, the material allocation is such that the marginal utility of consumption in each of the two sector is identical.

In equilibrium all markets clear, and the labor and material demand quantities coincide with labor supply choice of the consumer. so that

$$C_t = Y_t; \quad n_t^s = N_t^s; \quad n_t^i = N_t^i; \quad T^t = T_t^s + T_t^i,$$

where the condition $C_t = Y_t$ implies that all final output is consumed and there are no savings.

The SPME is very similar but it only features a price for the fixed materials p_t^T in place of the cost of capital R_t The limit system at time t for obtaining the BGP is made up by the following four equations. The growth rates are

$$\begin{aligned}
g_C &= \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)} g^s - (1-\alpha)\Omega(g^s - g^i) \\
g_{ns} &= \frac{1-\gamma}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)} g^s - \Omega(g^s - g^i) \\
g_\mu &= \frac{\sigma-1}{1+\alpha+\sigma(1-\alpha)} (g^s - g^i) \\
g_{T_i} &= -2 g_\mu
\end{aligned}$$

where the constant Ω is depends on the structural parameters of the model $\Omega = \frac{1}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)} \frac{\sigma-1}{1+\alpha+\sigma(1-\alpha)}$. The SMPGE is a a semi endogenous growth model, since its equilibrium growth rate crucially depends on the exogenous growth productivity g^s and g^i, but it fully interacts with the structural parameters of the model. For further characterizing its properties, it is necessary to solve for the corresponding optimal growth problem, to which we next turn.

The optimal growth problem yields the following equations and the growth rates are

$$\begin{aligned}
g_C^* &= \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)} g^s \\
g_{ns}^* &= \frac{1-\gamma}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)} g^s \\
g_{ni}^* &= \frac{1-\gamma}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)} g^s + (\sigma-1)(g^i - g^s) \\
g_{Ti}^* &= -(\sigma-1)(g^s - g^i)
\end{aligned}$$

We are thus now in a position to study the misallocation and growth effect of the SMPGE.