DISCUSSION PAPER SERIES

DP19652
(v. 4)

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LABOUR ECONOMICS AND MACROECONOMICS AND GROWTH

ISSN 0265-8003

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Discussion Paper DP19652
First Published 08 November 2024
This Revision 23 December 2025

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JEL Classification: O40, O41

Keywords: Monopsony, Growth, Misallocation

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Monopsony in Growth Theory

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December 23, 2025

Abstract

The secular decline in the labor share and the long-run reduction in labor supply suggest that imperfect labor markets can play a role in long-run economic growth. This paper introduces oligopsony and oligopoly power in a Neoclassical Growth Model with superstar firms. The endogenous markdown of productivity on wages is the key driver of growth misallocation in the asymptotic balanced growth path. The model can be calibrated to simultaneously rationalize the joint trends of GDP growth, declining labor share and hours worked. For the US, the consumption equivalent loss with respect to the optimal growth path is calibrated around 7.5 percent. The theory is also coherent with growing markdown in the US estimated from a simple accounting exercise. An extension of the model with hand-to-mouth workers and capitalists delivers balanced growth with increasing inequality. While - in this context- proportional taxation distorts equilibrium labor supply, a raising minimum wage can restore efficient growth.

Keywords: Monopsony, Growth, Unbalanced Growth, Labor Share, Misallocation.
JEL codes: O40, O41, J23, J30, J42 :

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[‡]We thank Chad Jones, Rachel Ngai, Victor Ríos-Rull and Gianluca Violante for their insightful feedback, and seminar participants at Collegio Carlo Alberto, Birkbeck College, Norwegian Business School, LUISS University, University of Pennsylvania, Princeton University, Bocconi University and the IMF for comments and suggestions.

1 Introduction

The neoclassical growth model (NGM) assumes competitive labor markets (Cass, 1965; Koopmans, 1965; Acemoglu, 2009). However, two long run macroeconomic facts suggest that imperfect labor markets can potentially also play a role in long-run economic growth. First, the secular decline in the labor share implies a long-run reduction in the share of total income accrued to labor (Bergholt et al., 2022; Karabarbounis, 2024). Second, long-run statistics on hours worked by the average individual point to a secular decline in labor supplied to the market (Huberman and Chris, 2007; Ramey and Francis, 2009; Boppart and Krusell, 2020).

Since the seminal contributions of Romer (1990) and Aghion and Howitt (1992), market power in the product market has been the focus of growth theory. Empirically, market power by so called "superstar firms" (Autor et al., 2020) is associated with stagnant wages (Deb et al., 2022) and declining labor share. Growth models devote less attention to market power in the hiring of labor, even though labor market imperfections too imply a wedge between the firm marginal product and the wage paid to the individuals. In recent years, the existence of monopsony power in the labor market has attracted considerable attention both in theoretical and in empirical work (Manning, 2021; Yeh et al., 2022; Berger et al., 2022).

The paper proposes and solves a multisector growth model with unbalanced TFP growth, and a superior sector in which "superstar" firms enjoy both oligopsonistic and oligopolistic power. Superstar firms derive oligopoly power from barriers to entry and from knowledge of the imperfect substitutability between different goods, and oligopsonistic power from knowledge of other firms' labor demand conditional on wages. As in traditional growth theory, the production functions are neoclassical and the demand for goods is isoelastic. When superstar firms enjoy both oligopoly and oligopsony power, labor market imperfections induce growth effects and the decentralized growth process is inefficient. With standard isoelastic product demand, there are no growth effects if superstar firms enjoy only oligopoly power, but only level effects, including lower labor share and lower hours worked. Instead, oligopsony power by superstar firms induces a drag on the growth rates of the labor share, hours worked and total production. Further, the decline in hours per capita is a feature of the model also when income and substitution effects from wage changes cancel each other out and preferences are coherent with the King et al. (1988) restriction. The negative effect of oligopsonistic power on growth in hours represents the growth equivalent of the classic static result of the negative effects on hours worked of a generic labor market wedge (Prescott, 2004).

In the solution of the model, the labor wedge increases over time, and is the key driver of the growth results. A simple accounting exercise that exploits the equilibrium equations of the model, suggests that rising markdown are fully coherent with rising sectoral labor concentration by superstar firms, as reported by Autor et al. (2020). In this perspective, the model takes as given the phenomenon of superstar firms suggested by Autor et al. (2020), and investigates the growth consequences of labor market power in those markets. Yeh et al. (2022) also reports evidence of rising markdown and monopsony power in US manufacturing.

We solve our model under standard capital accumulation and labor augmenting technological progress, and require TFP growth to be unbalanced in favor of the superstar sector. Yet, in a version of the model with Hicks-neutral technological progress, TFP growth differentials across sectors are not necessary to generate inefficient growth and declining labor share. Instead, a more capital intensive superstar sector is sufficient to generate all implications of the model.

The paper has several quantitative implications and results. In the first quantitative exercise in Section 5 we assume that a calibrated "representative advanced economy" moves along the asymptotic superstar equilibrium defined by the model. The section firstly shows that to quantitatively match the three long run facts concerning i) growth in GDP/consumption, ii) (negative) growth in hours worked and iii) (negative) growth of labor share (all reported empirically in Section 2) it is not enough to work with a NGM with preferences that imply an income effect of wage growth larger than the substitution effect, or preferences that belong to the Boppart and Krusell (2020) class. Indeed, we show that only the model with oligopsonistic labor markets can match all three growth moments. Secondly, the section quantifies the size of the income effect due to market imperfections and positive profits, and decomposes the part of the income effect that can be attributed to pure preferences (such as in Boppart and Krusell, 2020) from the part attributable to the non wage transfer in a growth setting (Prescott, 2004). In general, the size of the income effect is much larger if the model is used to match the (larger) negative trend in hours per worker than the trend in hours per person. Quantitatively, market imperfections implied by the model account for 16 percent of the income effect in the case of hours per worker and as much as 80 percent in the case of hours per person. Third, we quantify the size of the asymptotic "growth drag" due to market imperfections vis-à-vis the competitive economy, and find that oligopsony implies an asymptotic loss of 5 percent of GDP growth. The proportional loss of the growth rate of hours is 7 percent lower than the competitive economy in the case of hours per worker and fall from -0.02 percent to -0.1 percent in the case of hours per person, which correspond to an increase by a factor of 5 in proportional terms.

In the fourth quantitative exercise we focus on the US economy and solve quantitatively the transition of the economy along the asymptotic path. We calibrate a version of the model with a fixed factor of production like "materials". We match the level of the labor share, hours worked and other key macro moments to the "time 0" of the calibration to the US economy in the first ten years of the 2000s. Before calibrating the transitional dynamics, we perform a simple accounting exercise based on the employment dynamics of the superstar firms. The accounting exercise implies an upward trend of the monopsonistic markdown that can be used as an empirical benchmark. We then estimate the transition path of the US economy toward the asymptotic balanced growth path, and shows that a version of the calibrated economy perfectly matches the growth rate and can account for 60 to 80 percent of the (negative) growth observed in hours per worker and the labor share. Finally, we quantitatively assesses the consumption equivalent loss of the calibrated economy. With respect to a competitive economy, the consumption loss is approximately 7.5 percent. Lastly, the paper shows that in the transitional dynamics, an increase in market concentration induces level effects on hours worked and labor share, without influencing their long run growth properties.

The paper has also implications for long run inequality. A version of the model with capitalist and hand-to-mouth workers shows that the divergence between the capital share and the labor share is accompanied by increasing and long lasting income and consumption inequality. In addition, the model with heterogeneous agents has implications for the long run dynamics of labor supply, since the hand-to-mouth individuals work harder than what would be optimal for a central planner, and harder than the representative decentralized agent subject to oligopsony and oligopoly. We also discuss the role of policy in this setting. We first study the effects of a proportional tax on capitalists' income rebated to workers, showing that while it addresses the most severe effects of inequality, it depresses labor supply and GDP growth vis-à-vis the efficient levels. We also address the possibility that a growing minimum wage increases both growth of hours and growth of wages, in line with the classical intuition due to Robinson (1969).

The paper proceeds as follows. Section 2 reviews the two key stylized facts and the relevant literature. Section 3 presents the basic structure of the economy, the superstar sector, production technologies and preferences. It then solves for the balanced growth path with superstar oligopolists and oligopsonists, proving its stability, as well as for the competitive equilibrium and compares the two solutions highlighting the growth effects of oligopsony power. Section 4 solves the model with hand-to-mouth workers and capitalists and derives the implications for inequality. We also analyze two types of policy intervention, taxation of capitalists and minimum wage. Section 5 presents the calibration and quantifies the misallocation loss due to monopsony in consumption equivalent terms. It also presents the accounting exercise on the dynamics of markdown. Section 6 summarizes and concludes.

2 Two Long Run Facts and Literature Review

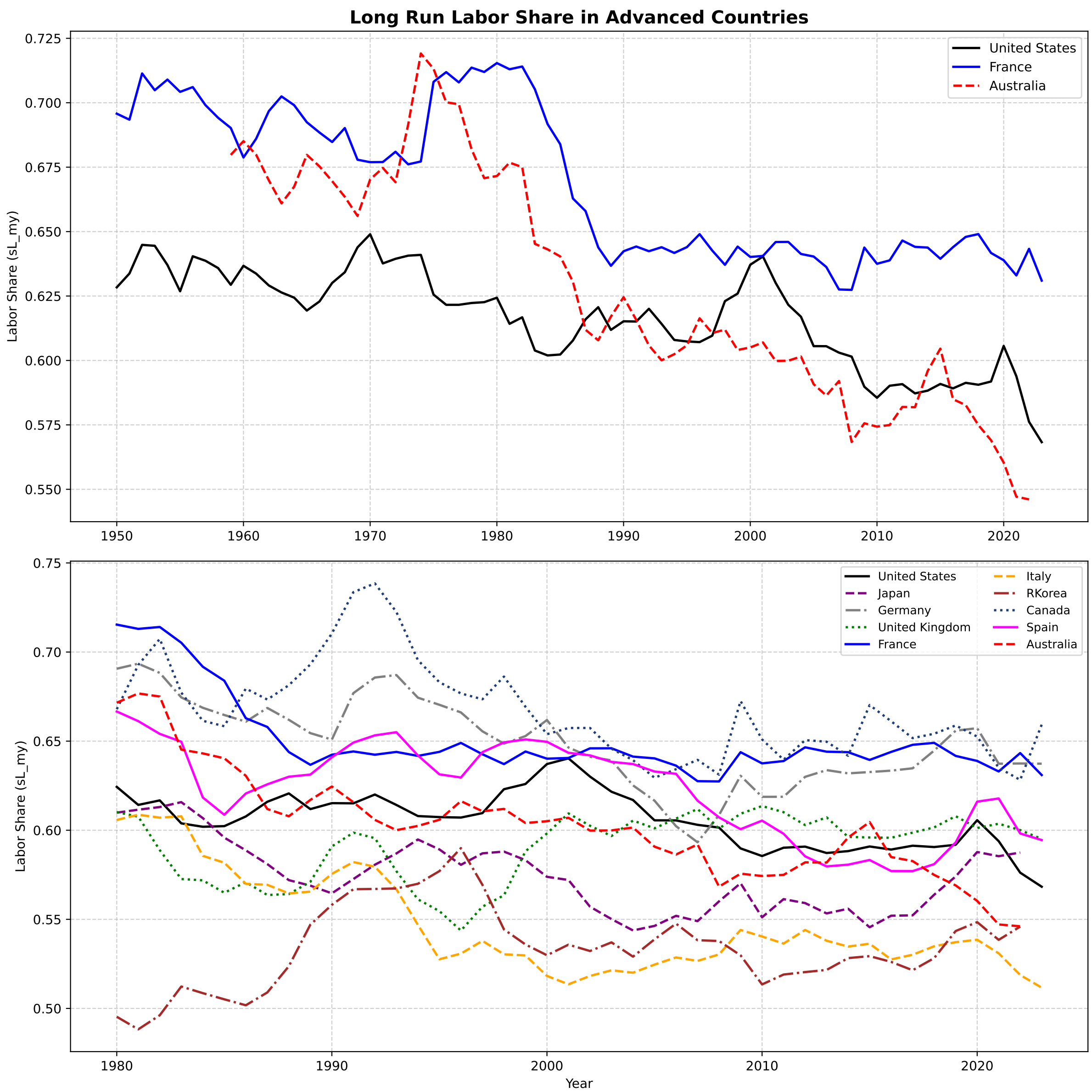
We document two facts that are well known in the literature, even though they are not typically considered jointly and part of a unified theory. The first fact concerns the long run dynamics of the labor share. The second fact is the long-run dynamic of labor supply. To organize the evidence we use the latest available data from World Penn Table (Feenstra et al., 2015). We focus on nine advanced economies for which data in the Penn World Table cover the period from 1950s to 2023 for labor supply, and from 1980 to 2023 for the labor share.[1] For three of these countries (United States, France and Australia) there are labor share statistics also from the 1950s.

2.1 The Decline of the Labor Share

The secular decline in the labor share has been first recognized in the seminal paper by Karabarbounis and Neiman (2014). Since then, the empirical literature on the measure of the labor share has been very vivid across the two sides of the Atlantic, and in most developing countries (Cette et al., 2019; Brooks et al., 2021). At the empirical methodological level, there is strong debate about measurement issues linked to the role of housing, self employment and proprietary income, and the role of government. Atkenson (2020)- for example- argues that the estimates of the falling labor share are largely due to changes in measurement details by the Bureau of Labor Statistics. More recently, Karabarbounis (2024) reviews the large work on the decline in the labor share and reports unambiguous evidence of its decline. Figure 1 uses data for the same nine countries of our sample from the Penn World Table and reports the long run trend in the labor share. Figure 1 replicates the results in Karabarbounis (2024). Table 1 summarizes the size of the downward trend for the nine countries in using both a linear trend and an HP filter with coefficient 100, a standard value for yearly data. The results of the two estimates are basically identical. Quantitatively, Table 1- in

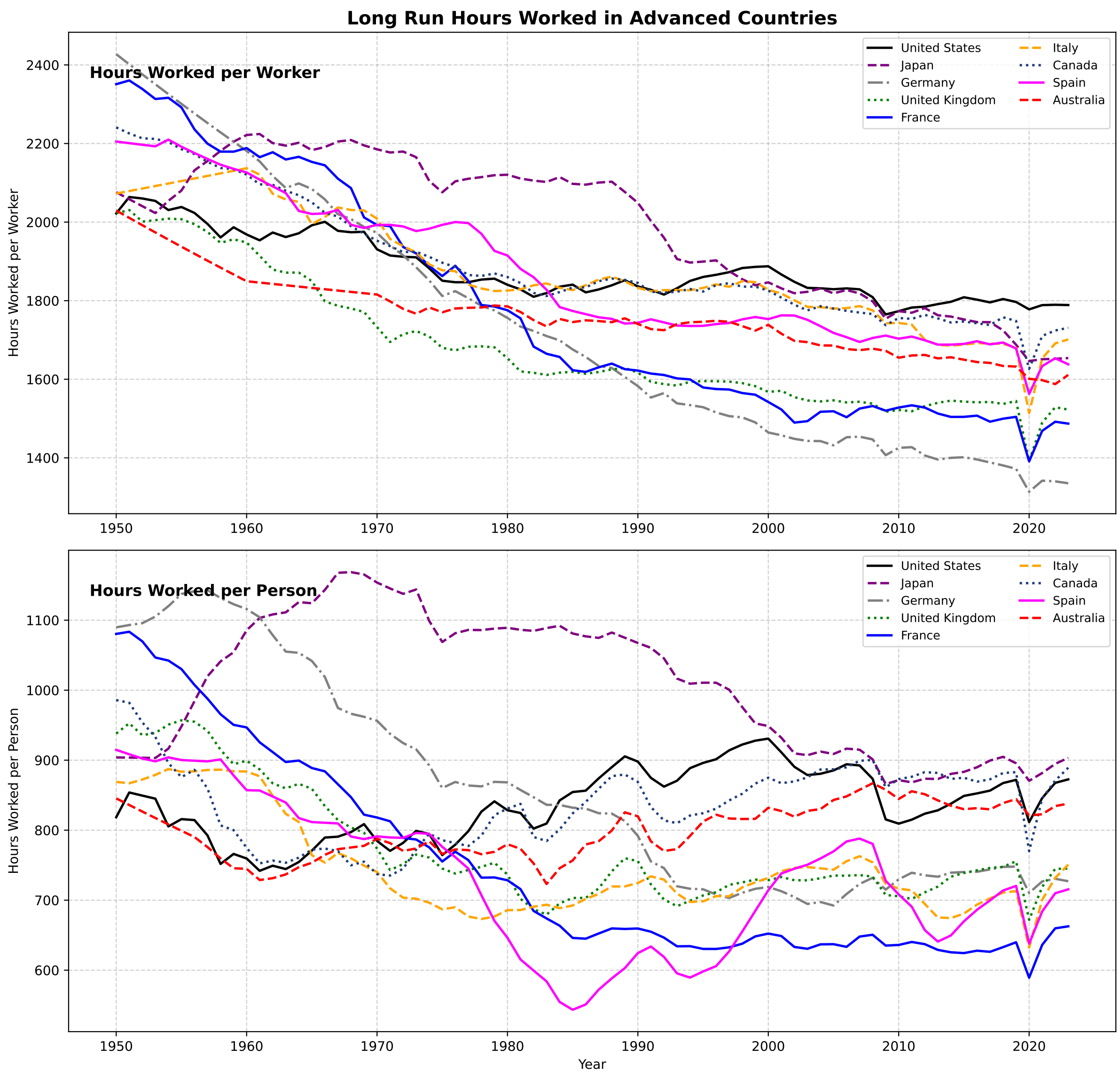
[1]Following Karabarbounis (2024) for the labor share and Boppart and Krusell (2020) for hours worked, it is possible to focus on slightly different advanced countries and on a longer period. In summarizing the evidence, the paper focuses on consistency and reports statistics on the nine countries listed in Table 1, 2 and 3. Adding single advanced economies does not change the nature of the stylized facts.

Figure 1: Dynamics of Labor Share



Source: World Penn Table (Feenstra et al., 2015)

Figure 2: Dynamics of Hours Worked



Source: World Penn Table (Feenstra et al., 2015) line with Karabarbounis (2024)- shows that that since the mid 80's for the representative advanced economy the labor share has been declining at a rate of −0.11 percent per year, a value that we will consider in the calibration exercise. The possible explanations for the secular fall in the labor share in the US and in most countries are different, and they relate to technology (Acemoglu and Rastrepo, 2022), cost of capital (Kaymak and Schott, 2023), market power by firms and rising concentration (Autor et al., 2020; Deb et al., 2022), changes in labor market institutions and firms' market power (Yeh et al., 2022), and globalization. While it is difficult to give weights to the possible concurrent factors, this paper examines the relationship between balanced growth, declining labor share and *monopsonistic* power by superstar firms. The existence of these types of firms is part of a separate literature that we review in section 2.4.

Table 1: Estimated Trend Coefficients for Labor Share

Country	*Linear Trend*[a]	*HP Filter*[b]
Period: 1950-2023		
United States	-0.0708***	-0.0708
France	-0.1119***	-0.1119
Australia	-0.2202***	-0.2202
Average[c]	-0.1343	-0.1343
Period: 1980-2023		
United States	-0.0781***	-0.0781
Japan	-0.1007***	-0.1007
Germany	-0.1304***	-0.1304
United Kingdom	0.0750***	0.0750
France	-0.1152***	-0.1152
Italy	-0.1666***	-0.1666
Canada	-0.1294***	-0.1294
Spain	-0.1514***	-0.1514
Australia	-0.2141***	-0.2141
Average[c]	-0.1123	-0.1123

Coefficients are expressed in percentage points per year.

[a] , Linear Trend column reports OLS slopes with significance.

Stars denote $^{***}p \leq 0.01$, $^{**}p \leq 0.05$, $^{*}p \leq 0.10$.

[b] , HP Filter column reports the slope of the HP-filtered trend ($\lambda = 100$).

[c] , Averages are simple means across countries within each subperiod.

Source: Authors' calculations based on Penn World Table (Feenstra et al., 2015).

2.2 Labor Supply in The Long Run

Traditional growth theory used to take the long run stability of total labor and leisure as a key stylized fact. Prescott (1986) argues that leisure shows no secular trend while the real wage has grown steadily. In terms of preferences, the stability of leisure and labor supply has been obtained by representative agent models in which the income and substitution effects of wage increases cancel each other out. The stability of hours worked over the long run has been challenged by Huberman and Chris (2007), Ramey and Francis (2009), and more recently by Boppart and Krusell (2020). Indeed, hours worked per worker have been declining in most countries when we consider a time horizon of more than a century, similarly to what is reported in the top part of Figure 2 for the nine

Table 2: Estimated Trend Coefficients for Hours per worker and Hours per Person

Country	*Hours per worker*		*Hours per Person*	
	Linear Trend[a]	*HP Filter*[b]	*Linear Trend*[a]	*HP Filter*[b]
Period: 1950-2023				
United States	-0.3406***	-0.3406	0.1352***	0.1352
France	-1.2975***	-1.2975	-0.5962***	-0.5962
Australia	-0.4429***	-0.4429	0.1101***	0.1101
Average[c]	-0.6937	-0.6937	-0.1169	-0.1169
Period: 1980-2023				
United States	-0.1382***	-0.1382	-0.0246	-0.0246
Japan	-1.1386***	-1.1386	-0.6064***	-0.6064
Germany	-0.9044***	-0.9044	-0.2630***	-0.2630
United Kingdom	-0.3196***	-0.3196	0.0572**	0.0572
France	-0.5361***	-0.5361	-0.1080***	-0.1080
Italy	-0.5033***	-0.5033	0.0144	0.0144
Canada	-0.3419***	-0.3419	0.1271***	0.1271
Spain	-0.4160***	-0.4160	0.3392***	0.3392
Australia	-0.3909***	-0.3909	0.2036***	0.2036
Average[c]	-0.5543	-0.5543	-0.0734	-0.0734

Coefficients are expressed in percentage points per year.

[a] , Linear Trend columns reports OLS slopes with significance.

Stars denote $^{***}p \leq 0.01$, $^{**}p \leq 0.05$, $^{*}p \leq 0.10$.

[b] , HP Filter columns reports the slope of the HP-filtered trend ($\lambda = 100$).

[c] , Averages are simple means across countries within each subperiod.

Source: Authors' calculations based on Penn World Table (Feenstra et al., 2015).

countries of our analysis. Hours worked per person have declined too for most countries, even if post World War II data for the US suggest a stable level of hours worked per person. Yet, even the US features a marked secular decline over two centuries (Boppart and Krusell, 2020).[2] The bottom part of Figure 2 reports the post world war dynamics in hours per capita, while Table 2 reports the trend estimates for hours worked. Table 2 shows that for the average country of our sample- in a way fully coherent with the evidence reported in Boppart and Krusell (2020)- hours worked per worker decline at rate approximately equal to -0.55 percentage point per year. With respect to hours per person, the sample used suggests a more modest decline, and the estimates we use for the average country is approximately -0.1 percentage point per year. Boppart and Krusell (2020) provide a class of preferences - used in this paper- that are coherent with balanced growth and declining hours worked. Boppart et al. (2023) explore the extensive and intensive margin of labor supply.

2.3 GDP Growth

Table 3 reports the average annual growth rates in GDP per capita and in GDP per worker for the nine countries. The average annual growth in GDP pe capita since the 1980 is approximately 1.5 percent for the average advanced economy. The same statistics for GDP per worker is slightly higher. If one considers the longer period 1950-1980 for the countries for which there are data on the labor share and hours worked (Table 1 and Table 2), Table 3 highlights the decline in GDP growth in advanced economies over the last 40 years. In the remainder of our analysis, we will consider GDP growth for the "representative advanced economy" as the growth rates reported in Table 3 for 1980 to 2023.

2.4 Superstar Firms

The rise of concentration in market power by firms has been recently addressed by Autor et al. (2020), Hsieh and Rossi-Hansberg (2023) and Kwon et al. (2024). The existence of superstar firms is linked to the increasing concentration of sales and employment in few large firms in each major industry. In addition, superstar firms have faster productivity growth than non-superstar firms and markups that rise over time. Autor et al. (2020) argue that the rise of superstar firms is associated with a marked decline in the labor share in the same industries. The key driver and mechanism emphasized by Autor et al. (2020) is rising monopoly power of these firms in the product market. Kwon et al. (2024) do not look at the labor market implications of the secular increase in concentration in the corporate sector. Our paper takes as given the existence of monopolistic superstar firms, and studies the growth consequences of superstar firms with labor market power, in a way consistent with the recent evidence on monopsony in the US reviewed below.

[2] Ramey and Francis (2009) argues that the decline in hours worked per person are mainly accounted for by an increase in schooling by young workers. Conversely, prime age individuals between the ages of 25 and 54 are working the same number of hours now as in 1900, as a combination of a rise in female hours worked and reduction in male hours.

Table 3: Average Growth Rates of GDP per capita and GDP per worker

Country	*Growth GDP per capita*	*Growth GDP per worker*
Period: 1950 onward		
United States	1.9816	2.1534
France	3.1102	3.2451
Australia	2.0221	2.0669
Average[a]	2.3713	2.4885
Period: 1980 onward		
United States	1.4776	1.6902
Japan	1.3484	1.4529
Germany	1.3891	1.5230
United Kingdom	1.9254	2.0556
France	1.3379	1.4666
Italy	1.0824	1.1962
Canada	1.4498	1.5861
Spain	1.5892	1.7137
Australia	1.6874	1.8442
Average[a]	1.4985	1.6920

Entries report compound annual growth rates over each subperiod.

Values are expressed in percentage points per year.

[a] , Averages are simple means across countries within each subperiod.

Source: Authors' calculations based on Penn World Table (Feenstra et al., 2015).

2.5 Rest of the Growth Literature

Monopsony in Growth and Aggregate Labor Markets

Economics of growth has paid remarkably little attention to the growth effects of labor market imperfections, even though market failures and monopolistic power played a key role in the endogenous growth revolution of the early 90s (Aghion and Howitt, 1992; Romer, 1990). Mortensen and Pissarides (1998) and Aghion and Howitt (1994) study the effect of growth on unemployment. More recently, Martellini and Menzio (2020) incorporates constant unemployment in a balanced growth model with declining search frictions. Research on the effects of monopsony power on aggregate labor market is vast theoretically and empirically, as documented- among others- by the recent survey by Manning (2021). Deb et al. (2022) study superstar firms in a static model in which firms enjoy marker power also in the hiring of labor. With respect to Deb et al. (2022), we focus on the growth effects of monopsony power, a topic that has received much less attention. One exception is Barr and Roy (2008). In their model, the supply of labor is driven by spatial labor mobility costs between an informal sector and a urban production, and the low wage in equilibrium leads to lower human capital accumulation and slow down growth. The resulting endogenous growth is suboptimal. Barr and Roy (2008) do not link monopsony to the dynamics of the labor share, to the dynamics of labor supply nor to the existence of superstar firms.

Growth Theory and the Labor Share

Several recent papers developed growth models with implications for the *level* of the labor share. Akcigit and Ates (2021, 2023) provide growth models with endogenous markups that imply negative correlation between market concentration and the level of the labor share. While these models generate balanced growth paths with different level of the labor share, we are not aware of growth models in which the labor share declines at a permanent constant rate, as in the current paper. Similarly, models with endogenous automation (Acemoglu, 2025) show that capital labor substitution can reduce the level of the labor share, but do not generate a constant and permanent decline. Similarly, Jones and Liu (2024) solve for a BGP with constant labor share and two types of capital-embodied technological change. Further, outside BGP they calibrate the two technological forces to obtain stagnant wages and declining labor share.

Growth Theory and Structural Change

The model we present features a labor reallocation across sectors driven by TFP growth differential. In this sense, our theory borrows some of the ideas from the structural change literature (Kongsamut et al., 2001; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). The long run asymptotic balanced growth path that we obtain when productivity growth in the superior sector is larger than productivity growth in the inferior sector is similar to the long run equilibrium of Acemoglu and Guerrieri (2008). With respect to both Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) we solve a fully decentralized model with imperfections in both the good and the labor markets, multiple firms in the superior sector and endogenous labor supply.

Misallocation

Jones (2022) argues that studying the misallocation of factors in production is the key research avenue in the economics of growth in the years to come. The key existing contributions on misallocation are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The intuition in this literature is that misallocation at the micro level aggregates up into TFP differentials. For example, Hsieh et al. (2022) argues that reduced misallocation of talent in the US as a result of the fall in labor market discrimination accounts for 0.3 of TFP growth in the last 20 years. In our model, the key driver of misallocation is the existence of market power in the hiring of labor by superstar firms, which slows down the reallocation of factors of production to the superior, more productive sector (compared to the optimal competitive outcome). In Section 5, the paper calibrates the consumption equivalent loss implied by the model for the US.

3 A Model of Superstar Monopsonistic Growth

3.1 Technology and Market Structure

We consider an economy with a single consumption good produced competitively by combining the output of two intermediaries. The intermediaries are called good s and good i and are produced in different sectors. The superscript s and i refer to a superior and inferior sector in a sense that we specify below. The elasticity of substitution between the two intermediate goods is indicated by σ. Total output is obtain by a Constant Elasticity of Substitution production function in the two goods and it is thus

$$Y_t = \left[\zeta^{\frac{1}{\sigma}}(y_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}(y_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where y_t^j with $j \in \{i, s\}$ are the output of the two intermediaries. We indicate with p_t^s and p_t^i the price of the two intermediaries and we normalize at all times the price of the final consumption good so that

$$1 \equiv P_t = \left[\zeta(p_t^s)^{1-\sigma} + (1-\zeta)(p_t^i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{2}$$

The intermediaries produce with a constant returns to scale technology that combines labor with capital. Labor in sectors j is indicated with N_t^j while capital is indicated with K_t^j, with $j = \{i, s\}$, and A_t^j is exogenous TFP level. The intermediaries produce with a neoclassical technology that has constant returns to scale in (K_t^j, N_t^j), and we assume that their production function is Cobb Douglas with a common elasticity of output to capital $\alpha \in (0, 1)$ in both sectors so that

$$y_t^s = (K_t^s)^\alpha (A_t^s N_t^s)^{1-\alpha}; \quad y_t^i = (K_t^i)^\alpha (A_t^i N_t^i)^{1-\alpha}.$$

In the paper we label the intermediary s as a sector that produces a superior good using a superior technology such that

$$A_t^j = A^j e^{g^j t}; \quad j = i, s; \quad g^s > g^i; \quad A^s > A^i\ .$$

For symmetry, the intermediate good i is labeled as the inferior good and sector. In words, the superior sector has higher level of output per unit of labor and capital, and grows faster. The fact that productivity at time zero is greater in the superior sector is not crucial, since the higher growth of sector s means that eventually its TFP will overtake that of sector i.

The intermediaries are produced not only with different technologies, but also with different market structures. The intermediate good s is produced by m superstar firms, and the superscript s refers to both the superior sector and the superstar firms. Output by firm m in the s sector is indicated by $y_t^{f,s}$ with $f \in \{1, ..., m\}$, so that total output of the intermediary s is

$$y_t^s = \sum_{f=1}^{m} y^{f,s}; \qquad y_t^{f.s} = (K_t^{f,s})^\alpha (A_t^s N_t^{f,s})^{1-\alpha},$$

where for simplicity and symmetry we assume that the level of technology by the m firms in the s sector is the same, or that $A_t^{f,s} = A_t^s\ \forall f$.[3] The idea is that sometime in the past the m firms in sector s discovered and patented an efficient way to produce the good s. The good produced by the m firms is perfectly homogeneous. The property rights to the technology s ensure that the m firms have oligopoly power in their good market and oligospony power in their labor market. This implies that a superstar firm f is a oligopolist in selling the output $y_t^{f,s}$ and a oligopsonist in the hiring of labor $N_t^{f,s}$. The paper does not study how superstar firms acquires their market power, but rather the growth consequences of being a superstar in the hiring of labor and the production of good s.[4]

[3]The oligopoly problem can easily be extended to the case in which $A_t^{f,s}$ differ across firms. What is necessary is that the m firms share the same growth rate g^s.

[4]Autor et al. (2020) shows that the superstar firms have higher likelihood to patent new product and larger investment in R&D with respect to ordinary firms.

The degree of product market power in sector s depends on the elasticity of substitution between the two intermediate goods. In the paper we study the case of imperfect substitutability of the two goods ($\sigma < \infty$), but the analysis extends to the case of perfect substitutability ($\sigma \to \infty$). Since the superstar firm is oligopolist, the paper rules out the case of complementarity between the two goods ($0 < \sigma \leq 1$). The inferior good is produced competitively and the firms hires labor in a competitive market. The capital market is competitive for all firms in the economy.

Regardless of the degree of substitutability between the two intermediaries, the superstar firm is always a *oligopsonist* in the hiring of labor in sector s. While the price of labor for the two sectors is indicated with w_t^s and w_t^i, the superstar firms have oligopsony power in the sense of Berger et al. (2022), who extend the classic model of Robinson (1969) to a market with m firms. While the classic monopsony model is inherently static, we study its long run consequences in a growth setting. The capital markets in the production of intermediaries are competitive.

3.2 Preferences

The economy is populated by a measure one of identical, infinitely-living (or perfectly altruistic) agents, and we abstract from population growth. The individual is endowed with 1 unit of time and we shall indicated with $h_t \leq 1$ the total hours supplied to labor, while we denote consumption of the final output good by C_t. The instantaneous utility function reads

$$U(C_t, h_t) = u(C_t) - v(h_t),$$

$U(C_t, h_t)$ is additive separable in consumption and total hours, and features standard assumptions. Thus $u(c)$- defined on $\Re_+$- is strictly increasing, concave and twice differentiable, with derivatives $u' > 0$ and $u'' < 0$ inside of the domain. $v(h_t)$ represents the dis-utility of labor and is defined on $\Re_+$. It is strictly increasing, concave and twice differentiable, with derivatives $v' > 0$ and $v'' > 0$ in its domain. The present discounted utility value of a stream $\{C_t, h_t\}_{t \geq 0}$ to a household is

$$\int_0^\infty e^{-\rho t} \left[u(C_t) - v(h_t) \right] dt,$$

where $\rho > 0$ is the subjective discounting. Within the Boppart and Krusell (2020) class preferences, from now on we consider the instantaneous utility $u(C, h)$ of the CRRA (Constant Relative Risk Aversion) class, originally proposed by MaCurdy (1981). The utility function is CRRA in consumption with coefficient $\gamma > 0$ and features a constant Frisch elasticity θ, so that

$$u(C_t) - v(h_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}; \quad \text{if } \gamma \neq 1,$$

and $u(C_t) = \log C_t$ if $\gamma = 1$. The parameter γ is the inverse of the elasticity of the marginal utility of consumption. To be coherent with the Boppart and Krusell (2020) class, the parameter γ should be strictly greater than one ($\gamma > 1$). The parameter θ is the percentage change in hours when the wage is changed by 1%, keeping the marginal utility of consumption constant. With this particular set of preferences, $\epsilon_{w_t, n_t} = \theta$, so that the firm sets wage using the $\lambda - \textit{constant}$ elasticity of labor supply.

Households own all factors of production. Given their time endowment, they supply labor in quantities n_t^s and n_t^i to the firms in both sectors so that

$$n_t^i + \sum_{f=1}^{m} n_t^{s,f} = n_t^i + n_t^s = h_t,$$
$$h_t \leq 1.$$

Household can save in two safe assets $\mathbb{A}_t^j$ that delivers a rate of return r_t^j, with $j \in \{i, s\}$. The safe assets give rights to capital stocks in the production of intermediaries.

3.3 Final Good Sector and the Problem of the Inferior Intermediary

We now present the simple decentralized model with superstar oligopsonists and oligopolists in the intermediate superior sectors. The final firm produces in a competitive sector with technology regulated by the CES production function (1) and by the price index (2). If we indicate with Π_t^f the profits in the final sector, the competitive firm problem concerns the choice of the two intermediaries y_t^s, y_t^i, so that

$$\Pi_t^f = \max_{y_t^s, y_t^i} Y - p_t^s y_t^s - p_t^i y_t^i. \tag{3}$$

The first order conditions imply the demand functions

$$y_t^s = \zeta (p_t^s)^{-\sigma} Y_t, \tag{4}$$
$$y_t^i = (1-\zeta)(p_t^i)^{-\sigma} Y_t, \tag{5}$$

where σ is the elasticity of substitution between the two intermediaries in the production of the final good. Note that equations (4) and (5) hold at the aggregate level since the single intermediate firm is price taker.

The competitive firms producing good i in the inferior sector maximize profits as price takers in both the good and the factor markets, and their period by period problem is

$$\Pi_t^i = \max_{K_t^i, N_t^i} = p_t^i y_t^i - w_t^i N_t^i - R_t^i K_t^i, \tag{6}$$

where N_t^i is the choice of labor by the competitive intermediaries w_t^i is the wage and R_t^i is the unit price of capital. Factor demands in the intermediate sectors, thanks to the Cobb-Douglas production assumption, are

$$w_t^i = p_t^i \frac{(1-\alpha) y_t^i}{N_t^i}; \quad R_t^i = p_t^i \frac{\alpha y_t^i}{K_t^i}; \tag{7}$$

and p_t^i is the price level that, at equilibrium, satisfies the demand function (5).

3.4 The Superstar Oligopolist and Oligopsonist

The core of the growth model is the problem of the superstar firms in the superior sector. Firms are both oligopolists and oligopsonists. On the product side, the oligopolist f competes with other $m-1$ firms and faces a downward sloping demand function given by equation (4). On the input side the firm is price taker in the choice of capital at cost R_t^s, while it is oligopsonist in the labor market. It faces an upward sloping relationship between the wage and its employment that we indicate at this stage with $w_t^{f,s}(N_t^{f,s})$, where $N_t^{f,s}$ is time t employment of the superior firm f. The firm f competes à-la-Cournot with other $m-1$ firms and chooses the quantity of output via the choice of labor and capital. In what follows we indicate the total quantity of employment and output produced by the firms in sector s different from f, respectively as $N_t^{-f,s} = \sum_{l=1;l\neq f}^{m} N_t^{l,s}$ and with $y_t^{-f,s} = \sum_{l=1;l\neq f}^{m} y_t^{l,s}$. Firm f takes as given such quantities and we can write

$$y_t^s = y_t^{f,s} + y_t^{-f,s}.$$

The market power of firm f is derived by full knowledge of the following fundamental conditions

$$\begin{aligned} w_t^{f,s}(N_t^{f,s}; N_t^{-f,s}) &= p_t^i(1-\alpha)(K_t^i)^{\alpha}(N_t^i)^{-\alpha}(A_t^i)^{1-\alpha}\ , \quad N_t^i = H_t - N_t^{f,s} - N_t^{-f,s} \\ p_t^s &= Y_t^{\frac{1}{\sigma}}(y_t^{f,s} + y_t^{-f,s})^{-\frac{1}{\sigma}}\zeta^{\frac{1}{\sigma}} \end{aligned} \tag{8}$$

While the firm takes as given the total hours H_t and total employment by other firms $N_t^{-f,s}$, the first equation above implies that the firm knows that the alternative labor condition for its potential labor force is employment in the inferior sector that hires N_t^i labor. The second equation is the demand for the intermediary y_t^s that implies that the firm faces a downward sloping output demand and it takes as given the production of the other oligopolists $y_t^{-j,s}$.

The superstar f firm maximizes profits period by period and its time t problem solves

$$\begin{aligned} \max_{N_t^{f,s}, K_t^{f,s}, W_t^{f,s}} \Pi_t^{f,s} &= p_t^s(y_t^{f,s}; y_t^{-f,s})y_t^s(K_t^{f,s}, N_t^{f,s}, A_t^s) - W_t^{f,s}N_t^{f,s} - R_t^s K_t^{f,s} \\ \text{s.t} \qquad & W_t^{f,s} \geq w_t^{f,s}(N_t^{f,s}), \end{aligned}$$

where the constraint highlights the fact that the wage set by the firm f must be as large as the outside option. With respect to the wage, the firm optimally sets a *limit wage* and satisfies the constraint with equality. In moving along the labor demand function in the inferior sector, the superstar firm faces an upward sloping relationship between the wage paid $w_t^{f,s}$ and its labor force. In light of equation (8) the firm moves along a (smooth) upward sloping wage employment schedule fully in line with the intuition of Robinson (1969). For given employment of other firms $N_t^{-f,s}$, the firm thus faces a classical monopsonistic problem, and the total labor costs to the firm are $w_t^{f,s}(N_t^{f,s}; N_t^{-f,s})N_t^{f,s} = p_t^i(1-\alpha)(K_t^i)^{\alpha}(H_t - N_t^{f,s} - N_t^{-f,s})^{-\alpha}(A_t^i)^{1-\alpha}N_t^{f,s}$, with marginal cost of labor to the superstar firm that can be expressed as

$$\begin{aligned} \frac{\partial w_t^{f,s}(N_t^{f,s}; N_t^{-f,s})N_t^{f,s}}{\partial N_t^{f,s}} &= p_t^i(1-\alpha)\frac{y_t^i}{H_t - N_t^{f,s} - N_t^{-f,s}}\left[1 + \frac{\alpha(N_t^{f,s} + N_t^{-f,s})}{H_t - N_t^{f,s} - N_t^{-f,s}}\right] \\ &= w_t^{f,s}\left[1 + \alpha\frac{(N_t^{f,s} + N_t^{-f,s})}{N_t^i}\right] \end{aligned}$$

Figure 3- in the top panel- plots the classic textbook choice of labor in the context of the model. Note that since $\epsilon_{N^{f,s},w^{f,s}} = \frac{dN^{f,s}}{dw^{f,s}}\frac{w^{f,s}}{N_t^{f,s}}$ we have that the marginal cost of labor to the firm is exactly in line with Manning (2021), since the firm elasticity of labor supply is defined as $\epsilon_{N^{f,s},w^{f,s}} = \frac{N^i}{\alpha(N^{f,s}+N^{-f,s})}$ so that $\frac{\partial w_t^{f,s}(N_t^{f,s})n_t^{f,s}}{\partial N_t^s} = w_t^s\left[1+\frac{1}{\epsilon_{N^{f,s},w^{f,s}}}\right]$. The expression in square brackets[5] represents the markdown of wages over the marginal product,

$$\mu_t^{f,s} = 1 + \frac{1}{\epsilon_{N^{f,s},w^{f,s}}} \tag{11}$$

and it plays a key role in the analysis.

The second dimension of market power to the superstar firm is related to the downward sloping marginal revenue. It implies a markup over the price that depends on the elasticity of substitution between the two intermediate goods so that

$$\frac{\partial\left(y_t^{f,s}p_t^s(y_t^{f,s}+y_t^{-f,s})\right)}{\partial y_t^{f,s}} = \left(1-\frac{1}{\sigma}\frac{y_t^{f,s}}{y_t^{f,s}+y_t^{-f,s}}\right)p_t^s(y_t^{f,s}+y_t^{-f,s})$$

Note that as $\sigma\to\infty$ the intermediate firm becomes a price taker, and the problem makes sense as long as $\sigma>1$, a standard restriction in monopoly theory, where σ is the constant demand elasticity. The final key first order condition for choice of labor for the superstar firms is the solution to

$$\underbrace{\underbrace{p_t^s(y_t^{f,s}(K_t^{f,s},N_t^{f,s},A_t^s))\left(1-\frac{1}{\sigma}\frac{y_t^{f,s}}{y_t^{f,s}+y_t^{-f,s}}\right)}_{MR}\frac{\partial y_t^s}{\partial N_t^{f,s}}}_{MRP} = \underbrace{\underbrace{p_t^i(y_t^i(K_t^i,N_t^i,A_t^i))\frac{\partial y_t^i}{\partial N_t^i}}_{w_t^i}\underbrace{\left[1+\alpha\frac{N_t^{f,s}+N_t^{-f,s}}{N_t^i}\right]}_{\mu_t}}_{MC}, \tag{12}$$

where the marginal revenue product of labor is equal to its marginal cost. The left hand side of equation (12) is linked to product market power while the right hand side is related to labor market oligopsonistic power. In what follows we assume that equation (12) leads to an interior solution $0\le N_t^{f,s}\le 1$, which is true as long as the following assumption 1 is satisfied.

Assumption 1. *The elasticity of output to capital is strictly positive, or* $\epsilon_{y_t^s,K_t^s}=\alpha\in(0,1)$.

Assumption 1 is a key condition for the existence of monopsonistic power, and implies that a model with constant returns to labor is not compatible with an interior solution. Assumption 1 ensures that in equation (12) the marginal revenue product to the oligopsonist is equal to its marginal cost. The

[5]We can derive the expression for the markdown when the inferior sector uses a general technology $G(K_t^i,N_t^i,A_t^i)$. In that case the expression for the markdown reads

$$\mu_t^{f,s} = 1+\frac{n_t^s}{mn_t^i}\frac{-G_{NN}(K_t^i,n_t^i,A_t^i)}{G_N(K_t^i,n_t^i,A_t^i)} ,$$

see Appendix B for details of the model with general technologies.

choice of capital by firm f is influenced by oligopoly power in the product market and its problem solves,

$$p_t^s \frac{\partial y_t^{f,s}}{\partial K_t^{f,s}} = \left(1 - \frac{1}{\sigma}\frac{y_t^{f,s}}{y_t^{f,s} + y_t^{-f,s}}\right) R_t^s, \tag{13}$$

where the left hand side is the marginal revenue product of capital and the left hand side is the marginal cost.

Symmetric Superstar Equilibrium

The symmetric Cournot equilibrium is a situation in which all m firms in the superior sector produce and hire the same quantity of capital and labor, so that

$$\begin{aligned} y_t^{f,s} = \frac{y_t^{-f,s}}{m-1} &= \overline{y}_t^s; \quad y_t^{-f,s} = (m-1)\overline{y}_t^s; \quad y_t^s = m\overline{y}_t^s; \\ N_t^{f,s} = \frac{N_t^{-f,s}}{m-1} &= \overline{N}_t^s; \qquad N_t^{-f,s} = (m-1)\overline{N}_t^s; \quad N_t^s = m\overline{N}_t^s \\ K_t^{f,s} &= \overline{K}_t^s; \quad K_t^s = m\overline{K}_t^s \end{aligned}$$

The superior firm first order conditions for the average firm becomes

$$p_t^s\left(\frac{\sigma m - 1}{\sigma m}\right)\frac{\partial y_t^s}{\partial N_t^s} = p_t^i \frac{\partial y_t^i}{\partial N_t^i}\left[1 + \alpha \frac{\overline{N}_t^s}{H_t - m\overline{N}_t^s}\right] = p_t^i \frac{\partial y_t^i}{\partial N_t^i}\left[1 + \alpha \frac{N_t^s}{mN_t^i}\right] \tag{17}$$

$$p_t^s\left(\frac{\sigma m - 1}{\sigma m}\right)\frac{\partial y_t^s}{\partial K_t^s} = p_t^i \frac{\partial y_t^i}{\partial K_t^i}, \tag{18}$$

the demand of the superior intermediary is

$$p_t^s = Y_t^{\frac{1}{\sigma}}(m\overline{y}_t^s)^{-\frac{1}{\sigma}}\zeta^{\frac{1}{\sigma}}; \quad m\overline{y}_t^s = \zeta(p_t^s)^{-\sigma}Y_t,$$

and the final good is

$$Y_t = \left[\zeta^{\frac{1}{\sigma}}(m\overline{y}_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}(y_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Equation (17) plays a key role in growth since it implies that the markdown can growth indefiently since $\lim_{(N_t^s)/(nN_t^i)\to\infty} \mu_t^{f,s} = \infty$. This result will be shown later. Equation (17) implies that the fraction between the revenue marginal product and the wage is the product between the markup on the product market and the markdown on the labor market,

$$\frac{p_t^s \frac{\partial y^s}{\partial N^s}}{w_t^s} = \underbrace{\frac{m\sigma}{m\sigma - 1}}_{markup}\underbrace{\left(1 + \frac{1}{\epsilon_{N_t^s, w_t^s}}\right)}_{markdown}. \tag{19}$$

While the oligopsonists sets the quantity of labor in equating marginal revenue to the marginal cost, the wage is set by the oligopsonists so as to satisfy the labor supply given by equation (8). Equation

(18) highlights the link between the markup of oligopoly and its effect on the capital market. While our model with CES implies a constant markup of the superstar firms, one can potentially build models with oligopolistic markups that grow over time, as in Autor et al. (2020). Equation (18) points out why such models are at odds with a constant capital output ratio. Indeed, in equation (18), constant values of R_t^s and p_t^s- a feature of the equilibrium we will solve and of most growth models- imply that a growing markup must necessarily be associated with a growing capital output ratio, a variable that is typically constant in long run growth (Jones, 2016).[6]

The inferior sector has similar conditions for labor and capital that read

$$p_t^i \frac{\partial y_t^i}{\partial N_t^i} = w_t^i \ , \qquad p_t^i \frac{\partial y_t^i}{\partial K_t^i} = R_t^i \ .$$

When the two sectors pay the same wage and rate of return (i.e. when capital depreciation is the same across the two sectors), taking the ratio of the conditions of the two sectors leads to a fundamental factor allocation

$$\left[1 + \alpha \frac{N_t^s}{m N_t^i}\right] \frac{N_t^s}{K_t^s} = \frac{N_t^i}{K_t^i} \ ,$$

where it is clear that as long as $\alpha > 0$, the factor allocation of the superstar firm is distorted vis-á-vis a pure neoclassical factor ratio equilibrium that implies $\frac{N_t^s}{K_t^s} = \frac{N_t^i}{K_t^i}$. The latter condition characterizes the competitive equilibrium that will be discussed later. The superstar firm makes strictly positive profits that are fully distributed to the consumers who owns the firm, to which we turn next.

3.5 Household Problem and Budget Constraint

The representative agent maximizes lifetime utility. Agents can save in safe assets $\mathbb{A}^{f,s}, \mathbb{A}_t^i$ that yield rights to capital in sectors i, and to any firm in sector s with $f \in \{1, ..., m\}$. Agents own all firms in the economy and are thus entitled to a rebate of profits from the superstar sector. Agents can obtain wages in each sector that we label as w_t^i and $w_t^{f,s}$, with $t \in \{1, ..., m\}$. These amounts, together with rate of returns on assets $r_t^i, r_t^{f,s}$, are time varying and taken as given in the maximization problem. We denote the household supply of labor in sector j at time t by n_t^j, again with $j \in \{i, s\}$. Since the total endogenous hours supplied are indicated with h_t, this implies $n_t^i = h_t - n_t^s$, where $n_t^s = \sum_{f=1}^m n_t^{f,s}$ is the sum of employment in each superstar firm f in sector s. The total asset holdings of the representative household is the sum of the safe assets $\mathbb{A}_t^j$.

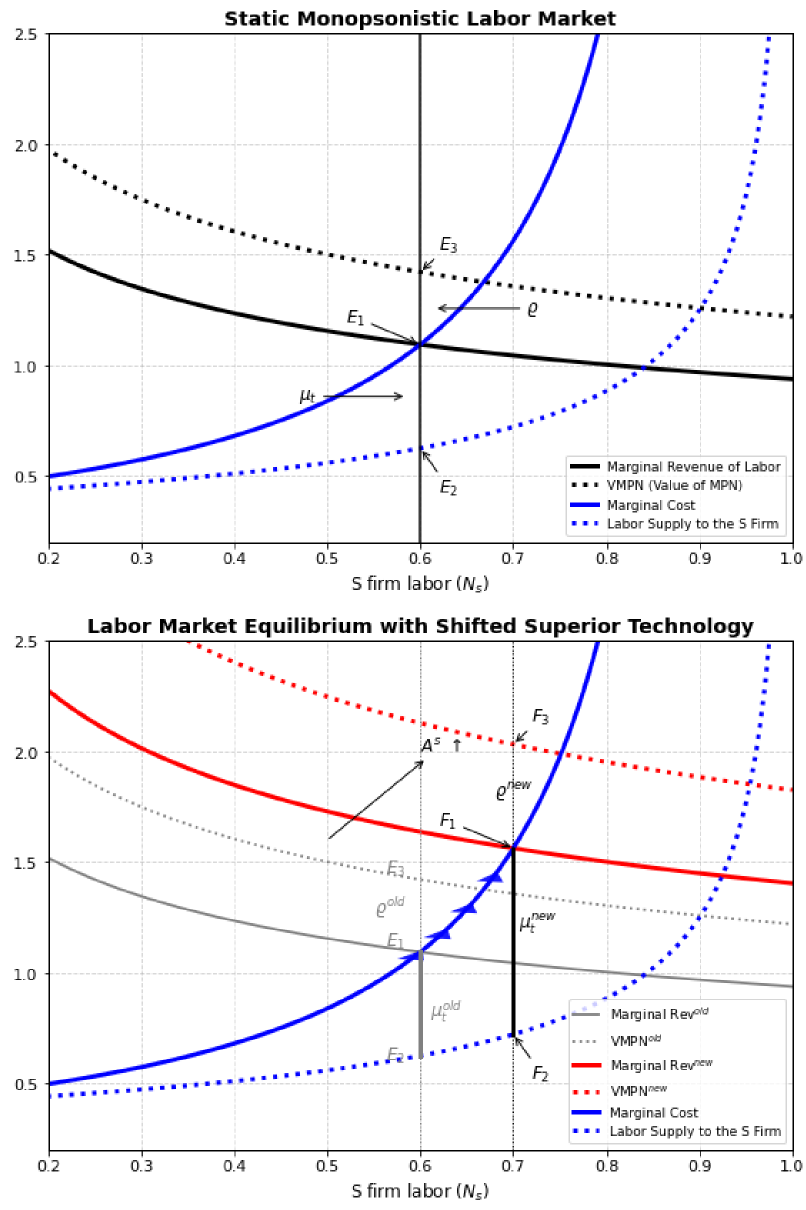
The household solves

$$\begin{aligned} \max_{[C_t, \{\mathbb{A}_t^{f,s}, n_t^{f,s}\}_{f=1}^m, \mathbb{A}_t^i, n_t^i]_{t \geq 0}} \quad & \int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \right] \mathrm{d}t \\ s.t. \quad & P_t C_t + \textstyle\sum_{f=1}^m \dot{\mathbb{A}}_t^{f,s} + \dot{\mathbb{A}^i}_t = \sum_{f=1}^m w_t^{f,s} n_t^{f,s} + \sum_{f=1}^m r_t^{f,s} \mathbb{A}_t^{f,s} + \sum_{f=1}^m \Pi_t^{f,s} + w_t^i n_t^i + r_t^i \mathbb{A}_t^i \\ & \textstyle\sum_{f=1}^m n_t^{f,s} + n_t^i = h_t; \quad n_t^{f,s}, n_t^i \geq 0 \ , \ \mathbb{A}_t^{f,s} \geq 0 \ , \mathbb{A}_t^i \geq 0, \end{aligned} \tag{20}$$

where $\sum_{f=1}^m \Pi_t^s$ denotes the sum of superstar firm profits and P_t is the natural price index of equation (2) normalized to one. The budget constraint in (20) deserves some comments. Factors allocated to

[6] Appendix F.3 makes this point explicit.

Figure 3: Monopsonistic Labor Market with the Superior Technology



the each sector yield wage income $\sum_{f=1}^{m} w_t^{f,s} n_t^{f,s}, w_t^i n_t^i$ and capital income $\sum_{f=1}^{m} r_t^{f,s} \mathbb{A}_t^{f,s}$. Indirectly, renting labor and capital to the superstar firms in sector s yields also profits $\sum_{f=1}^{m} \Pi_t^{f,s}$. Note that the budget constraint implies that the safe assets and the capital in sector i can be converted one to one in consumption. In order to have finite utility at equilibrium, we need to impose the following parametric restriction.

Assumption 2. *Parameters are such that* $-\rho + (1-\gamma)g^s < 0$.

To write the Hamiltonian, we define investments $q_t^{f,s} = \dot{\mathbb{A}}_t^{f,s}$ and $q_t^i = \mathbb{A}_t^i$, so that we have the states $x_t = (\{\mathbb{A}_t^{f,s}\}_{f=1}^m, \mathbb{A}_t^i)'$ and the controls $z_t = (C_t, \{n_t^{f,s}, q_t^{f,s}\}_{f=1}^m, n_t^i, q_t^i)'$. The current value Hamiltonian reads

$$\hat{H} = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \lambda_t \left(-C_t + \sum_{f=1}^{m} [-q_t^{f,s} + w_t^{f,s} n_t^{f,s} + r_t^{f,s} \mathbb{A}_t^{f,s} + \Pi_t^{f,s}] - q_t^i + w_t^i n_t^i + r_t^i \mathbb{A}_t^i \right) +$$

$$+ \sum_{f=1}^{m} \mu_t^{f,s} q_t^{f,s} + \mu_t^i q_t^i + \phi_t \left(-h_t + \sum_{f=1}^{m} n_t^{f,s} + n_t^i \right)$$

where we kept implicit the inequality constraints. For an interior solution we obtain the system

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma} \tag{21}$$

$$\psi (h_t)^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^{f,s}; \quad \forall f = 1...m \tag{22}$$

$$\psi (h_t)^{\frac{1}{\theta}} = C_t^{-\gamma} w_t^i \tag{23}$$

$$r_t^s = r_t^i; \qquad \forall f = 1...m \tag{24}$$

$$P_t C_t + \sum_{f=1}^{m} \dot{\mathbb{A}}_t^{f,s} + \dot{\mathbb{A}^i}_t = \sum_{f=1}^{m} w_t^{f,s} n_t^{f,s} + \sum_{f=1}^{m} r_t^{f,s} \mathbb{A}_t^{f,s} + \sum_{f=1}^{m} \Pi_t^{f,s} + w_t^i n_t^i + r_t^i \mathbb{A}_t^i$$

$$\lim_{t\to\infty} e^{-\rho t} u'(C_t) \mathbb{A}_t^{f,s} = \lim_{t\to\infty} e^{-\rho t} u'(C_t) \mathbb{A}_t^i = 0; \quad \forall f = 1...m,$$

where we used the notation $r_t = r_t^{f,s} = r_t^i \quad \forall f$. These first order conditions deliver some key features and properties of the oligopsonistic labor market. Equation (21) is a standard Euler condition for consumption, as in the Neoclassical Growth literature. Conditions (22) and (23) imply a *fundamental arbitrage condition in the labor market* for which $w_t^s = w_t^{f,s} = w_t^i \quad \forall f = 1..m$: along an equilibrium path the wage obtained in each firm of the superior sector and paid by the oligopsonist must coincide with the wage paid in the inferior sector. Equation (24) is a similar condition for capital and can be labeled as *fundamental arbitrage condition in the capital market.* The last two conditions are just the budget constraint and the transversality conditions.

3.6 Superstar Monopsonistic Equilibrium

Equilibrium definition In a market clearing equilibrium the total claims held by the individual must be equal to the amount of formal capital. In addition, aggregate employment by the superstar firms must be equal to household labor allocated in each sector. Further, since household assets are the same as the capital stock and capital used in each sector depreciates at rate $\delta^{f,s}$ $(f \in \{1....,m\}$ and δ^i, the market rate or return must be equal to the cost of capital net of depreciation. Finally, the wage function of the firm f oligopsonistic superstar firm must be coherent with the wage arbitrage condition of the household. This implies that at equilibrium of the asset and labor markets the following conditions apply to prices

$$r_t = R_t^{f,s} - \delta^{f,s} = R_t^i - \delta^i; \quad f \in 1,,,m\} \tag{25}$$

$$w_t = p_t^i(1-\alpha)(K_t^i)^{\alpha}(H_t - N_t^s)^{-\alpha}(A_t^i)^{1-\alpha}; \quad N_t^s = \sum_{f=1}^{m} N_t^{f,s},$$

as well as quantities

$$\mathbb{A}_t^{f,s} = K_t^{f,s}; \quad \mathbb{A}_t^i = K_t^i; \quad n_t^{f,s} = N_t^{f,s}; \quad n_t^i = N_t^i; \quad H_t = h_t; \quad f = 1, ..., m. \tag{26}$$

We are now in a position to define the symmetric equilibrium.

Definition 1. *Given a path for productivity* $\{A_t^s, A_t^i\}_{t=0}^{\infty}$ *and a set of superstar firms* m*, a superstar symmetric monopsonistic equilibrium (SSMPE) is a set of sequences of labor allocations* $\{\overline{n}_t^s, n_t^s, n_t^i, h_t\}_{t=0}^{\infty}$*, capital allocations,* $\{\overline{K}_t^s, K_t^s, K_t^i\}_{t=0}^{\infty}$ *intermediate and final good production* $\{\overline{y}_t^s, y_t^s, y_t^i, Y_t\}_{t=0}^{\infty}$*, consumption* $\{C_t\}_{t=0}^{\infty}$*, factor prices* $\{w_t^s, w_t^i, R_t^s, R_t^i\}_{t=0}^{\infty}$*, and intermediate goods prices* $\{p_t^s, p_t^i\}_{t=0}^{\infty}$ *such that*

1. *symmetric super employment* $\overline{n}_t^s$*, capital* $\overline{K}_t^s$ *and output* $\overline{y}_t^s$ *solve the oligopolistic and oligopsonistic symmetric firm problem (equations* (17) *and* (18)*);*
2. *intermediate prices* $\{p_t^s, p_t^i\}_{t=0}^{\infty}$ *and quantities* $\{y_t^s, y_t^i, Y^t\}_{t=0}^{\infty}$ *solve the final good maximization problem (Equations* (3)*,* (4) *and* (5)*);*
3. *labor allocations* $\{n_t^s, n_t^i, h_t\}_{t=0}^{\infty}$*, capital allocations* $\{K_t^s, K_t^i\}_{t=0}^{\infty}$ *and consumption* $\{C_t\}_{t=0}^{\infty}$ *solve the representative consumer problem (Equations* (20)*,* (22)*,* (23)*,* (21)*);*
4. *wages* $\{w_t^s, w_t^i\}_{t=0}^{\infty}$*, cost of capital* $\{R_t^s, R_t^i\}_{t=0}^{\infty}$*, labor and capital allocation* $\{n_t^s, n_t^i\}_{t=0}^{\infty}$ $\{K_t^s, K_t^i\}_{t=0}^{\infty}$ *solve the superstar monopsonistic firm problem- equations* (19) *and* (13)*,* (10)*; the competitive inferior market (equations* (6) *and* (7)*); and the arbitrage condition* (25)
5. *all markets clear, Eq.* (26)*.*

Fundamental system The evolution of the SSMPE is given by the following system of nine equations, in which two of them have time derivatives

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma} p_t^i (1-\alpha)(K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha}$$
$$n_t^s + n_t^i = h_t$$
$$\gamma \frac{\dot{C}_t}{C_t} = \frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s - \rho$$
$$C_t + \dot{K}_t^s + \dot{K}_t^i = Y_t - \delta^s K_t^s - \delta^i K_t^i$$
$$Y_t = \left[\zeta^{\frac{1}{\sigma}} ((K_t^s)^\alpha (A_t^s n_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} ((K_t^i)^\alpha (A_t^i n_t^i)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$
$$(K_t^s)^\alpha (A_t^s n_t^s)^\alpha = \zeta (p_t^s)^{-\sigma} Y_t \ ,$$
$$(K_t^i)^\alpha (A_t^i n_t^i)^\alpha = (1-\zeta)(p_t^i)^{-\sigma} Y_t$$
$$\frac{\sigma m - 1}{\sigma m} p_t^s (1-\alpha)(K_t^s)^\alpha (n_t^s)^{-\alpha} (A_t^s)^{1-\alpha} = p_t^i (1-\alpha)(K_t^i)^\alpha (n_t^i)^{-\alpha} (A_t^i)^{1-\alpha} \left[1 + \alpha \frac{1}{m} \frac{n_t^s}{n_t^i} \right] \tag{27}$$
$$\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (K_t^s)^{\alpha-1} (A_t^s n_t^s)^{1-\alpha} - \delta^s = p_t^i \alpha (K_t^i)^{\alpha-1} (A_t^i n_t^i)^{1-\alpha} - \delta^i$$

where $K_t^s = m\overline{K}_t^s, y_t^s = m\overline{y}_t^s, n_t^s = \overline{n}_t^s$ are defined in equation (15) with the symmetric Nash equilibrium. To study the dynamics of the model in a way coherent with standard growth theory, we would like to solve the system above for a balanced growth path. However, since the two sectors feature asymmetric growth, in the limit the superior sector prevails in the allocation of factors of production, and this generates structural reallocation in finite time. For this reason, we will characterize an *asymptotic balanced growth path* (a solution in which all variables grow at constant rates).

Definition 2. *A (asymptotic) balanced growth path (BGP) is a SSMPE that features constant aggregate consumption and GDP growth.*

We will now show that there exists a unique BGP solution to the model, that it is locally stable and describe its properties. In what follows we indicate with g_m the growth rate of variable m along the BGP. As usual, in order to solve the growth model we transform the system of equations in intensive form, by defining capital and consumption in efficiency units. In our case we have two types of capital and two types of labor, so we will define

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s} \ , \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i}, \quad c_t = \frac{C_t}{A_t^s n_t^s} \ .$$

Details on the definition of variables and the resulting system are presented in Appendix A.1.

Proposition 1. *The superstar monopsonistic growth problem solves for a unique asymptotic balanced growth with the following key features*

1. *the growth rate of consumption and output per-capita are constant $g_C = g_Y = g_{y^s} > g_{y^i}$, and the intermediate good sector absorbs the entire final good production and converges to a constant term ($y_t^s/Y_t \to \frac{1}{\delta^{1-\sigma}}$) and $y_t^i/Y_t \to 0$;*

2. *the relative labor allocation in the superior sector grows indefinitely* $\frac{1}{m}\frac{n_t^s}{n_t^i} \to \infty$ *but total hours grow at the constant rate* g_h *equal to the growth rate of hours to the superior sector* $g_h = g_{n^s} > g_{n^i}$*;*
3. *the growth rate of wages is lower than the growth of output* $g_w < g_Y$*;*
4. *the interest rate* r_t *and the capital per unit of efficiency in the superior sector are constant* ($r_t^s \to r$ *and* $x_t^s \to x^s$) *while the capital per unit of efficiency tends to infinity in the inferior sector* ($x_t^i \to \infty$).

For the sake of completeness, the balanced growth equilibrium has further implications. Specifically, the price of intermediate superior good tends to a constant ($p_t^s \to \delta^{\frac{1}{\sigma-1}}, g_{p^s} \to 0$), while the inferior good price grows indefinitely ($p_t^i \to \infty$) at an asymptotically constant rate. While the fundamental system of nine equations in nine unknowns looks hard to solve, its asymptotic growth analogous is a linear system in growth rates. Before looking at the solution of such system it is useful to study the asymptotic properties of the markdown, the key driver of the SSMPE.

Proposition 2. *The markdown of the monopsonistic superstar firm has a constant asymptotic growth rate* $\frac{\dot{\mu}}{\mu} = g_{n^s} - g_{n^i}$

The proof of Proposition 2 is straightforward and it just follows from the definition of the markdown. Indeed, the growth rate of the markdown implies that

$$\frac{\dot{\mu}_t}{\mu_t} = \frac{\alpha(g_{n^s}(t) - g_{n^i}(t))}{\frac{1}{m}\frac{n_t^i}{n_t^s} + \alpha}; \quad g_\mu = \lim_{t\to\infty}\frac{\dot{\mu}_t}{\mu_t} = g_{n^s} - g_{n^i},$$

where the result highlights that the indefinite growth of the markdown is a natural byproduct of the fact that in the limit $\frac{n_t^i}{n_t^s} \to 0$. Some intuition of the mechanism can be obtained from the bottom part of Figure 3, where we plot the implications of unbalanced growth in A^s and A^i along the firm labor supply in assuming that $m = 1$. The equilibrium growth rates can be obtained by the following linear system (zeroing all growth rates of converging values, $g_{p^s} = g_{x^s} = 0$, and imposing $g_C = g_Y$ and $g_h = g_{n^s}$)

$$\begin{aligned} g^s &= g^i + g_{p^i} + \alpha g_{x^i} + g_{n^s} + g_{n^i} \\ g_{p^i} + (\alpha-1)g_{x^i} &= 0 \\ g^i + g_{n^i} + \alpha g_{x^i} &= -\sigma g_{p^i} + g^s + g_{n^s} \\ \frac{1}{\theta}g_{n^s} &= -\gamma(g^s + g_{n^s}) + g_{p^i} + g^i + \alpha g_{x^i} \end{aligned}$$

Once the system is solved, one can recover $g_C = g^s + g_h$. The solution of the system leads to the final expression of the three key growth rates of consumption/GDP, hours and markdown that read

$$g_h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}g^s - \frac{1}{\left(\frac{1}{\theta}+\gamma\right)}\frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \tag{28}$$

$$g_C = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma}g^s - \frac{1}{\left(\frac{1}{\theta}+\gamma\right)}\frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \tag{29}$$

$$g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \tag{30}$$

The SSMPE is a a semi endogenous growth model, since its equilibrium growth rate crucially depends on the exogenous growth productivity g^s and g^i, but its endogenous rate fully interacts with the structural parameters of the model. For further characterizing its properties, we next solve for the corresponding competitive economy.

3.7 The Competitive Economy

In the competitive decentralized economy the superior sector is price taker in the production of the intermediary y_t^s and wage taker in the hiring of labor n_t^s. The rest of the economy is very similar to the structure of the SSMPE, since the final sector and the inferior intermediate sectors were already competitive. The representative consumer problem is substantially similar to the one solved in Section 3.5, with the only difference that in the budget constraint there are no rebated profits in equilibrium, since in the competitive superior sector $\Pi_t^s = 0$

The competitive economy can be studied in terms of growth rates or in level, exactly as the SSMPE. The competitive economy is a decentralized version of the unbalanced growth equilibrium studied by Acemoglu and Guerrieri (2008). While both intermediaries act in a competitive setting, their technology feature asymmetric growth path and in the limit the superior sector prevails in the asset allocation. We indicate with g_m^* the growth rate of variable m, where the superscript $*$ emphasizes that we are solving for the competitive economy. The intermediate goods produced in the superior sector absorb the entire production of the final good and- in terms of labor allocation- we look for a balanced growth path in which all labor moves to the superior sector and $n^s/n^i \to \infty$. The following proposition summarizes qualitatively the balanced growth of the competitive economy.

Proposition 3.

1. *The growth rate of consumption and output per-capita are constant $g_C = g_Y = g_{y^s} > g_{y^i}$; and the intermediate good sector absorbs the entire final good production ($y_t^s/Y_t \to \frac{1}{\delta^{1-\sigma}}$) and $y_t^i/Y_t \to 0$);*

2. *the relative labor allocation in the superior sector grows indefinitely $\frac{n_t^s}{n_t^i} \to \infty$ but total hours grow at the constant rate g_h equal to the growth rate of hours to the superior sector $g_h = g_{n^s} > g_{n^i}$;*

3. *the growth rate of wages coincide with the growth of output $g_w = g_Y$;*

4. *the interest rate r_t and the capital per unit of efficiency in the superior sector are constant (so that $r_t^s \to r_t$ and $x_t^s \to x^s$) while $x_t^i \to \infty$ in the inferior sector.*

Note that also in the competitive economy the price of intermediate superior good tends to a constant ($p_t^s \to \delta^{\frac{1}{\sigma-1}}, g_{p^s} \to 0$), while the inferior good price grows indefinitely at an asymptotically constant rate ($p_t^i \to \infty$).

Comparing the results in Propositions 3 and 1, the qualitative behavior of the two economies looks very similar. At this stage, the only exception is related to the growth of wages, that in the competitive economy coincides with the growth of output, a standard result in traditional balanced growth equilibria. This - in turn- has important effects on equilibrium growth rates.

The system of ODEs of the competitive economy, is analogous to the one of the monopsonistic economy, with which it shares most equations with one notable exception: Eq. (27) is substituted with

$$p_t^s(1-\alpha)(K_t^s)^\alpha(n_t^s)^{-\alpha}(A_t^s)^{1-\alpha} = p_t^i(1-\alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha} \ . \tag{31}$$

The full system is presented in Appendix A.2. Comparing Eq. (31) with Eq. (27), it is immediately apparent that the key difference concerns the absence of the monopsonistic markdown and monopolistic markup in the equation of the optimal economy. The question is whether these two distortions create misallocation in the growth process. Note that compared to the monopsonistic case there is also no markup in the Euler equation and equality of returns to capital. It turns out that the key distortion to the growth problem concerns the existence of monopsonistic power.[7] To derive such result one can obtain the corresponding system of equation in growth rates (presented in Appendix C), which is the same as the monopsonistic case apart from the first equation, which now reads

$$g^s = g^i + g^*_{p^i} + \alpha g^*_{x^i} \ .$$

The system solves for the following optimal growth rates

$$g^*_C = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma} g^s \ , \quad g^*_h = g^*_{n^s} = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s \ , \quad g^*_{n^s} - g^*_{n^i} = (1-\alpha)(\sigma-1)(g^s - g^i) \tag{32}$$

We are thus now in a position to study the misallocation and growth effect of the SSMPE.

3.8 Misallocation and Consumption Equivalent Loss

We can formally show that the imperfect labor markets induce negative growth effects in the SSMPE. The following proposition summarizes the main result.

Proposition 4. *The balanced growth path of the SSMPE is inefficient and both output per capita and total hours grow slower than in the corresponding optimal growth problem.*

To prove the proposition it is sufficient to compare the balanced growth path for hours and consumption in the decentralized monopsonistic equilibrium of Eq. (28) and (29) with the optimal ones in Eq. (32). The growth expressions can be written as

$$g_C = g^*_C - \frac{1}{\frac{1}{\theta}+\gamma} g_\mu \ , \quad g_h = g^*_h - \frac{1}{\frac{1}{\theta}+\gamma} g_\mu \ , \tag{33}$$

where $g_\mu = g_{n^s} - g_{n^i} > 0$ as reported in equation (30). Equation (33) immediately shows that as long as $g_\mu > 0$ (which is always the case at the decentralized equilibrium), monopsony power distorts both hours and consumption growth, generating a "growth drag" ($g_h < g^*_h$ and $g_c < g^*_c$). This implies that both the growth rate of consumption and the growth rate of total hours are negatively affected by the growth of markdown, and thus imperfect labor market induce an additional negative channel in the long run dynamics of labor supply. The total growth misallocation depends on the preferences parameters γ and θ as well as from the technological parameters α and σ. We first discuss the role of technological parameters. In assumption 1 we highlighted that the presence of diminishing returns to capital - and thus $0 < \alpha < 1$- is an essential prerequisite for the existence of monopsony. In addition, equation (33) shows that misallocative effects on output are present as long $\alpha < 1$, or as

[7] Appendix F solves a model with monopoly power only and shows that there are no growth distortions compared to the optimal when monopoly power is the only imperfection in the economy.

long as labor has a productive role in the superior technology. The role of σ operates through its effect on the growth of the markdown g_μ, and clearly a higher value of σ implies higher markdown growth ($\frac{\partial \mu}{\partial \sigma} > 0$). The parameter σ in the model has a dual role, since it reflects both the elasticity of substitution between the two intermediaries as well as the monopolist markup in the superstar firm. The model requires $\sigma > 1$ since the markup is simply $\varrho = \frac{\sigma}{\sigma - 1}$. Higher σ implies a lower markup and a lower monopoly power. Yet, one case easily establish that there is no *direct* growth effect of monopoly power in model. Indeed, a simple monopoly model without monopsony- as illustrated in Appendix F.2- shows that $g_c^m = g_c^*$, where g_c^m is the growth rate of the version of the model with only monopoly in the product market an no monopsony in the labor market. Yet, in the SSMPE a higher σ implies lower growth, through an *indirect* effect that operates via the markdown μ_t. The mechanism is the following. Higher σ implies higher elasticity of substitution between the goods produced in the superior and in the inferior sector. A larger substitution implies larger growth in markdown g_μ and thus larger monopsony power. This- in turn- leads to higher overall distortion and lower growth.

An alternative explanation to declining hours

The preference parameters θ and γ play an important role in the growth process, and in the associated long run decline in hours per person and hours worked. In the recent literature surveyed in Section 2, much of the attention has been devoted to the income effects of long run wage increases. Boppart and Krusell (2020) propose a class of preferences coherent with the observed long run decline in hours worked in balanced growth. With respect to the MaCurdy (1981) preferences specified in our model, the Boppart and Krusell (2020) class implies a value of the elasticity of marginal utility of consumption strictly larger than one (i.e. $\gamma > 1$). In other words, $\gamma > 1$ is a necessary condition for having a neoclassical growth model with declining labor supply, assuming competitive labor markets. Indeed, this is apparent from equation (32), in which the growth rate of hours is zero under $\gamma = 1$.

In the theory of labor supply, the MaCurdy (1981) utility function with $\gamma = 1$ implies a set of preferences in which income and substitution effect cancel each other out, as suggested in the literature by King et al. (1988). The SSMPE has an important result in this respect, as highlighted by the following proposition.

Proposition 5. *In the asymptotic balanced growth path of the SSMPE, the growth rate of hours is negative even when the income and substitution effect of wage changes cancel each other out (i.e.* $\gamma = 1$*).*

To show the latter we can simple set $\gamma = 1$ in the monopsonistic growth rate of hours to find

$$g_h = -\frac{1}{\frac{1}{\theta} + 1} \frac{(1-\alpha)(\sigma - 1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i) < 0$$

The result is due to the additional income effect generated by returns to capital and the profit rebate Π_t^s. In the macroeconomic literature, Prescott (2004) used this mechanism to highlight the steady state differences in hours worked across the two sides of the Atlantic. In this respect, our result is the dynamic and growth counterpart of the “classic” labor supply result.

Labor Share Dynamics

The model has also qualitative implications for the second macro fact outlined in Section 2, namely the dynamics of the aggregate labor share. In the competitive equilibrium the labor share is time invariant and its level is simply

$$\alpha^*_{L,t} = \frac{w_t^{*s} n_t^{*s} + w_t^{*i} n_t^{*i}}{Y_t^*} = 1 - \alpha \ ,$$

where the decentralized optimal wages w_t^{*s} and w_t^{*i} coincide with the marginal product of labor in each sector. This implies that there is no labor share dynamics in the optimal growth problem. Conversely, in the SSMPE the labor share is time variant and its level depends on the state of the economy. In the monopsonistic decentralized economy, again by definition the labor share equals

$$\alpha_{L,t} = \frac{w_t^s n_t^s + w_t^i n_t^i}{Y_t} \ .$$

Yet, since we know the balanced growth properties from Proposition 1, there is a clear result for the labor share dynamics.

Proposition 6. *The growth of the labor share is negative in the SSMPE and the capital share (which includes profits) tends to 1.*

The result follows directly from the fact that along the asymptotical balanced growth path, the growth rate of the labor share equals the opposite of the growth rate of the markdown,

$$\frac{\dot{\alpha}_L}{\alpha_L} = -g_\mu < 0 \ .$$

The Role of Market Concentration

Market concentration in the model has a dual interpretation, since it can be described as market size of top firms within the superior sectors or as a share of the superior sector in the total economy. In the superior sector there are m symmetric firms that grow faster than the inferior firms and eventually absorb the entire production of the economy. With respect to the asymptotic growth rates, changing the number of firms m does not imply any *growth effect*, as evident from equation (29). Yet, the number of firms m in the superior sector has key *level* effect on the economy. The number of oligopolists m in the superior sector reduce the level of the markup and at the limit, when the number of firms m growth indefinitely in the symmetric equilibrium we have that (for any fixed level of n^s and n^i)

$$\lim_{m\to\infty} \mu(m; n^s, n^i) = \lim_{m\to\infty} \left(1 + \alpha \frac{1}{m} \frac{n^s}{n^i}\right) = 1,$$

so that the oligopolistic sector tends to the competitive economy. In the same spirit, the markup charged by superstar firms decreases in m and goes to one as m diverges,

$$\lim_{m\to\infty} \frac{\sigma m - 1}{\sigma m} = 1 \ .$$

In this context, an increase in market concentration within the superior sector can be described by a reduction in m. Formally, the SSMPE can be defined for a given path $\{m_t\}_{t=0}^{\infty}$, and one way to model increase in concentration within sector s is an exogenous reduction over time in the number of firms. Such increase in concentration implies various level effects along the transitional dynamics, as highlighted in the quantitative section.

The alternative interpretation of market concentration is related to the size of the superior sector in the economy. Since in sector s firms are superstar, the competitive fringe in the economy represents ordinary firms. In this interpretation, market concentration can be measured as the share of sales (or employment) of the top z firms in the economy. The time t concentration of the first z firms in the economy (with $z \leq m$) reads,

$$\text{conc}_t^z = \frac{p_t^s z \overline{y}_t^s}{Y_t^s} = \frac{z}{m}\frac{p_t^s y_t^s}{Y_t^s} = \frac{z}{m}\left(1 - \frac{p_t^i y_t^i}{Y_t}\right) \ , \qquad \lim_{t\to\infty} \text{conc}_t^z = \frac{z}{m} \ .$$

Concentration rises endogenously in the transitional dynamics as the superior sector grows to dominate the entire economy and the value of production from the inferior sector becomes negligible. In section 5.2 we take this alternative interpretation for deriving accounting implications of our theory.

Consumption Equivalent Loss

To estimate the misallocation effects of the superstar model we perform the Lucas exercise of Consumption Equivalent Losses (CEL) due to misallocation. The exercise rests on the numerical simulation of the model, and it is obtained in Section 5. Here we just highlight the logic. We simulate a path T of the competitive economy and we label the sequences as $\{c_t^*, h_t^*\}_{t=0}^{T}$. Similarly, the path for the monopsonistic equilibrium is $\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^{T}$. The corresponding utility level is

$$\tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^{T}) = \sum_{t=0}^{T} \beta^t \left[\frac{\tilde{c}_t^{1-\gamma}}{1-\gamma} - \psi \frac{\tilde{h}_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1}\right]$$

where $\beta = \frac{1}{1+\rho dt}$ is the discount rate and dt is the length of the period. We also approximate TFP growth for a period dt so that $A_{t+1}^s - A_t^s \approx dt g^s A_t^s$. The question for estimating the consumption equivalent loss is what fraction λ of the path of consumption is the representative agent willing to give up in order to obtain an utility similar to what the misallocated economy produces. In other words we seek a λ such that

$$\sum_{t=0}^{T} \beta^t \left[\frac{[(1-\lambda)c_t^*]^{(1-\gamma)}}{1-\gamma} - \psi \frac{(h_t^*)^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1}\right] = \tilde{U}(\{\tilde{c}_t, \tilde{h}_t\}_{t=0}^{T}) < U(\{c_t^*, h_t^*\}_{t=0}^{T})$$

The results are reported in the numerical Section 5.

3.9 Stability of the SSMPE

We now investigate whether the SSMPE will approach the BGP. We focus on allocations in the neighborhood of the BGP, thus investigating only local (saddle-path) stability. To prove stability, we cannot use the formulation of the system above with x_t^s and x_t^i. Since capital per unit of efficiency is unbounded ($x_t^i \to \infty$), we ca not write a Jacobian for the dynamical system evaluated at the BGP. We thus introduce variables capturing the share of capital and labor in the two sectors, in line with Acemoglu and Guerrieri (2008),

$$K_t = K_t^s + K_t^i \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad x_t = \frac{K_t}{A_t^s h_t} \ , \quad c_t = \frac{C_t}{A_t^s h_t} \ .$$

Along the BGP, both κ_t, ν_t and h_t converge to zero, while x_t and c_t converge to positive finite values $\bar{x}$ and $\bar{c}$. We can study the behavior of the system in a neighborhood of these values. When proving stability, the production function of the intermediaries is Cobb-Douglas, and the depreciation rates are symmetric (i.e. $\delta^s = \delta^i = \delta$). The full system of equations is presented in Appendix D.

Theorem 1. *If capital depreciates at the same rate in both sectors ($\delta^s = \delta^i$), the SSMPE exhibits local saddle-path stability.*

The details of the proof are in Appendix D. The key idea behind the proof is to rewrite the system of equations as

$$\dot{z} = \Phi(z)$$

where $z = (c, h, \nu, x)$ and Φ is the transition of the dynamical system. Four equations is the minimum number of equations we can reduce to: the Euler equation and the aggregate resource constraint, plus the consumption-leisure tradeoff and the monopsonistic labor supply differentiated with respect to time. We can substitute out κ since the equation stemming from equality of rates of returns to capital is time-invariant (and for the opposite reason, we ca not dispense with the consumption-leisure and labor demand equations since they are time dependent). Since we have endogenous labor supply, we end up having one more equation than Acemoglu and Guerrieri (2008). In our setting, we have three predetermined variables: h_0, κ_0 and ν_0. It is intuitive that one of these, κ_0, comes from $K_0 = K_0^s + K_0^i$ being given. We have two more predetermined values since two of our differential equations come from differentiating intertemporal constraints: hence once C_0 is chosen, these equations determine h_0 and ν_0. Hence we need to show first that the BGP is hyperbolic, i.e. that all eigenvalues of the Jacobian of the transition evaluated at the BGP have non-zero real part, and then that three eigenvalues are negative, while one is positive.

We are able to show that the determinant of the Jacobian of the transition evaluated at the BGP equals the determinant of the following matrix (in which $*$ denotes a finite, non zero constant)

$$\hat{J}_\Phi = \begin{pmatrix} 0 & 0 & * & \frac{\bar{c}}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha(\alpha-1)\bar{x}^{\alpha-2} \\ * & g_h & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * \end{pmatrix},$$

where asterisks denote finite, possibly non-zero values. Clearly

$$\det J_\Phi = \det \hat{J}_\Phi = -\left(-\frac{1}{1+\theta\alpha}\right) g_h (g_{n^i} - g_{n^s}) \frac{c}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha(\alpha-1)x^{\alpha-2} < 0 \ ,$$

so that we can establish that the BGP is hyperbolic. Since the determinant is negative, either three or one eigenvalues are negative. We can then compute the characteristic polynomial of the Jacobian, whose roots are the eigenvalues. In short we find (letting λ be the variable of the polynomial and again $*$ denote a finite, non-zero constant)

$$P(\lambda) = \det(J_\Phi - \lambda I) = \begin{pmatrix} * - \lambda & 0 & * & * \\ * & g_h - \lambda & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} - \lambda & 0 \\ -\frac{1}{1+\theta\alpha} & 0 & * & * - \lambda \end{pmatrix}$$

Again it is straightforward to see that

$$P(\lambda) = (g_h - \lambda)(g_{n^i} - g_{n^s} - \lambda) \det \begin{pmatrix} * - \lambda & * \\ -\frac{1}{1+\theta\alpha} & * - \lambda \end{pmatrix}$$

Now since both g_h and $g_{n^i} - g_{n^s}$ are negative, so must be two of the eigenvalues. But then the only possibility to have a negative determinant of the Jacobian is that three eigenvalues are negative and one positive, thus establishing stability.

3.10 Implications of Different Capital Intensities

In our model, results are driven by unbalanced TFP growth rates between the superior superstar sector and the inferior competitive sector. Autor et al. (2020) provide some evidence on the growth productivity differential between superstar and normal firms.

Our analysis however suggests that an alternative mechanism might be playing a role, namely different capital intensities between superstars and fringe firms. Indeed, in Autor et al. (2020) there is also evidence of higher capital intensity by superstar firms. While the model and the SSMPE were solved under the assumption of symmetric capital intensity across sectors ($\alpha_i = \alpha_s$) and asymmetric labor augmenting technological progress, one can solve the model also in the case of $\alpha_s > \alpha_i$ and Hicks-neutral TFP growth, as in Acemoglu and Guerrieri (2008). Rewriting the model with Hicks neutral technological progress, all the growth result presented hold as long

$$\frac{g^s}{1 - \alpha^s} > \frac{g^i}{1 - \alpha^i} \ .$$

Hence, to obtain a model with a declining labor share, declining hours worked and growth drag is sufficient to assume $g^s = g^i$ and $\alpha^s > \alpha^i$.

4 A Worker-Capitalist Model and Policy

Monopsonistic worker-capitalist model One concern related to the decline of the labor share is the increase inequality across individuals and the distribution of income across different segments of the population. In a representative agent (RA) framework such mechanisms are muted: while payments shift from labor to profits, the RA is the sole owner of labor and capital, and she receives all capital income in the form of interests and dividends. As discussed in Section 3.8, from the labor supply standpoint the rebate of profits to the RA generates an additional income effect beyond that of a pure wage increase. Such income effect is one of the primary source of growth misallocation in the SSMPE, and is related to the classic labor supply effect of public spending highlighted in the macro literature by Prescott (2004).

This section explores how an economy populated by workers and capitalists changes the predictions of our model. Workers are the only suppliers of labor, can not save, and are hand-to-mouth consumers who affect growth through the choice of labor supply. Capitalists save in a safe asset that guarantees full claims to capital and profits. Thus they earn both the marginal productivity of capital and profits. The model is in the spirit of Straub and Werning (2020), who build on the classic paper by Judd (1985) in the context of optimal taxation.

There is a unit measure of workers and a unit measure of capitalists. Workers face a static problem: maximize utility period by period by supplying labor and consuming the final good. The problem with MaCurdy utility reads

$$\max_{C_t^w, h_t} \frac{(C_t^w)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} , \qquad s.t. \quad C_t^w = \sum_{i=1}^{m} w_t^{f,s} n_t^{f,s} + w_t^i n_t^i \ , \quad \sum_{i=1}^{m} n_t^{f,s} + n_t^i = h_t \ ,$$

where the budget constraint shows that the only source of financing for the period t consumption (indicated by C_t^w, where the apex w refers to workers) is labor income from the two sectors. The worker problem delivers a standard consumption-leisure tradeoff and the arbitrage conditions for wages so that

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma} w_t \ , \qquad w_t^{f,s} = w_t^i = w_t \quad f = \{1, 2...., m\}.$$

Capitalists do not work and their utility depend only on consumption C_t^c, where the apex c refers to capitalists. They face a standard forward looking problem with CRRA utility that reads

$$\max_{\{C_t^c\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{(C_t^c)^{1-\gamma} - 1}{1-\gamma} dt \ , \qquad s.t. \quad C_t^c + \sum_{f=1}^{m} \dot{\mathbb{A}}_t^{f,s} + \dot{\mathbb{A}}_t^i = \sum_{f=1}^{m} r_t^s \mathbb{A}_t^s + r_t^i \mathbb{A}_t^i + \sum_{f=1}^{m} r_t^s \Pi_t^{f,s}$$

The problem delivers a standard Euler equation and equality of return across assets

$$\gamma \frac{\dot{C}_t^c}{C_t^c} = r_t - \rho \ , \qquad r_t^{f,s} = r_t^i = r_t \quad f = \{1, 2...., m\}.$$

The production side of the economy is unchanged with respect to the superstar economy and the SSMPE. The firm problem is identical to the model with the representative consumer and the symmetric Cournot equilibrium implies $K_t^s = m\overline{K}_t^s, n_t^s = m\overline{n}_t^s, y_t^s = m\overline{y}_t^s$ exactly as in Section 3.4. The equilibrium system of ODEs of the model is

$$
\begin{aligned}
C_t^w &= p_t^i(1-\alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha}h_t \\
\psi h_t^{\frac{1}{\theta}} &= (C_t^w)^{-\gamma}p_t^i(1-\alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha} \\
n_t^s + n_t^i &= h_t \\
\gamma\frac{\dot{C}_t^c}{C_t^c} &= \frac{\sigma m-1}{\sigma m}p_t^s\alpha(K_t^s)^{\alpha-1}(A_t^s n_t^s)^{1-\alpha} - \delta^s - \rho \\
C_t^c + C_t^w + \dot{K}_t^s + \dot{K}_t^i &= Y_t - \delta^s K_t^s - \delta^i K_t^i \\
Y_t &= \left[\zeta^{\frac{1}{\sigma}}((K_t^s)^\alpha(A_t^s n_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}((K_t^i)^\alpha(A_t^i n_t^i)^{1-\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
(K_t^s)^\alpha(A_t^s n_t^s)^{1-\alpha} &= \zeta(p_t^s)^{-\sigma}Y_t\ , \qquad (K_t^i)^\alpha(A_t^i n_t^i)^{1-\alpha} = (1-\zeta)(p_t^i)^{-\sigma}Y_t \\
\frac{\sigma m-1}{\sigma m}p_t^s(1-\alpha)(K_t^s)^\alpha(n_t^s)^{-\alpha}(A_t^s)^{1-\alpha} &= p_t^i(1-\alpha)(K_t^i)^\alpha(n_t^i)^{-\alpha}(A_t^i)^{1-\alpha}\left[1+\frac{\alpha}{m}\frac{n_t^s}{n_t^i}\right] \\
\frac{\sigma m-1}{\sigma m}p_t^s\alpha(K_t^s)^{\alpha-1}(A_t^s n_t^s)^{1-\alpha} - \delta^s &= p_t^i\alpha(K_t^i)^{\alpha-1}(A_t^i n_t^i)^{1-\alpha} - \delta^i
\end{aligned}
$$

While a system of nine ODE looks at first untractable, the solution can be handled once we express the problem in terms of share of factors employed in the inferior sector compared to the total (this intensive form system is described in Appendix A.3). Then a subset of the equations pins down factor shares and prices as a function of the technological ratio A_t^s/A_t^i and independently of the overall capital and hours input. Since the production side of the economy is the same as in our benchmark setting, in equilibrium the growth rate of the markdown (and hence of employment in the inferior sector compared to the superior sector), as well as that of the inferior intermediate price p_t^i coincides with the representative agent solution, so that

$$
g_\mu = g_{n^s} - g_{n^i} = \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i)\ , \qquad g_{p^i} = 2\frac{1-\alpha}{1+\alpha+\sigma(1-\alpha)}(g^s-g^i)\ .
$$

The growth rate of wages is identical to the representative agent monopsonistic case, since it depends only on the inferior price and parameters,

$$
g_w = g^s - g_\mu = g^s - \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s-g^i) < g^s\ .
$$

Yet, the overall macro-economy differ from the standard SSMPE, since the growth rate of workers' consumption is now proportional to labor income growth, and does not depend on profits and returns to capital. The equilibrium growth rate of the labor supply in the worker capitalist economy reads

$$
g_h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}g_w = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}\left[g^s - \frac{(1-\alpha)(\sigma-1)}{1+\alpha+\sigma(1-\alpha)}(g^s-g^i)\right]\ . \tag{34}
$$

The asymptotic balanced growth path of consumption displays a dramatic growth inequality between two agents, as the following proposition specifies.

Proposition 7. *In the balanced growth equilibrium there exists unbalanced consumption growth between workers and capitalists and in the asymptotic path the capitalists absorb the entire consumption level in the economy.*

Formally, we just need to observe that

$$g_{C^w} = g_w + g_h < g^s + g_h = g_{C^c} \ ,$$

so that in the limit capitalist consumption absorb the entire share of aggregate consumption and

$$\frac{C_t^c}{C_t^w + C_t^c} \to 1 \ .$$

Utilitarian efficient allocation The labor supply result in equation (34) deserves some discussion. To evaluate the welfare properties of the capitalist–worker setup, we need a simple efficient benchmark. We therefore look at an optimal growth model where the planner is utilitarian and gives capitalists a welfare weight $\omega > 0$.[8] The problem is

$$\max_{\{C_t^c, C_t^w, h^t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \left[\frac{(C_t^w)^{1-\gamma} - 1}{\gamma - 1} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} + \omega \frac{(C_t^c)^{1-\gamma} - 1}{\gamma - 1} \right] dt$$

subject to

$$C_t^w + C_t^c + \dot{K}_t^s + \dot{K}_t^i = Y_t - \delta^s K_t^s - \delta^i K_t^i$$

$$Y_t = \left[\zeta^{\frac{1}{\sigma}} ((K_t^s)^{\alpha} (A_t^s n_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} ((K_t^i)^{\alpha} (A_t^i n_t^i)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$n_t^s + n_t^i = h_t$$

The planner makes direct use of the two CRS production technologies and hence abstracts from the number of firms in the superior sector. The solution clearly implies (indicating with superscript * the optimal growth solution)

$$\omega^{1/\gamma} C_t^{*w} = C_t^{*c}$$

so that both types of agents consume a constant fraction of output. The consumption-leisure tradeoff implies the standard condition of neoclassical growth models, or

$$\psi (h_t^*)^{\frac{1}{\theta}} = (C_t^{*w})^{-\gamma} \frac{\partial Y_t}{\partial h_t}.$$

This implies that in the limit path the marginal productivity of labor, which asymptotically grows at rate g^s, is enjoyed by workers and there is no unbalanced consumption growth. Formally, the optimal balanced growth for consumption and hours read

$$g_h^* = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s \ , \qquad g_{C^w}^* = g_{C^c}^* = g^s + g_h^* = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma} g^s \ , \tag{35}$$

[8]We assume the planner cannot make capitalists work. This affects the levels but not the growth rates of the solutions.

which are exactly the optimal growth rates in the representative agent model.

We are thus in a position to compare the results of the optimal and decentralized monopsonistic economy. There are two results. First, the effect on inequality is dramatic: whereas the decentralized superstar monopsonistic market implies an ever increasing consumption inequality between the two groups of agents, with $g_{C^w} > g_{C^c}$, the central planner delivers a balanced consumption growth between the two groups and a constant relative weight that depends on the structural parameter ω. The second result concerns the effect on labor supply, which come from comparing equations (34) and (35).

Proposition 8. *If preferences are coherent with the Boppart and Krusell (2020) class (and $\gamma > 1$) the growth rate of labor supply in the decentralized worker-capitalist monopsonistic economy is higher than the optimal growth rate. If preferences are coherent with King et al. (1988), hours are constant in both the efficient and decentralized monopsonistic equilibrium.*

Indeed, the growth rates are

$$g_h = g_h^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu \ , \qquad g_{C^c} = g^s + g_h = g_{C^c}^* + \frac{\gamma - 1}{\frac{1}{\theta} + \gamma} g_\mu \ , \quad g_{C^w} = g_w + g_h = g_{C^w}^* - \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g_\mu \ , \quad (36)$$

Equations (36) highlight how preferences interact with the labor market distortion arising from monopsony power. The results of Proposition 8 emphasize in a sharp way the effect of profits on labor supply. In the SSMPE with representative agents, labor supply falls faster than in the optimal growth model. We claimed that the result was due to an additional income effect linked to the rebate of profits in the representative agent labor supply problem. In the capital worker model of this section, the additional income effect due to profits is shut down, since workers decide their labor supply uniquely on the basis of their labor income. The result in Proposition 8 shows that in this case workers dynamically supply *more* hours than the efficient level. This result represents a natural theoretical experiment for the labor supply channel of the paper, that we claimed to be a long-run version of the classic Prescott (2004) paper on labor supply in Europe.

Redistributive taxation The analysis of the worker-capitalist economy naturally raises the question of policy. A simple experiment considers the welfare and growth effects of a proportional tax τ on capitalists' income Ω_t^c (to be defined below), whose proceeds are fully rebated to workers. Such tax directly targets the distributional imbalance between workers and capitalists. However, its implications for labor supply and hours growth are more complex. The transfer reintroduces the additional income effect channel present in the representative agent economy, thereby reintroducing the downward distortion in hours growth emphasized by Prescott (2004) and formalized in Proposition 8.

Consider a proportional tax τ on capitalists' income in the monopsonistic worker-capitalist model. Capitalists' total income Ω_t^c consists of both marginal returns to capital and monopoly profits. These two components together equal total production of the final good net of labor payments and capital depreciation,

$$\Omega_t^c = Y_t - w_t h_t - \delta^s K_t^s - \delta^i K_t^i,$$

where w_t is the oligopsonistic wage, $h_t = n_t^i + m\overline{n}_t^s$ and $K_t^s = m\overline{k}_t^s$. The only difference in the system of equations of the model is the workers' budget constraint, which now reads[9]

$$C_t^w = (1-\tau) w_t h_t + \tau\big(Y_t - \delta^s K_t^s - \delta^i K_t^i\big)\,. \tag{37}$$

Equation (37) implies that, asymptotically, workers consume a constant fraction of output net of capital depreciation, so that $C_t^w = \tau(Y_t - \delta^s K_t^s - \delta^i K_t^i)$. The growth rates of the markdown and of the inferior good price are the same as in the representative agent model. However, the transfer induces an additional income effect for workers, qualitatively identical to the profits effects on labor supply in the representative agent economy.

The growth rates of output, consumption, and hours worked with the tax τ read

$$g_h^\tau = g_h^* - \frac{1}{\frac{1}{\theta}+\gamma} g_\mu = \frac{1-\gamma}{\frac{1}{\theta}+\gamma} g^s - \frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s - g^i), \qquad g_{C^w}^\tau = g_{C^c}^\tau = g_C^* - \frac{1}{\frac{1}{\theta}+\gamma} g_\mu. \tag{38}$$

Comparing equations (38) with the efficient outcomes of equation (35) yields two main results. First, a tax on capitalists' income eliminates the growth inequality between workers' and capitalists' consumption. Second, the tax reduces the growth rate of labor supply, in a way consistent with the standard negative income effect on labor supply of redistributive policies (Prescott, 2004). Lower labor input, in turn, implies lower total output. We summarize these findings in the following proposition.

Proposition 9. *A tax that ensures that workers consume a nonzero constant fraction of output eliminates the growth inequality between workers' and capitalists' consumption. However, such a tax depresses hours worked and total production relative to the optimal utilitarian outcome.*

As a final remark, a tax levied on profits only would have the same qualitative effects, since asymptotically the marginal product of labor accrues to profits.

Minimum wage In the presence of monoposny, a binding minimum wage can potentially induce efficient outcomes. In this section we show that along the balanced path equilibrium (when superstar employment eventually absorbs the entire labor supply), a rising minimum wage that grows at rate g^s is able to achieve efficient growth rates. Yet, in the transition the optimal growth rates of the minimum wage needs not be exactly g^s and would need to be carefully crafted to produce efficiency. The argument can be formally addressed in a model with an exogenous minimum wage. The problem of the inferior sector and the household problem are the same as before, while that of the superstar superior sector for a given minimum wage $\bar{w}_t$ is

$$\begin{aligned} \max_{N_t^{f,s}, K_t^{f,s}, W_t^{f,s}} \Pi_t^{f,s} &= p_t^s(y_t^{f,s}; y_t^{-f,s}) y_t^s(K_t^{f,s}, N_t^{f,s}, A_t^s) - W_t^{f,s} N_t^{f,s} - R_t^s K_t^{f,s} \\ \text{s.t} \qquad & W_t^{f,s} \geq \bar{w}_t \end{aligned}$$

Notice that the minimum wage needs not be binding in this problem. Since the superstar firms take the capital allocated to the inferior sector as given, it might be that the marginal product

[9] By Walras' law, the consumption level of capitalists is pinned down by the aggregate resource constraint and does not appear explicitly in the system of equations

in the inferior sector is above the minimum wage also when all labor is supplied to it (i.e. when $p_t^i(1-\alpha)(H_t)^{-\alpha}(K_t^i)^{\alpha}(A_t^i)^{1-\alpha} > \bar{w}_t$). Whenever this is not the case, there exists a unique $\hat{N}_t^{f,s} \in (0, H_t)$ that characterizes the optimal labor demand of the inferior sector at the minimum wage. In this case, the superstar's constraints become

$$w_t^s = \begin{cases} p_t^i(K_t^i)^{\alpha}(H_t - N_t^{f,s})^{-\alpha}(A_t^i)^{1-\alpha}(1-\alpha) & N_t^{f,s} \in [H_t - \hat{N}_t^{f,s}, H_t] \\ \bar{w}_t & N_t^{f,s} \in [0, H_t - \hat{N}_t^{f,s}] \end{cases}$$

$$y_t^{f,s} = \zeta(p_t^s)^{-\sigma}Y_t - y_t^{-f,s}$$

$$y_t^{f,s}(K_t^{f,s}, N_t^{f,s}, A_t^s) = (K_t^{f,s})^{\alpha}(A_t^s N_t^{f,s})^{1-\alpha}$$

where the minimum wage is binding when each superstar firm chooses optimally to set $w_t^{f,s} = \overline{w}_t, \quad f = \{1, 2..., m\}$. In this latter case superstar firms turn out to be only oligopolists, since the wage is exogenously set at $\overline{w}_t$. While this logic holds for a given intratemporal problem, a continuously binding minimum wage must growth indefinitely. In addition, in an economy with only monopolistic power, growth should be at its optimal level, as argued in Section F.2. Formally, to bind indefinitely the minimum wage must grow faster then the (shadow) monopsonistic wage in the absence of it. From the labor supply standpoint, the consumption leisure tradeoff with a binding minimum wage reads

$$\psi h_t^{\frac{1}{\theta}} = (C_t^w)^{-\gamma}\bar{w}_t \ .$$

Since we know that in this economy the optimal and first best growth rates for hours and consumption from Eq. (35), we can find a growth condition for the minimum wage to achieve the efficient growth rates:

$$g_{\bar{w}}^* = \frac{1}{\theta}g_h^* + \gamma g_{C^w}^* = g^s \ .$$

We can then summarize the analysis above in the following statement.

Proposition 10. *A growing minimum wage that matches the growth rate of the superior monopsonistic sector achieves efficient growth rates of hours worked and consumption.*

Even though the minimum wage achieves efficient growth rate of hours and consumption, it depressed the growth rate of hours worked with respect to a decentralized monopsonistic equilibrium. The result is a byproduct of the fact that monopsony causes hand-to-mouth workers to *work too much*, not too little as it usually happens in models of monopsony power. If one were to solve for the optimal minimum wage in a complete monopsonistic RA model (as in Section 3), the classic result of Robinson (1969) would be obtained in a growth setting. In other words, with respect to a decentralized SSMPE, a growing minimum wage would increase both the growth of labor supply and wages.

5 Quantifying the Growth Misallocation and the Consumption Equivalent Loss

We are now in a position to quantify the misallocative effects of the distortions induced by market power in the superior sector. We perform four quantitative exercises. First, in section 5.1 we calibrate the asymptotic BGP of our "representative advanced economy" and show that only the SSMPE can perfectly match the key three growth moments: GDP growth, hours growth and labor share growth. Second, we estimate the size of the income effect necessary to match the fall in hours that can be attributed to misallocations. Third, we quantify the growth drag between the SSMPE and a competitive economy. Fourth (in section 5.2), we focus on the US economy and solve quantitatively the transition of the calibrated US economy along the asymptotic path. While in section 5.1 we rely on the version of the model with capital, in section 5.2 we face the well known instability problems that affect shooting simulations of growth models with capital and forward looking consumption (Judd, 1998; Miranda and Fackler, 2002).[10] In the context of our model, such instability problems are more severe because of the complexity of the labor and capital allocation between the two sectors. To overcome such instability, we solve and calibrate a version of the model in which capital is substituted with a factor in fixed supply. In such a model there are no savings and consumption absorbs period t output. The model can be easily solved and simulated for an exogenous sequence of productivity path $\{A_t^s, A_t^i\}_{t=0}^T$. The fixed material can be thought as "materials" or "land". The details of the model and the very few differences with respect to the full model with capital are reported in Appendix E. The simulations in Section 5.2 refer to the model with a fixed factor.

5.1 Asymptotic Calibration and Growth Drag

In the first quantitative exercise we assume that the "representative advanced economy" features a long-run growth behavior summarized by the three moments reported in Section 2, and moves along the asymptotic superstar equilibrium defined by the model. The three long-run facts concern growth in GDP/consumption, (negative) growth in hours worked and (negative) growth of labor share. Table 4 in the Column labeled "data" reports the target of the three moments for both GDP per capita and GDP per worker. The growth moments corresponds to the rounded values of the average empirical moments for the period between 1980 and 2023.

In what follows we let $\hat{g}_C^{pc}, \hat{g}_h^{pc}$ and $\hat{g}_{\alpha_L}^{pc}$ represent the empirical estimates of the three moments. When such estimates refer to per worker we indicated them with $\hat{g}_C^{pw}, \hat{g}_{ns}^{pw}$ and $\hat{g}_{\alpha_L}^{pw}$. The quantitative goal is to find values of the parameters γ, g^s, θ to match the three theoretical moments indicated in section 3.6. From the literature we match the λ-constant elasticity of labor supply from Chetty et al. (2011), and we are thus left with the two key parameters g^s and γ.

Let us first focus on the monopsonistic model $SSMPE$. For given θ, using the empirical growth rate derived in section 3.6, given the three moments, one can back out the value of $\hat{\gamma}$ as

$$\hat{\gamma}^j = 1 - \frac{1}{1+\theta}\frac{\hat{g}_h^j}{\hat{g}_C^j} + \frac{\hat{g}_{\alpha_L}^j}{\hat{g}_C^j}, \quad j = pc, pw \tag{41}$$

Once we obtain a value of $\hat{\gamma}$, the value of $\hat{g}^{s,j}$ can be obtained as

$$\hat{g}^{s,j} = \frac{\hat{g}_h^j \left(\frac{1}{\theta} + \gamma^j\right)}{1 - \hat{\gamma}^j} \quad j = pc, pw \tag{42}$$

[10]ChatGPT reports "The shooting algorithm can be unstable for solving growth model boundary value problems (BVPs) because numerical errors can be magnified, leading to slow or failed convergence. This instability stems from the iterative process of guessing initial conditions and integrating the differential equations, where incorrect guesses or sensitive model dynamics can result in disproportionate growth in numerical errors, especially for models with delays or complex dynamics."

Finally, using the properties of the SSMPE one has

$$\hat{g}_{\mu} = -\hat{g}_{\alpha_L}, \tag{43}$$

where $\hat{g}_{\mu}$ is the backed out value of growth of the markdown, which coincides with the opposite of the asymptotic growth rate of the labor share. Note that in equation (43) the estimate of $\hat{g}_{\alpha_L}$ is independent of the fact that one uses per capita or per worker data. Given the calibration of equation (41), (42) and (43), it is not surprising that in Table 4, the SSMPE perfectly match the three moments in both the per capita and the per worker case.

Next we turn to the column labeled *NGM Model* in Table 4. Using equations (41) and (42), one can perfectly match the growth of GDP and the growth in hours. Yet, the growth rate of the labor share can not be matched, since the NGM implies a constant asymptotic value of the labor share that depends on α, the elasticity of output to capital that we set to the standard value of $\hat{\alpha} = 0.33$. Such property was also outlined in Section 3.8. While there is a potential degree of freedom in choosing the value of such elasticity, the implied growth rate of the labor share is zero, as indicated in Table 4. The bottom line of this first exercise is that to match the three growth moments of Table 4, it is not sufficient to assume preferences in which the income effect of wage changes is larger than the substitution effect, as it is done by Table 4 in line with with Boppart and Krusell (2020). Indeed, it is necessary to work with model in which the labor share declines permanently, as in the SSMPE.

Table 4 performs an additional exercise with respect to the income effect and the value of γ in the MaCurdy (1981) preferences. Working with the NGM, Boppart and Krusell (2020) show that for matching the declining trend of labor supply, it is necessary to work with a value of $\gamma > 1$. This is evident in Table 4, where the value of $\hat{\gamma}^j$ is always greater than one. In general, the larger is the downward trend of hours worked, the larger is the income effect necessary to match the growth moment $\hat{g}_h^j$. In the SSMPE there are profits that are transferred to the representative consumer. Such profits represent a non wage transfer to the consumer that reduces hours worked. Such effect represents the standard result of Prescott (2004) obtained in a growth context. Since in the SSMPE there is a profit channel that depends on market imperfections, it is less important to rely on income effects due to preferences. We measure such extra income effect as

$$\Delta IE = \left| \frac{(\gamma^{ngm,j} - \gamma^{ssmpse,j})}{\gamma^{ngm,j}} \right|, \qquad j = pc, pw$$

where ΔIE describes-in absolute terms- the share of the income effect that can be imputed to misallocation effects. Note that- in the definition of ΔIE- 1 is the value of $\gamma^{ngm,j}$ in a model without negative trend in hours, in line with the traditional preferences used in the business cycle literature since King et al. (1988). γ^{ngm} and γ^{ssmpse} are the value of γ in the two models. Table 4 shows that the additional size of the income effect due to preferences (ΔIE) is 16 percent of hours per worker and as much as 81 percent in the case of hours per person.

The third quantitative exercise linked to the asymptotic equilibrium is to estimate the size of the "growth drag" due to market imperfections. In the SSMPE equilibrium we obtained a growth drag for both GDP and hours worked. Such values are obtained analytically in Section 3.8. In Table 5 we quantify such growth drag using the calibrated values of Table 4. We find that oligopsony implies an asymptotic loss of 5 percent of GDP growth in the case of GDP per capita, and 10 percent of GDP growth in the case of GDP per worker. The growth of hours is 8 percent lower than the competitive economy in the case of hours per worker, and fall from -0.02 percent to -0.1 percent in the case of hours per person, which corresponds to an increase by a factor of 5 in proportional terms.

Table 4: Calibration of Asymptotic Balanced Growth Path

	Model SSMPE[a]		*NGM Model with MaCurdy preferences*[b]		*Data*	
	per capita	per worker	per capita	per worker	per capita	per worker
Moments						
GDP growth	0.0150	0.0169	0.0150	0.0169	0.0150	0.0169
Growth of Hours	-0.0010	-0.0050	-0.0010	-0.0050	-0.0010	-0.0050
Growth Labor Share	-0.0011	-0.0011	0.0000	0.0000	-0.0011	-0.0011
Parameters[c]						
γ	1.017	1.335	1.090	1.400		
g^s	0.016	0.022	0.016	0.022		
θ	2.84	2.84	2.84	2.84		
Share of Income Effect due to Misallocation (percentage points)[d]						
	81.350	16.270				

[a] , SSMPE is the calibrated asymptotic Superstar Monopsonistic Equilibrium.
[b] , in NGM there are no market imperfections and the labor share is constant, so we calibrate GDP and hours growth only.
[b] , a special case of Boppart and Krusell (2020).
[c] , γ is the elasticity of the marginal utility of consumption in MaCurdy (1981) preferences.
[c] , g^s is the growth rate of the superior technology.
[c] , θ is the elasticity of the marginal disutility of labor in MaCurdy (1981) preferences.
[d] , The relative increase in income effect due to γ between a Boppart and Krusell (2020) economy and the model SSMPE.
Source: Authors' calculations.

Table 5: Asymptotic Growth Drag

	Model SSMPE[a]		*Competitive Economy*[b]		*Data*	
	per capita	Per Person	per capita	Per Person	per capita	Per Person
Moments						
Gdp growth	0.0150	0.0169	0.0159	0.0188	0.0150	0.0169
Growth of Hours	-0.0010	-0.0050	-0.0002	-0.0047	-0.0010	-0.0050
Growth Labor Share	-0.0011	-0.0011	0.0000	0.0000	-0.0011	-0.0011
Parameters[b]						
γ	1.017	1.335	1.017	1.335		
g^s	0.016	0.022	0.016	0.022		
θ	2.84	2.84	2.84	2.84		
Growth Drag[c]						
Gdp Growth	-5.47	-10.09				
Hours	406.96	7.39				

[a], SSMPE Is the asymptotic Superstar Monopsonistic Equilibrium.
[b], Optimal Competitive Growth with the parameters of the SSMPE.
[c] , γ is the elasticity of the marginal utility of consumption in MaCurdy (1981) preferences.
[c] , θ is the elasticity of the marginal disutility of labor in MaCurdy (1981) preferences.
[c] , g^s is the growth rate of the superior technology.
[d] , Growth Drag is the loss in asymptotic growth rate between SSMPE and Optimal Growth.
Source: Authors' calculations.

5.2 Calibration of the Growth Transition in the US and Consumption Equivalent Loss

In the fourth quantitative exercise we focus on the US economy and we solve numerically the transition of the economy along the asymptotic path. The ultimate goal of the section is to quantify the consumption equivalent loss for the US economy induced by the growth misallocation of the SSMPE.

To address the numerical instability discussed at the beginning of the section, we calibrate a version of the model with a fixed factor of production like "materials".[11] The logic of the exercise is to calibrate the model to match some relevant features of the US economy at the beginning of the 21st century, and then simulate the dynamic transition toward the asymptotic SSMPE. Before discussing the details of the calibration, we perform an accounting exercise aimed at obtaining a value of the markdown $\hat{\mu}_0$ that we will use in the calibration.

Accounting for Rising Markdown

The key driver of the growth theory we presented is the dynamics of the markdown. The dynamics in hours and in the labor share ultimately depends on the dynamics in μ_t. This, in turn, depends on the ratio of employment in the superior sector to employment in the inferior sector from equation (11). If one denotes with $\hat{n}_t^{s,j}$ and $\hat{n}_t^{i,j}$ employment estimates in the two sectors in some industry j, an estimate of the markdown in period t for such industry can be obtained as

$$\hat{\mu}_t^j = 1 + \alpha \frac{\hat{n}_t^{i,j}}{m \hat{n}_t^{s,j}} ,$$

where m is the number of firms in the oligopsonistic market.

In this section we use the recent evidence on the rise of superstar firms to calibrate the dynamics of $\hat{\mu}_t^j$ in different sectors. Autor et al. (2020) estimate a time trend for the concentration ratio for various variables of the top superstar firms. They also estimate the concentration ratio for employment for both the top 4 and top 20 firms in each industry. In this respect, if we use the estimate of $\hat{n}_t^{s,j}$ for industry j, where $\hat{n}_t^{s,j}$ is estimated in percentage term, then we can use the simple identity $\hat{n}^{i,j} = 1 - \hat{n}^{s,j}$ to estimate the employment share in the inferior fringe of the sector. To obtain an estimate of the markdown we then just need values α and m. In Figure 4 and Table 6 we use a value of $\alpha = 0.2$ that is coherent with the calibration exercise for the transitional dynamics that is performed in the next section.

The results of the accounting exercise are reported in Table 6 and Figure 4 for six major industries, and in particular for Retail Trade, Services, Utilities and Transportation, Wholesale Trade, Finance and Manufacturing. Original data are from Autor et al. (2020) and refer to point estimates from 1982 until 2010 for most industries, even though the time span vary across sectors. In the Figure the left y-axis refers to the estimates based on the C20 ratio while the right y-axis refer to the C4 ratio. While the level estimates varies across industry, the Figure reveals an unambiguous upward trend in the estimated markup in all major industries, with the only possible exception of manufacturing. In terms of level, the average value at the beginning of the period is around 1.017. For manufacturing, even though the estimates is V-shape, the concentration is clearly rising since the beginning of the century, in line with the trend estimates provided by Yeh et al. (2022). Although based on accounting exercise, the estimates of Figure 4 are certainly coherent with the key driver of of the SSMPE presented in the paper. Table 6 reports the value for $\hat{\mu}_{2002}^j$ for the six industries. It reports also the average value that we use as a benchmark value for the US moments that we follow in Table 7. Finally, the column labeled growth $CR4$ and $CR20$ report the average growth rate of the markdown implied by

[11] In Appendix E we report the equations of the model with fixed factor.

Table 6: Markdown Estimates and Average Growth by Industry

Industry	$CR4^{a}_{2002}$	$CR20^{a}_{2002}$	Growth CR4 (%)	Growth CR20 (%)	Period[b]
Finance	1.020	1.011	0.030	0.014	1982-2002
Retail Trade	1.021	1.007	0.056	0.019	1982-2002
Services	1.007	1.003	0.005	0.002	1982-2012
Utilities & Transportation	nan	1.016	0.067	0.034	1982-1997
Wholesale	1.010	1.004	0.017	0.006	1982-2007
Manufacturing	1.028	1.018	-0.002	-0.003	1982-2007
Average	1.017	1.010	0.029	0.012	

[a] , $CR4_{2002}$ and $CR20_{2002}$ refer to the employment concentration ratio for C4 and C20 respectively.
[b] , Each industry has a different sample period as indicated in the last column.

Source: Authors' calculations and Autor et al. (2020).

the accounting exercise. Remarkably, the order of magnitude is comparable to that of the growth of the labor share of the US from Table 1, particularly in the case of the C20 measure.

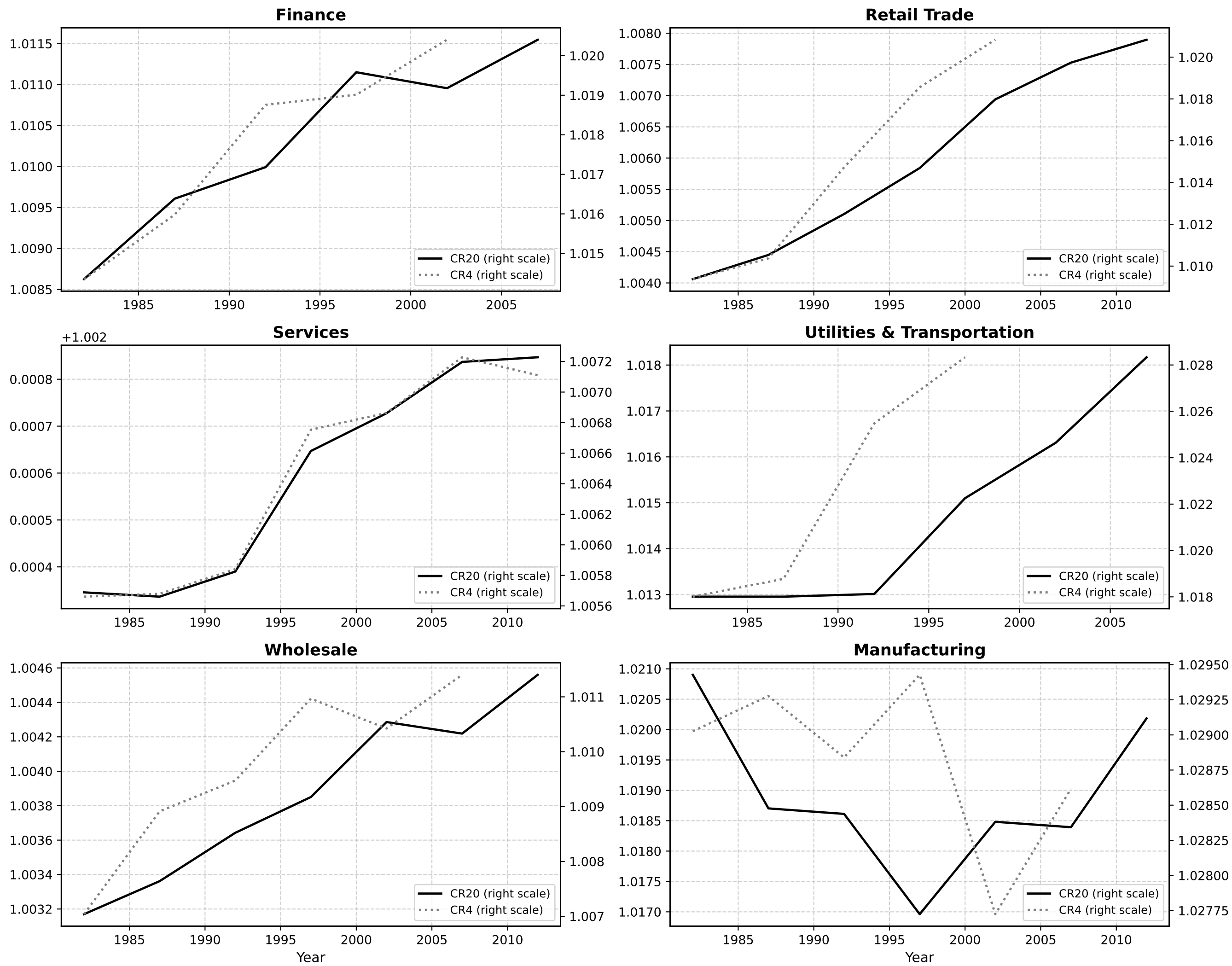
US Calibration and Consumption Equivalent Loss

To perform the fourth quantitative exercise and to calibrate the transitional dynamics for the US, four steps are required. First, we assume that the US economy features asymptotic growth moments described by the model. In other words $\hat{g}_C^{j,us}, \hat{g}_h^{j,us}, \hat{g}_{\alpha_L}^{j,us}$ describe the asymptotic behavior of the economy and - for given initial condition- the calibrated US economy will converge to such stable asymptotic path. Implicitly- following the logic of Table 8- such assumption pins down values for θ, γ, g^s and g^i. Table 8 implies that the value of $\gamma^{US,pc}$ is lower than that of the representative advanced economy, and also lower than one in absolute value. This result is not surprising since it depends on the slightly upward trend of hours worked per capita in the US. When we focus on hours worked per worker, Table 8 suggests that $\gamma^{US,pw} > 1$

Second, we set few parameters of the model from standard values in the literature, as indicated in the second part of Table 8. The combination of σ and m determine the value of markup, since the markup equals $\frac{\sigma m}{\sigma m-1}$. We target a initial value of the markup from Autor et al. (2020). While m is a natural number in the model, to keep flexibility in the calibration we set m as a continuous measure of concentration. Since $\sigma > 1$ is a key condition for the mechanism of the model and the growth of the markdown to be at work, we set a reference value of $m = 1.25$ and obtain the corresponding value of σ. The discount rate ρ is assumed to match a standard value for yearly long-run interest rate. The initial level of technology in the inferior sector is normalized to $A^i = 1$. The value of α is particularly sensitive for the transitional dynamics. While the model converges to the numerical asymptotic SSMPE, the speed of transition is a cause of concern. With a benchmark value of $\alpha = .33$, the speed of transition is far too low. Yet, in the model with fixed materials, the elasticity of output to material is not necessarily linked to the elasticity of output to capital. In order to increase the speed of transition, we set $\alpha = .2$ in Table 8.

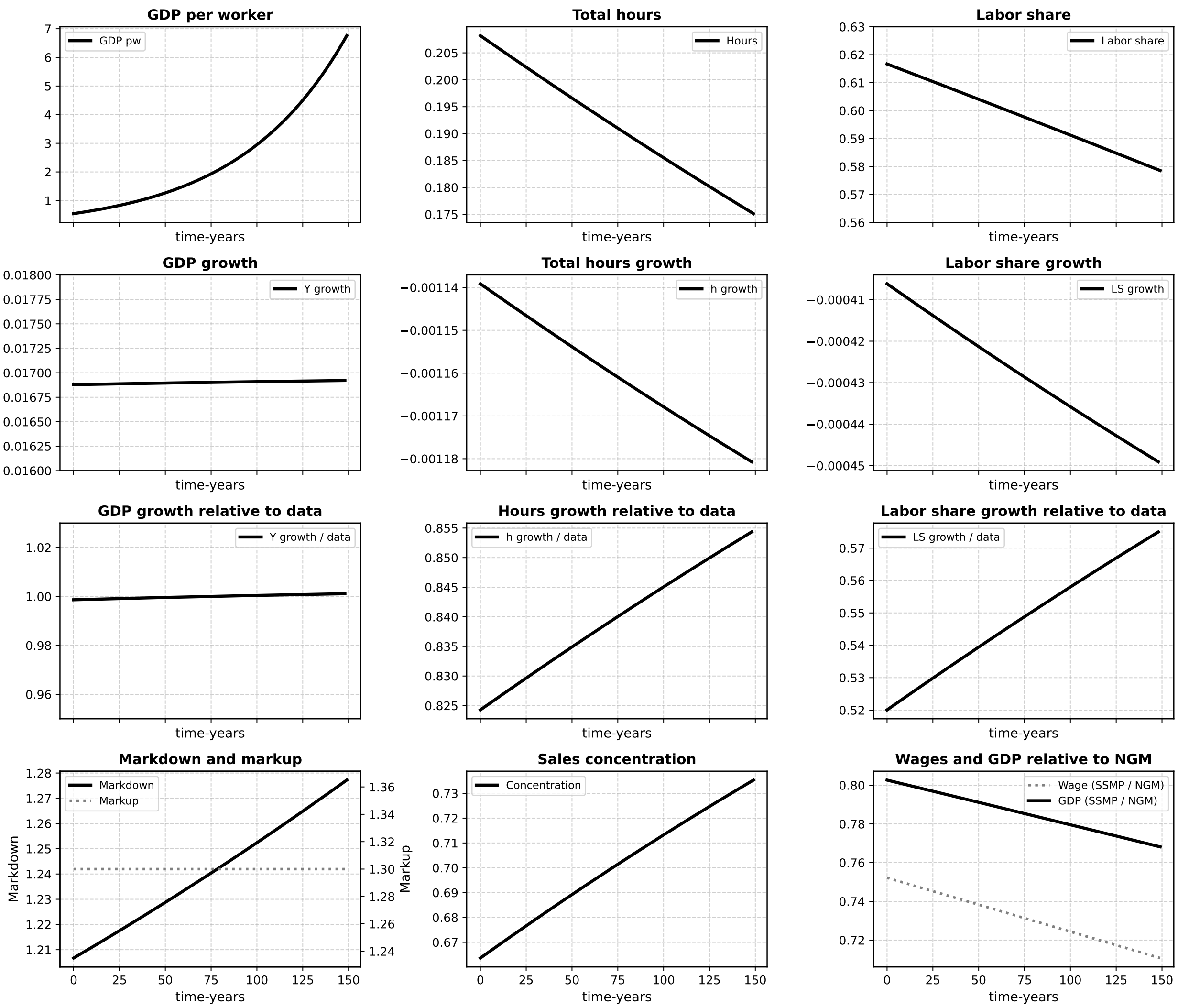
Third, the core of the calibration concerns choosing the remaining four parameters, that correspond to A^s, ψ, T, ζ. Ideally, we would like to match four key moments of the US economy: the average value of markdown $\hat{\mu}_0$, the initial level of the labor share (Karabarbounis, 2024) $\hat{\alpha}_0$, the initial level of hours worked $\hat{h}_0$ and the initial level of concentration in the superior sector summarized by the concentration ratio in the superior sector (Autor et al., 2020), $\hat{C}_{m0}$. Yet, in the analytics of the model such moments are not independent from each other. In particular, indicating with $\hat{\mu}_0$, $\hat{C}_{m0}$

Figure 4: Markdown Estimates from Concentration Ratio Across Six Main US Sectors



The panels report our estimates of the markdown by sector from the concentration data in Autor et al. (2020). CR20 refers to the employment of the top 20 firms, C4 to the employment of the top 4 firms. The markdown is computed according to $\mu = 1 + \alpha \frac{n_t^s}{n_t^i}$

Figure 5: Transitional Dynamics of the Calibrated US Economy



and $\hat{\alpha}_{L0}$ the initial markdown, concentration ratio and the labor share, the analytics of the model implies

$$\begin{aligned} \hat{\mu}_0 &= 1 + \frac{\alpha}{m}\frac{n_i^s}{n_0^s} \\ \frac{\sigma m}{\sigma m - 1}\frac{n^s}{n^i}\left(1 + \frac{\alpha}{m}\frac{n^s}{n^i}\right) &= \frac{\hat{C}_{m0}}{1 - \hat{C}_{m0}} \\ \frac{\sigma m}{\sigma m - 1}\frac{n^s}{n^i}\left(1 + \frac{\alpha}{m}\frac{n^s}{n^i}\right) + 1 &= \frac{(1-\alpha)}{\hat{\alpha}_{L0}}\left(1 + \frac{n^s}{n^i}\right) \end{aligned} \tag{44}$$

Equation (44) implies that we can match only one of the three moments among markdown, labor share and concentration. In the logic of the paper we take as a key macro fact the declining labor share (Karabarbounis, 2024). As a result, in what follows, we calibrate $\hat{\alpha}_{L0}$ to the level of the labor share in 2002, and use endogenously the corresponding level of markdown $\hat{\mu}_0$ and concentration $\hat{C}_{m0}$.

Further, one has also to realize that the four parameters to be identified (A^s, ψ, T, ζ) are not independent from each other, since only some nonlinear combinations of them can be pinned down from data: $i)$ $\frac{\zeta}{1-\zeta}\left(\frac{A^s}{A^i}\right)^{\sigma-1}$ and $ii)$ $\frac{T^{\alpha(1-\gamma)}}{\psi}$ (the details of the derivation are left in Appendix). Hence, the final and fourth step of the calibration is to select values of A^s and ψ, for given T and ζ, so that through the relationships among parameters $i)$ and $ii)$- as well as from the equations above- we perfectly match initial hours worked $\hat{h}_0$ and the initial labor share $\hat{\alpha}_{L0}$.

Once the four steps outline above are completed, the model is fully calibrated. It is still true that some values, such as the level of output at time zero, will depend on the exact choices of A^s, ψ, T and ζ, and one might wonder if we can then compute the consumption equivalent loss at all. As it turns out, the consumption equivalent loss does not depend on the exact values of the four parameters above as long as the combinations that can be identified from data are indeed calibrated correctly. The root of this result lies in the fact that the consumption equivalent loss involves comparing utility in the monopsonistic and competitive case, so that values such as the level of output do not affect the result, as we summarize in the following proposition.

Proposition 11. *Given combinations of ζ and A^s and of T and ψ that reproduce the moments from the data (labor share and hours worked), the consumption equivalent loss can be computed and it does not depend on the values of these parameters (just on their combinations that can be identified from the data).*

The calibration exercise is performed over 150 periods/years, and the results are plotted in Figure 5. The model replicates the three growth facts during the transition, and the calibrated economy experiences positive GDP growth with negative growth in both hours and labor share. While in the asymptotic equilibrium the facts are matched perfectly, during the transition the quantitative behavior of the economy features an adjustment path at different speed. Figure 5 suggests that GDP growth matches the actual historical growth also during the transition. Conversely, the growth in total hours and labor share starts slower and matches approximately 80 and 50 percent of the actual growth rate. Figure 5 reports also the dynamics of the implied moments, and particularly the level of markdown and concentration. As anticipated, in order to obtain a reasonable convergence, the model needs a level of α around what is used in Table 8, which implies an initial value of the markdown that is twenty percent larger than what is obtained in the accounting exercise of Table 6. Similarly, the simulated model features a concentration ratio that is larger than the simple C4 estimate reported by Autor et al. (2020). The final chart of Figure 5 reports the difference between the wage and GDP level in the model relative to the first best. While such difference is growing over time, the quantitative exercise is connected to the consumption equivalent loss, to which we turn now.

The quantitative results for the Consumption Equivalent Loss (CEL) are reported in Table 9. Overall, growth misallocations imply a CEL around 7.6 percent. The analytics of the exercise is derived in Section 3.8. Note that in Table 9 the quantitative results is similar in the case of hours worked per worker and per person, and is not sensitive to the time horizon of the simulation.

As a final exercise, we perform a transitional dynamics with a change in concentration. Formally, we run the same transitional dynamics of the US economy performed in Figure 5, in assuming that along the transition- in a totally unexpected way- the concentration in the superstar sector falls by 20 percent. Figure 6 reports the dynamic transition. The sub-chart with markdown and markup show the effects of one-off increase induced by the twenty percent fall in the measure m. The main message of Figure 6 concerns the difference between level effect and growth effects, a key feature in growth theory since the seminal contribution of Lucas (1988). In this respect, the increase in concentration induces negative "level effects" in the labor share and hours worked, but does not induce any significant growth effects, as highlighted by the growth dynamics in Figure 6.

Table 7: Calibrated Moments of Monopsonistic Material-Labor Growth Path - US

Moment	Data		Model	
	per capita	per worker	per capita	per worker
	Growth moments			
GDP growth	0.0148	0.0169	0.0148	0.0169
Total hours growth	-0.0002	-0.0014	-0.0002	-0.0014
Labor share growth	-0.0008		-0.0008	
	Time 0 moments			
Moments from the literature				
λ-constant elasticity of l.s.	2.8400		2.8400	
Monopolistic markup	1.3000		1.3000	
Matched moments				
Total hours worked	0.0954	0.2145	0.0954	0.2145
Labor share	0.6167		0.6167	
Implied moments				
Concentration[a]	0.2750		0.6478	
Monopolistic markdown[b]	1.0170		1.1902	

[a] , C4 concentration ratio from Autor et al. (2020).
[b] , Authors' calculations from Section 5.2.

Source: Authors' calculations.

6 Conclusion

Long-run dynamics of labor supply and imperfect labor markets are becoming more relevant in macroeconomics and in growth theory. Models of growth should address the long-run decline in hours per worker and the fall in the labor share observed in most advanced economies. This paper takes first steps in this direction, proposing and solving a standard growth model with declining hours worked and declining labor share driven by oligopsonistic power by superstar firms. In a

Figure 6: Dynamic Simulation of Calibrated Economy with Rising Concentration

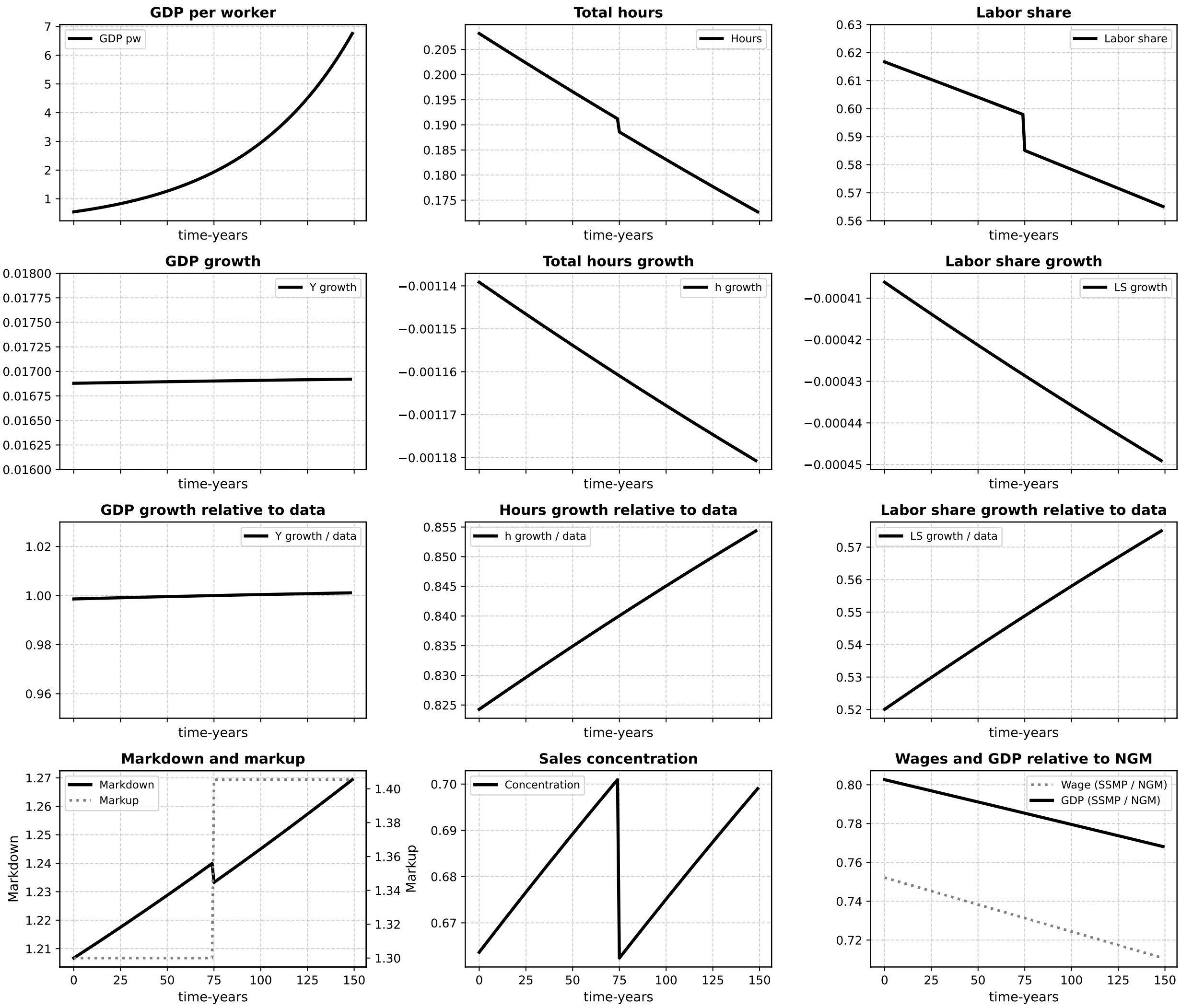


Table 8: Calibrated Parameters of Monopsonistic Material-Labor Growth Path - US

Parameter	Notation	per capita	per worker
Parameters from the literature and normalized			
Discount rate	ρ	0.0500	0.0500
Frisch elasticity	θ	2.8400	2.8400
Cobb-Douglas technology	α	0.2000	0.2000
CES share	ζ	0.1250	0.1250
Fixed materials supply	T	1.0000	1.0000
Time 0 productivity, inferior	A^i	1.0000	1.0000
Number of superstar firms	m	1.2500	1.2500
Calibrated parameters			
Utility curvature	γ	0.9697	1.0643
Productivity growth, superior	g^s	0.0150	0.0180
Productivity growth, inferior	g^i	0.0137	0.0167
CES elasticity	σ	3.4667	3.4667
Time 0 productivity, superior	A^s	4.2106	4.2106
Disutility of work	ψ	14.2265	5.1457
Source: Authors' calculations.			

Table 9: Consumption equivalent loss in percentage terms - US

Time periods	per capita	per worker
100	7.69	7.60
500	7.69	7.60
1000	7.69	7.60

Discrete time approximation with $dt = 1$.

Source: Authors' calculations.

broader perspective, the paper calls for expanding the intersection between growth theory and labor economics. Market power by firms, well established in economics of growth, should be extended to factor markets, incorporating empirically relevant features of real life firms. This research avenue is likely to uncover novel and interesting policy implications. The paper studied the growth and distributive effects of proportional taxation and rising minimum wage in economies with declining hours worked and labor share. In such economies, both policies can be welfare improving.

Much remains to be done. Once monopsonistic power by firms is recognized as part of growth theory, it should be studied how its existence shapes incentives for innovation and technology adoption. Imperfect labor markets have implications also for the empirics of growth. Almost seventy years ago, Solow (1957) introduced growth accounting under the assumption of perfect labor and capital markets. In his remarkable and influential analytical derivation, Solow argued that the empirical exercise should be extended to the case of imperfect labor markets. These concerns should now be taken seriously by national and international statistical offices.

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A Systems of equations

A.1 Monopsonistic economy

The system in intensive form for the monopsonistic economy is the following, where we define

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s} \ , \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i} \ , \quad c_t = \frac{C_t}{A_t^s n_t^s} \ , \quad y_t = \frac{Y_t}{A_t^s n_t^s}$$

$$\psi h_t^{\frac{1}{\theta}} = c_t^{-\gamma} p_t^i (1-\alpha) \frac{A_t^i}{(A_t^s n_t^s)^\gamma} (x_t^i)^\alpha$$

$$\frac{\dot{c}_t}{c_t} = \frac{\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s}$$

$$c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} = y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s}\right) x_t^s - \left(\delta^i + g^i + \frac{\dot{n}_t^i}{n_t^i}\right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s}$$

$$y_t = \left[\zeta^{\frac{1}{\sigma}} ((x_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$(x_t^s)^\alpha = \zeta (p_t^s)^{-\sigma} y_t \ ,$$

$$\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^\alpha = (1-\zeta)(p_t^i)^{-\sigma} y_t$$

$$\frac{\sigma m - 1}{\sigma m} p_t^s (1-\alpha) A_t^s (x_t^s)^\alpha = p_t^i (1-\alpha) A_t^i (x_t^i)^\alpha \left[1 + \frac{\alpha}{m} \frac{n_t^s}{n_t^i}\right]$$

$$\frac{\sigma m - 1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s = p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i$$

A.2 Competitive economy

The competitive economy is very similar to the one above, without markup and markdown.

$$\psi h_t^{\frac{1}{\theta}} = c_t^{-\gamma} p_t^i (1-\alpha) \frac{A_t^i}{(A_t^s n_t^s)^\gamma} (x_t^i)^\alpha$$
$$\frac{\dot{c}_t}{c_t} = \frac{p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{n}_t^s}{n_t^s}$$
$$c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s} = y_t - \left(\delta^s + g^s + \frac{\dot{n}_t^s}{n_t^s}\right) x_t^s - \left(\delta^i + g^i \frac{\dot{n}_t^i}{n_t^i}\right) x_t^i \frac{A_t^i n_t^i}{A_t^s n_t^s}$$
$$y_t = \left[\zeta^{\frac{1}{\sigma}} ((x_t^s)^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
$$(x_t^s)^\alpha = \zeta (p_t^s)^{-\sigma} y_t \ ,$$
$$\frac{A_t^i n_t^i}{A_t^s n_t^s} (x_t^i)^\alpha = (1-\zeta)(p_t^i)^{-\sigma} y_t$$
$$p_t^s (1-\alpha) A_t^s (x_t^s)^\alpha = p_t^i (1-\alpha) A_t^i (x_t^i)^\alpha$$
$$p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s = p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i$$

A.3 Worker-capitalist in intensive form

Define

$$c_t^c = \frac{C_t^c}{A_t^s h_t} \ , \quad c_t^w = \frac{C_t^w}{A_t^i h_t} \quad x_t^s = \frac{K_t^s}{A_t^s h_t} \ , \quad x_t^i = \frac{K_t^i}{A_t^i h_t} \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$c_t^w = p_t^i (1-\alpha)(x_t^i)^\alpha$$
$$\psi h_t^{\frac{1}{\theta}+\gamma} = (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma} (1-\alpha)(x_t^i)^\alpha$$
$$n_t^s + n_t^i = h_t$$
$$\frac{\dot{c}_t^c}{c_t^c} = \frac{\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h}$$
$$c_t^c + \frac{A_t^i}{A_t^s} c_t^w + \dot{x}_t = y_t - \delta^s x_t^s + \delta^i x_t^i$$
$$y_t = \left[\zeta^{\frac{1}{\sigma}} ((1-\nu_t)(x_t^s)^\alpha)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} \left(\frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
$$(1-\nu_t)(x_t^s)^\alpha = \zeta (p_t^s)^{-\sigma} y_t \ , \qquad \frac{A_t^i}{A_t^s} \nu_t (x_t^i)^\alpha = (1-\zeta)(p_t^i)^{-\sigma} y_t$$
$$\frac{\sigma m-1}{\sigma m} p_t^s (1-\alpha) A_t^s (x_t^s)^\alpha = p_t^i (1-\alpha) A_t^i (x_t^i)^\alpha \left[1 + \frac{\alpha}{m} \frac{1-\nu_t}{\nu_t}\right]$$
$$\frac{\sigma m-1}{\sigma m} p_t^s \alpha (x_t^s)^{\alpha-1} - \delta^s = p_t^i \alpha (x_t^i)^{\alpha-1} - \delta^i$$

Alternatively, define

$$K_t = K_t^s + K_t^i \ , \quad c_t^c = \frac{C_t^c}{A_t^s h_t} \ , \quad c_t^w = \frac{C_t^w}{A_t^i h_t} \quad x_t = \frac{K_t}{A_t^s h_t} \ , \quad \kappa_t = \frac{K_t^i}{K_t} \ , \quad \nu_t = \frac{n_t^i}{h_t} \ , \quad y_t = \frac{Y_t}{A_t^s h_t}$$

then the worker-capitalist system in intensive form reads

$$\begin{aligned}
c_t^w &= p_t^i(1-\alpha)\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^\alpha \\
\psi h_t^{\frac{1}{\theta}+\gamma} &= (c_t^w)^{-\gamma} p_t^i (A_t^i)^{1-\gamma}(1-\alpha)\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^\alpha \\
n_t^s + n_t^i &= h_t \\
\frac{\dot{c}_t^c}{c_t^c} &= \frac{\frac{\sigma m-1}{\sigma m}p_t^s\alpha\left(\frac{1-\kappa_t}{1-\nu_t}x\right)^{\alpha-1} - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h} \\
c_t^c + \frac{A_t^i}{A_t^s}c_t^w + \dot{x}_t &= y_t - ((1-\kappa_t)\delta^s + \kappa_t\delta^i)x_t \\
y_t &= \left[\zeta^{\frac{1}{\sigma}}\left((1-\nu_t)\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^\alpha\right)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}\left(\frac{A_t^i}{A_t^s}\nu_t\left(\frac{\kappa_t A_t^s}{\nu A_t^i}x_t\right)^\alpha\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
(1-\nu_t)\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^\alpha &= \zeta(p_t^s)^{-\sigma}y_t \ , \qquad \frac{A_t^i}{A_t^s}\nu_t\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^\alpha = (1-\zeta)(p_t^i)^{-\sigma}y_t \\
\frac{\sigma m-1}{\sigma m}p_t^s(1-\alpha)A_t^s\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^\alpha &= p_t^i(1-\alpha)A_t^i\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^\alpha\left[1+\frac{\alpha}{m}\frac{1-\nu_t}{\nu_t}\right] \\
\frac{\sigma m-1}{\sigma m}p_t^s\alpha\left(\frac{1-\kappa_t}{1-\nu_t}x_t\right)^{\alpha-1} - \delta^s &= p_t^i\alpha\left(\frac{\kappa_t A_t^s}{\nu_t A_t^i}x_t\right)^{\alpha-1} - \delta^i
\end{aligned}$$

the study of this system transformed in growth rates at infinity delivers the growth rates in the main text and is consistent with the notation used to prove stability of the main model.

B The System with general Technology

With general constant returns to scale production function for the superior and inferior sectors indicated respectively with $F(K^s, N^s, A^s)$ and $G(K^i, N^i, A^i)$, the general system for the SMPE

$$\frac{\sigma m-1}{\sigma m}p_t^s F_N(K_t^s, n_t^s, A_t^s) = p_t^i\left[G_N(K_t^i, n_t^i, A_t^i) - \frac{1}{m}\frac{n_t^s}{n_t^i}G_{NN}(K_t^i, n_t^i, A_t^i)\right]$$

$$\frac{\sigma m-1}{\sigma m}p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s = p_t^i G_K(K_t^i, n_t^i, A_t^i) - \delta^i$$

$$G(K_t^i, n_t^i, A_t^i) = (1-\zeta)(p_t^i)^{-\sigma}\left[\zeta^{\frac{1}{\sigma}}F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$F(x_t^s, n_t^s, A_t^s) = \zeta(p_t^s)^{-\sigma}\left[\zeta^{\frac{1}{\sigma}}F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$\psi h_t^{\frac{1}{\theta}} = C_t^{-\gamma}p_t^i G_N(K_t^i, n_t^i, A_t^i)$$

$$h_t = n_t^s + n_t^i$$

$$\gamma\frac{\dot{C}_t}{C_t} = \frac{\sigma m-1}{\sigma m}p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho$$

$$C_t + \dot{K}_t^s + \dot{K}_t^i = \left[\zeta^{\frac{1}{\sigma}}F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \delta^s K_t^s - \delta^i K_t^i$$

where from the first equation we can see that we can find a very nice expression for the markdown, i.e.

$$\mu_t = 1 - \frac{1}{m}\frac{n_t^s}{n_t^i}\frac{G_{NN}(K_t^i, n_t^i, A_t^i)}{G_N(K_t^i, n_t^i, A_t^i)}\ .$$

It is simple to show that with general CRS technologies one can't solve for the balanced growth path since for p_t^s and x_t^s converging to a finite number and for $p_t^i \to \infty$ it must be that $x_t^i = \frac{K_t^i}{A_t^i n_t^i}$ tend to ∞. To study this, one needs to know the shape of G (while in principle, we could keep F a neoclassical production function that satisfies Uzawa's theorem).

There is one special case which is tractable though: if the two intermediaries are perfect substitutes, the complications arising from prices disappear since the two goods must trade at the same price. With no diverging price for the inferior good, x_t^i will converge to a constant as well, and we can solve for the balanced growth path with a general inferior technology G.

C Growth rates systems

Monopsonistic Economy The system of growth rates for the monopsonistic decentralized economy is (at infinity already zeroing all growth rates of converging values, $g_{p^s} = g_{x^s} = 0$ and imposing

$g_C = g_Y$)

$$
\begin{aligned}
g^s &= g^i + g_{p^i} + \alpha g_{x^i} + g_{n^s} - g_{n^i} \\
g_{p^i} + (\alpha - 1) g_{x^i} &= 0 \\
g^i + g_{n^i} + \alpha g_{x^i} &= -\sigma g_{p^i} + g^s + g_{n^s} \\
\frac{1}{\theta} g_{n^s} &= -\gamma (g^s + g_{n^s}) + g_{p^i} + g^i + \alpha g_{x^i} \\
g_h &= g_{n^s} \\
g_C &= g^s + g_h
\end{aligned}
$$

The decentralized growth rates are

$$
\begin{aligned}
g_C &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1-\alpha)(\sigma - 1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i) \\
g_h = g_{n^s} &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s - \frac{1}{\frac{1}{\theta} + \gamma} \frac{(1-\alpha)(\sigma - 1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i) \\
g_{n^s} - g_{n^i} &= \frac{(1-\alpha)(\sigma - 1)}{1 + \alpha + \sigma(1-\alpha)} (g^s - g^i)
\end{aligned}
$$

Growth rates of additional variables of interest are

$$
g_\mu = g_{n^s} - g_{n^i} \ , \quad g_w = g_{p^i} + g^i + \alpha g_{x^i} \ .
$$

Optimal-Competitive Economy The corresponding system of growth rates for the competitive economy (equivalent to the optimal growth problem) is

$$
\begin{aligned}
g^s &= g^i + g^*_{p^i} + \alpha g^*_{x^i} \\
g^*_{p^i} + (\alpha - 1) g^*_{x^i} &= 0 \\
g^i + g^*_{n^i} + \alpha g^*_{x^i} &= -\sigma g^*_{p^i} + g^s + g^*_{n^s} \\
\frac{1}{\theta} g^*_{n^s} &= -\gamma (g^s + g^*_{n^s}) + g^*_{p^i} + g^i + g^*_{n^i} + \alpha g^*_{x^i} \\
g^*_h &= g^*_{n^s} \\
g^*_C &= g^s + g^*_h
\end{aligned}
$$

The optimal growth rates are

$$
\begin{aligned}
g^*_C &= \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s \\
g^*_h &= \frac{1 - \gamma}{\frac{1}{\theta} + \gamma} g^s \\
g^*_{n^s} - g^*_{n^i} &= (1-\alpha)(\sigma - 1)(g^s - g^i) \ .
\end{aligned}
$$

Notice that if we let the substitutability between goods increase, as we approach the perfect substitutes limit the growth rate of labor in the inferior sector goes to minus infinity, which is coherent with the intuition (that can be verified directly) that whenever goods are perfect substitutes, the inferior sector closes right away in the optimal solution, since there is no reason to use an inefficient inferior technology when there are constant returns to scale in factors of production.

D Stability

The system we use to prove stability is the following.

$$\frac{\dot{c}}{c} = \frac{\frac{\sigma m-1}{\sigma m} p^s F_K\left(\frac{1-\kappa}{1-\nu}x,1,1\right) - \delta^s - \rho}{\gamma} - g^s - \frac{\dot{h}}{h}$$

$$\psi h^{\frac{1}{\theta}+\gamma} = c^{-\gamma} p^i \frac{A^i}{(A^s)^\gamma} G_N\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right)$$

$$\frac{\sigma m-1}{\sigma m} p_s F_K\left(\frac{1-\kappa}{1-\nu}x,1,1\right) - \delta^s = p_i G_K\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right) - \delta^i$$

$$\frac{\sigma m-1}{\sigma m} p^s A^s F_N\left(\frac{1-\kappa}{1-\nu}x,1,1\right) = p^i A^i \left[G_N\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right) - \frac{1}{m}\frac{1-\nu}{\nu} G_{NN}\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right)\right]$$

$$c+\dot{x} = \left[\zeta^{\frac{1}{\sigma}}\left\{(1-\nu)F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)\right\}^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}}\left\{\nu\frac{A^i}{A^s}G\left(\frac{\kappa}{\nu}\frac{A^s}{A^i}x,1,1\right)\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \left((1-\kappa)\delta^s + \kappa\delta^i + g^s + \frac{\dot{h}}{h}\right)$$

$$1 = \delta p_s^{-\sigma}\left[\delta^{\frac{1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}}\left\{\frac{A_i}{A_i}\frac{\nu}{1-\nu}\frac{G\left(\frac{\kappa}{\nu}\frac{A_s}{A_s}x,1,1\right)}{F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$1 = (1-\delta)p_i^{-\sigma}\left[\delta^{\frac{1}{\sigma}}\left\{\frac{A_s}{A_i}\frac{1-\nu}{\nu}\frac{F\left(\frac{1-\kappa}{1-\nu}x,1,1\right)}{G\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x,1,1\right)}\right\}^{\frac{\sigma-1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Theorem 2. *Assume that $\delta^s = \delta^i$, then the model is locally saddle path stable.*

We will prove stability in five steps (all values are assumed at the steady state $(c^*, 0, 0, x^*)$).

1. Show that $\frac{\partial \dot{h}}{\partial h} = g_h < 0$ and that for all other variables, the derivative with respect to h is zero.
2. Show that $\frac{\partial \dot{\nu}}{\partial \nu} = g_{n^i} - g_{n^s} < 0$ and that with respect to all other variables, the derivative of $\dot{\nu}$ is zero.
3. Show that $\frac{\partial \dot{x}}{\partial c} = -\frac{1}{1+\theta\alpha_i}$ (under $\alpha_s = \alpha_i$, in general one should get $\frac{\partial \dot{x}}{\partial c} < 0$).
4. Show that $\frac{\partial \dot{c}}{\partial c} = -c^* \frac{\partial(\dot{h}/h)}{\partial c}$ and that $\frac{\partial \dot{c}}{\partial x} = \frac{c^*}{\gamma}\frac{\sigma-1}{\sigma}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2} - c^*\frac{\partial(\dot{h}/h)}{\partial x}$.

These steps together will generate a Jacobian of the type

$$J_\Phi = \begin{pmatrix} -c^*\frac{\partial(\dot{h}/h)}{\partial c} & 0 & * & \frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2} - c^*\frac{\partial(\dot{h}/h)}{\partial x} \\ * & g_h & * & * \\ 0 & 0 & g_{n^i} - g_{n^s} & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & * \end{pmatrix}$$

The determinant of this matrix is equal to that of the matrix (we are basically subtracting the third row from the first, this leaves the determinant unchanged.)

$$\begin{pmatrix} 0 & 0 & * & \underbrace{\frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2}}_{<0} \\ * & \underbrace{g_h}_{<0} & * & * \\ 0 & 0 & \underbrace{g_{n^i}-g_{n^s}}_{<0} & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & * \end{pmatrix}$$

which is strictly negative. Then the matrix is non-singular and the steady state is hyperbolic. Also, either one or three eigenvalues have positive real part. But eigenvalues are roots of the characteristic polynomial which solves

$$\det\begin{pmatrix} k\frac{\partial \dot{x}}{\partial c}-\lambda & 0 & * & (<0)+k\frac{\partial \dot{x}}{\partial x} \\ * & g_h-\lambda & * & * \\ 0 & 0 & g_{n^i}-g_{n^s}-\lambda & 0 \\ -\frac{1}{1+\theta\alpha_i} & 0 & * & *-\lambda \end{pmatrix} = (g_h-\lambda)(g_{n^i}-g_{n^s}-\lambda)\begin{pmatrix} k\frac{\partial \dot{x}}{\partial c}-\lambda & (<0)+k\frac{\partial \dot{x}}{\partial x} \\ -\frac{1}{1+\theta\alpha_i} & *-\lambda \end{pmatrix}$$

which implies that two of these are negative, and thus three eigenvalues must have negative real part.

Proof. The system of equations with Cobb-Douglas production functions reads as follows (assuming $\delta^s=\delta^i=\delta$, but leaving $\alpha_s\neq\alpha_i$)

$$\frac{\dot{c}}{c}=\frac{1}{\gamma}\frac{\sigma m-1}{\sigma m}\zeta^{\frac{1}{\sigma}}\left\{\zeta^{\frac{1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\left[\frac{A_i}{A_s}\frac{1-\nu}{\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{1}{\sigma-1}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}-\frac{\rho+\delta}{\gamma}-g^s-\frac{\dot{h}}{h} \tag{45}$$

$$\psi h^{\frac{1}{\theta}+\gamma}=c^{-\gamma}\frac{A_i}{A_s^{\gamma}}(1-\alpha_i)\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}(1-\zeta)^{\frac{1}{\sigma}}\left\{\zeta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\right\}^{\frac{1}{\sigma-1}} \tag{46}$$

$$\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right)^{\frac{1}{\sigma}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}=\alpha_i\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i-1}$$

$$A_s\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right)^{\frac{1}{\sigma}}(1-\alpha_s)\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s}=(1-\alpha_i)A_i\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}\left[1+\frac{1}{m}\frac{1-\nu}{\nu}\alpha_i\right]$$

$$c+\dot{x}=\left[\zeta^{\frac{1}{\sigma}}\left\{(1-\nu)\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s}\right\}^{\frac{\sigma-1}{\sigma}}+(1-\zeta)^{\frac{1}{\sigma}}\left\{\nu\frac{A_i}{A_s}\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_i}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-\left(\delta+g^s+\frac{\dot{h}}{h}\right) \tag{47}$$

Combining (taking the ratio of) the third and the fourth equations delivers

$$\frac{1-\alpha_s}{\alpha_s}\frac{1-\kappa}{1-\nu} = \frac{1-\alpha_i}{\alpha_i}\frac{\kappa}{\nu}\left[1+\frac{1-\nu}{\nu}\alpha_i\right]$$

which allows us to express κ as a function ν. In growth rates this implies

$$\frac{\dot{\kappa}}{\kappa}\frac{1}{1-\kappa} = \frac{\dot{\nu}}{\nu}\left[1+\frac{\nu}{1-\nu}+\frac{(1-\alpha_i)\nu}{\alpha_i+(1-\alpha_i)\nu}\right] .$$

We can also rewrite the third equation as

$$\frac{\sigma m-1}{\sigma m}\left(\frac{A_i}{A_s}\frac{\nu}{1-\nu}\right)^{\frac{1}{\sigma}}\left(\frac{\delta}{1-\delta}\right)^{\frac{1}{\sigma}}\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{1-\alpha_i\frac{\sigma-1}{\sigma}}\frac{\alpha_s}{\alpha_i}\left(\frac{1-\kappa}{1-\nu}x\right)^{-1+\alpha_s\frac{\sigma-1}{\sigma}} = 1 . \tag{48}$$

Differentiating with respect to time and imposing $\alpha_s = \alpha_i$ gives in a straightforward manner an equation that depends on ν and κ only (thus ultimately on ν only) and that when evaluated at $\nu = \kappa = 0$ returns

$$\frac{\dot{\nu}}{\nu} = -\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i) = g_{n^i}-g_{n^s}$$

We are now ready to proceed with our four steps of proof. Notice that whenever the ratio A_s/A_i shows up in equations (45) and (47) (which we are not going to differentiate), to make the equation time-invariant we need to substitute it out using (48).

1. Notice that the only equations in which h appears are (45), (46) and (47). We can transform (46) in growth rates. Once one substitutes $\frac{\dot{c}}{c}$ out using (45), the expression reads

$$\frac{1}{\theta}\frac{\dot{h}}{h} = -\delta^{\frac{1}{\sigma}}\left\{\delta^{\frac{1}{\sigma}}+(1-\delta)^{\frac{1}{\sigma}}\left[\frac{A_i}{A_s}\frac{1-\nu}{\nu}\frac{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{1}{\sigma-1}}\alpha_s\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_s-1}+\rho+\delta+g^i+\alpha_i\left[\frac{\dot{\kappa}}{\kappa}-\frac{\dot{\nu}}{\nu}+g^s-g^i+\frac{\dot{x}}{x}\right]+$$

$$+\frac{\delta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}}{\delta^{\frac{1}{\sigma}}\left[\frac{A_s}{A_i}\frac{\nu}{1-\nu}\frac{\left(\frac{1-\kappa}{1-\nu}x\right)^{\alpha_i}}{\left(\frac{\kappa}{\nu}\frac{A_s}{A_i}x\right)^{\alpha_s}}\right]^{\frac{\sigma-1}{\sigma}}+(1-\delta)^{\frac{1}{\sigma}}}\frac{1}{\sigma}\left[g^i-g^s-\frac{1}{1-\nu}\frac{\dot{\nu}}{\nu}+(\alpha_s-\alpha_i)\frac{\dot{x}}{x}-\alpha_i(g^s-g^i)+\alpha_s\left(-\frac{\kappa}{1-\kappa}\frac{\dot{\kappa}}{\kappa}+\frac{\nu}{1-\nu}\frac{\dot{\nu}}{\nu}\right)-\alpha_i\left(\frac{\dot{\kappa}}{\kappa}-\frac{\dot{\nu}}{\nu}\right)\right]$$

   We then notice that only the growth rate of h appear, not h itself. We can then substitute $\frac{\dot{h}}{h}$ out of (45) and (47). This establishes that no time derivative except $\dot{h}$ depends on h. Next, consider that we are left with an equation arising from differentiating (46) which is of the type

$$\frac{\dot{h}}{h} = \mathcal{H}(\nu, x, \kappa(\nu), \dot{x}(c, \nu, x, \kappa(\nu)))$$

   which must be such that

$$\mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

   but then clearly

$$\frac{\partial\dot{h}}{\partial h} = \mathcal{H}(0, x^*, 0, 0) = g_h < 0$$

   at the steady state. This establishes the first point.

2. For the second point, by differentiating (48) we find

$$\left(1-\alpha_i\frac{\sigma-1}{\sigma}\right)\left(\frac{\dot\kappa}{\kappa}-\frac{\dot\nu}{\nu}\right)-\left(1-\alpha_s\frac{\sigma-1}{\sigma}\right)\left(-\frac{\kappa}{1-\kappa}\frac{\dot\kappa}{\kappa}+\frac{\nu}{1-\nu}\frac{\dot\nu}{\nu}\right)+\frac{1}{\sigma}(g^i-g^s)+$$
$$+\frac{1}{\sigma}\left(\frac{\dot\nu}{\nu}+\frac{\nu}{1-\nu}\frac{\dot\nu}{\nu}\right)+\left(1-\alpha_i\frac{\sigma-1}{\sigma}\right)(g^s-g^i)+\frac{\sigma-1}{\sigma}(\alpha_s-\alpha_i)\frac{\dot x}{x}=0$$

Clearly for $\alpha_s=\alpha_i$ this gives an equation for $\frac{\dot\nu}{\nu}$ that depend on ν only. Also, again it is an expression for the growth rate of ν, and $\nu^*=0$, as above

$$\frac{\dot\nu}{\nu}=\mathcal{N}(\nu)$$

where

$$\mathcal{N}(0)=-\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i)=g_{n^s}-g_{n^i}$$

so that

$$\frac{\partial\dot\nu}{\partial\nu}=\mathcal{N}(0)=-\frac{(\sigma-1)(1-\alpha_i)}{1+\alpha_i+\sigma(1-\alpha_i)}(g^s-g^i)<0\ .$$

This completes the second step.

3. For the third point, notice that once we substitute out $\frac{\dot h}{h}$ for (47) using (1), we find

$$(1+\alpha_i\theta)\dot x=-c+\text{other terms independent of } c$$

This directly implies

$$\frac{\partial\dot x}{\partial c}=-\frac{1}{1+\alpha_i\theta}<0\ .$$

4. Since (45) is in the form

$$\frac{\dot c}{c}=\mathcal{C}(\nu,x,\kappa(\nu),\dot h/h)\ ,$$

we will have

$$\frac{\partial\dot c}{\partial c}=\mathcal{C}(0,x^*,0,g_h)-c^*\frac{\partial(\dot h/h)}{\partial c}$$

but since c converges to a constant, we must have $\mathcal{C}(0,x^*,0,g_h)=0$. Notice than that $\dot h/h$ depends on c only through $\dot x$, since

$$\frac{\partial(\dot h/h)}{\partial c}=\theta\alpha_i\frac{\partial(\dot x/x)}{\partial c}=-\frac{\theta\alpha_i}{1+\theta\alpha_i}\frac{1}{x^*}$$

On the other hand we have

$$\frac{\partial\dot c}{\partial x}=\frac{c^*}{\gamma}\frac{\sigma m-1}{\sigma m}\delta^{\frac{1}{\sigma-1}}\alpha_s(\alpha_s-1)(x^*)^{\alpha_s-2}-c^*\frac{\partial(\dot h/h)}{\partial x}$$

□

E The Model with Fixed Materials

The only difference in the model is the absence of capital and the fact that the intermediaries produce with a fixed material T_t^s and T_t^i so that

$$y_t^s = A_t^s (T_t^s)^\alpha; \quad y_t^i = A_t^i (T_t^i)^\alpha$$

The choice of materials is only influenced by the monopoly power in the labor market and its problem solves.

$$\frac{\partial y_t^s}{\partial T_t^s} p_t^s \frac{\sigma m - 1}{\sigma m} = p^T$$

The fundamental factor allocation for the superstar firms implies

$$\left[1 + \frac{\alpha}{m} \frac{N_t^s}{N_t^i}\right] \frac{N_t^s}{T^s} = \frac{N_t^i}{T^i}$$

where it is clear that as long as $\alpha > 0$, the factor allocation of the superstar firm is distorted vis-a-vis a pure neoclassical factor ratio equilibrium that implies $\frac{N_t^s}{T_t^s} = \frac{N_t^i}{T_t^i}$ that characterizes the optimal growth problem that will be discussed later. The superstar firm makes strictly positive profits that are fully distributed to the consumers who owns the firm, to which we turn next. Using the key conditions of the superstar firms for both labor and land,

The representative consumer problem maximize utility period by period and takes as given the wages w_t^j $(j = s, i)$ in the two sectors, and needs to choose the total hours h_t as well as the hours worked in every sector. She is also endowed with the materials T that that she rents to the firm at price P_t^T. Finally, the consumers obtains per capita profits/dividends from the superstar firm that we indicate with π_t^s. The consumer problem is

$$\begin{aligned} &\max_{C_t, h_t, n_t^s, n_t^i, T_t^s, T_t^i} \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t^{\frac{1}{\theta}+1}}{\frac{1}{\theta}+1} \\ &\text{s.t.} \quad P_t C_t = w_t^s n_t^s + w_t^i n_t^i + P_t^T (T_t^s + T_t^i) + \pi_t^s \\ &(n_t^s + n_t^i) \leq h_t; \quad T_t^s + T_t^i = \overline{T}_t \end{aligned}$$

where P_t is the natural price index of equation 2 normalized to one and $\overline{T}_t$ are the fixed material at time t. The key first order conditions for the consumers for the labor and material allocation are

$$\begin{aligned} C_t^{-\gamma} w_t^j &= \psi (n_t^s + n_t^i)^{\frac{1}{\theta}} \quad j = i, s \\ C_t^{-\gamma} \frac{\partial C_t}{\partial T_t^j} &= p_t^T \qquad\quad j = i, s \end{aligned}$$

and imply the standard neoclassical labor supply problem for which the marginal rate of substitution between labor and consumption is equal to the wage rate, and the marginal utility of consumption is equal to price of material. Since these factor conditions holds for each sector, the labor supply of the consumer implies a fundamental arbitrage condition so that the wage in the two sectors has to be identical for he worker to be in equilibrium, and $w_t^s = w_t^i$. Similarly, the material allocation is such that the marginal utility of consumption in each of the two sector is identical.

In equilibrium all markets clear, and the labor and material demand quantities coincide with labor supply choice of the consumer. so that

$$C_t = Y_t; \quad n_t^s = N_t^s; \quad n_t^i = N_t^i; \quad T^t = T_t^s + T_t^i,$$

where the condition $C_t = Y_t$ implies that all final output is consumed and there are no savings.

The SPME is very similar but it only features a price for the fixed materials p_t^T in place of the cost of capital R_t The limit system at time t for obtaining the BGP is made up by the following four equations. The growth rates are

$$\begin{aligned}
g_C &= \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)}g^s - (1-\alpha)\Omega(g^s - g^i) \\
g_{ns} &= \frac{1-\gamma}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)}g^s - \Omega(g^s - g^i) \\
g_\mu &= \frac{\sigma-1}{1+\alpha+\sigma(1-\alpha)}(g^s - g^i) \\
g_{T_i} &= -2g_\mu
\end{aligned}$$

where the constant Ω is depends on the structural parameters of the model $\Omega = \frac{1}{\frac{1}{\theta}+\alpha+\gamma(1-\alpha)}\frac{\sigma-1}{1+\alpha+\sigma(1-\alpha)}$. The SMPGE is a a semi endogenous growth model, since its equilibrium growth rate crucially depends on the exogenous growth productivity g^s and g^i, but it fully interacts with the structural parameters of the model. For further characterizing its properties, it is necessary to solve for the corresponding optimal growth problem, to which we next turn.

The optimal growth problem yields the following equations and the growth rates are

$$\begin{aligned}
g_C^* &= \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)}g^s \\
g_{ns}^* &= \frac{1-\gamma}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)}g^s \\
g_{ni}^* &= \frac{1-\gamma}{\frac{1}{\theta}+\gamma+\alpha(1-\gamma)}g^s + (\sigma-1)(g^i - g^s) \\
g_{T^i}^* &= -(\sigma-1)(g^s - g^i)
\end{aligned}$$

We are thus now in a position to study the misallocation and growth effect of the SMPGE.

F Monopoly in Growth Theory

This appendix discusses in details three comments on monopoly power and growth. For ease of exposition, in this section we assume that the superior sector features only one superstar firm. All results carry through in the case $m > 1$.

F.1 Dynamics of the aggregate markup in our model

The first fact is related to the aggregate markup in our model of superstar firm and structural transformation. While our discussion highlights the role of increasing markdown as a key driver to generate a declining labor share and the growth drag, one should distinguish between the constant markup of superstar $\frac{\sigma}{\sigma-1}$ and the aggregate markup in the economy. Indeed, the aggregate markup increases over time and converges to the one of the superior sector, mechanically driven by the production shift from the competitive fringe toward the superior sector.

The aggregate markup of the economy is defined as the weighted average

$$\mu_t^{agg} = \frac{p_t^s y_t^s + p_t^i y_t^i}{mc_t^s y_t^s + mc_t^i y_t^i}$$

Since both intermediaries are produced using Cobb-Douglas, the marginal cost in sector j reads

$$mc_t^j = \frac{1}{(A_t^j)^{1-\alpha_j}} \frac{r_t^{\alpha_j} W_t^{1-\alpha_j}}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}}; \quad j \in [i, s]$$

Furthermore

$$p_t^s = \frac{\sigma}{\sigma - 1} mc_t^s \ , \qquad p_t^i = mc_t^i$$

so that in the aggregate for $\alpha_s = \alpha_i$

$$\mu_t^{agg} = \frac{\frac{\sigma}{\sigma-1}\frac{y_t^s}{(A_t^s)^{1-\alpha}} + \frac{y_t^i}{(A_t^i)^{1-\alpha}}}{\frac{y_t^s}{(A_t^s)^{1-\alpha}} + \frac{y_t^i}{(A_t^i)^{1-\alpha}}}$$

now notice that $\frac{y_t^j}{(A_t^j)^{1-\alpha}}$ grows at rate $\alpha g_j + g_{n^j} + \alpha g_{x_j}$ and that $\alpha g^s + g_{n^s} > \alpha g^i + g_{n^s} + \alpha g_{x^i}$ so that over time as A_t^s / A_t^i grows, the aggregate markup

$$\mu_t^{agg} \to \frac{\sigma}{\sigma - 1}$$

converges to the markup of the superior sector from below.

F.2 Monopoly power has no growth effects with CES

The second fact we want to highlight is that the description of an economy in which the superior sector enjoys monopoly power only. A byproduct of the CES production assumption in the competitive final good sector implies that market power by superstar firms has no growth effects. The model with monopoly only is defined by the following seven equations.

$$
\begin{aligned}
p_t^s F_N(K_t^s, n_t^s, A_t^s) &= \frac{\sigma}{\sigma-1} p_t^i A_t^i G_N(K_t^i, n_t^i, A_t^i) \\
\frac{\sigma-1}{\sigma} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s &= p_t^i G_K(K_t^i, n_t^i, A_t^i) - \delta^i \\
G(K_t^i, n_t^i, A_t^i) &= (1-\zeta)(p_t^i)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
F(K_t^s, n_t^s, A_t^s) &= (1-\zeta)(p_t^s)^{-\sigma} \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
\psi h_t^{\frac{1}{\theta}} &= C_t^{-\gamma} p_t^i G_N(K_t^i, n_t^i, A_t^i) \\
\gamma \frac{\dot{C}_t}{C_t} &= \frac{\sigma-1}{\sigma} p_t^s F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho \\
C_t + \dot{K}_t^s + \dot{K}_t^i &= \left[\zeta^{\frac{1}{\sigma}} F(K_t^s, n_t^s, A_t^s)^{\frac{\sigma-1}{\sigma}} + (1-\zeta)^{\frac{1}{\sigma}} G(K_t^i, n_t^i, A_t^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \delta^s K_t^s - \delta^i K_t^i \\
n_t^s + n_t^i &= h_t
\end{aligned}
$$

where the only different with respect to the SMPE is the absence of the markdown in the right hand side of the first equation. Furthermore, these equations differ from those of a competitive model just for the presence of the markup $\frac{\sigma}{\sigma-1}$ in the first two equations and in the Euler equation. When passing to growth rates (recall that p_t^s and $x_t^s = \frac{K_t^s}{A_t^s n_t^s}$ converge to constants), the markup in the first to equations disappears, while the markup in the Euler equation only changes the BGP level of x_t^s. This establishes that growth rates in the economy with monopoly power only coincide with those of the competitive economy, hence monopoly power with CES demand has no growth effects.

F.3 A note on rising markups and balanced growth paths

One may wonder why we restrict ourself to CES demand for the superstar intermediate input. Indeed, the literature on superstar firms emphasizes the role of increasing aggregate markups. Similarly to what outlined in Appendix F.1, also Autor et al. (2020) the increase in aggregate markup is a byproduct of production moving to high markup firms. Even though CES is standard feature of growth theory (with structural transformation), when can propose different demand structure that leads to different predictions for the dynamics of markup that may be consistent with a balanced growth path (BGP) featuring declining labor share. Unlike the case of rising markdown, an economy with rising markup would distort also capital market and appears at odds with a constant capital output ratio in BGP. Such feature of BGP is part of the original Kaldor facts as well confirmed in the key facts on growth (Jones, 2016).

Specifically, in a an economy with diverging markup, the marginal productivity of capital needs to fall, requiring a declining capital output ratio. To make this point explicit, consider a simple Neoclassical Growth Model with one producer only that is able to charge a time varying markup ϱ_t, and whose profits are rebated to the representative agent of the economy. The model boils down to

(with Cobb-Douglas technology)

$$\frac{\dot{c}}{c} = \frac{\frac{1}{\varrho_t}\alpha x^{\alpha-1} - \delta - \rho}{\gamma} - g$$
$$c + \dot{x} = F(x, 1, 1) - \delta x$$

The labor share in this economy reads

$$\alpha_L = \frac{\alpha}{\varrho}$$

so that a constantly declining labor share requires ϱ to increase a constant rate. (equivalently one can consider a CES demand ans assume $\sigma \to 1$ so that $\sigma - 1$ has a constant growth rate). But a BGP requires a constant rate of return, and thus a constant consumption in efficiency unit c. The Euler equation implies

$$-g_\varrho + (\alpha - 1)g_x = 0 \Longrightarrow g_x = -\frac{g_\varrho}{1-\alpha} < 0$$

In other words, in a simple model with time varying markup the capital-output ratio declines too over time, contradicting one of the key empirical facts of long run growth, as discussed in Jones (2016).