Monopsony in Growth Theory

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Abstract

The neoclassical growth model assumes fixed labor supply and competitive labor markets. Is it harmless to ignore monopsonistic power in the neoclassical growth model? The paper argues that it is not, especially if a growth model needs to be consistent with the long-run dynamics of the labor share. This paper solves a minimalist growth model with monopsonistic power at the firm level and two production technologies with different degrees of efficiency. The paper shows that monopsonistic power by the representative firm implies either a "level" or a "growth" effect in the determination of the labor share. If the two sectors feature unbalanced growth, the economy converges to a an asymptotic balanced growth in which the labor share asymptotically decline, in line with secular evidence on labor share dynamics. The paper shows also that the monopsonistic equilibrium has sizeable "misallocative" effects, since it implies the use of less efficient technologies that are not used by the optimal growth problem. Finally, the paper shows that the negative welfare effect of monopsony is larger when the model accounts for endogenous labor supply as the redistribution from wages to profits induces a reduction in hours worked. The generalized model is also consistent with recent evidence on balanced growth with declining labor supply.

Keywords: Monopsony, Growth, Unbalanced Growth, Labor Share, Misallocation.

JEL codes: O40, O41. J23, J30, J42 :

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1 Introduction

The neoclassical growth model (NGM) assumes competitive labor markets (Cass, 1965; Koopmans, 1965; Acemoglu, 2009). Recent research on advanced economies shows that monopsony power in the labor market is a significant, widespread and sizeable phenomenon (Manning, 2021; Yeh et al., 2022). This paper asks a basic question. Is it harmless to ignore monopsonistic power in the neoclassical growth model? The paper argues that it is not, especially if a growth model needs to be consistent with the long-run dynamics of the labor share (Bergholt et al., 2022; Karabarbounis, 2024).

Classical monopsony needs two key ingredients: i) firm market power in setting wages and ii) an upward sloping positive relationship between the wage offered and the quantity of labor that the representative firm can hire (Robinson, 1969). To obtain an upward sloping relationship at the firm level the paper assumes that the representative agent can arbitrage the total quantity of labor offered across two sectors. Beyond the labor offered in a monopsonistic labor market, the agent has access to an alternative less efficient sector and technology in which she obtains marginal returns. This paper thus proposes and solves a minimalist NGM with monopsonistic power at the firm level.

In the model proposed, there are two neoclassical production functions with constant returns to scale in capital and labor that produce the same identical good. One technology is more efficient than the other one. Unsurprisingly, in a optimal growth problem the model collapses to a simple NGM that exploits only the superior technology. If the firm that has access to the superior technology enjoys also monopsonistic power in the hiring of labor, in the spirit of Robinson (1969), the equilibrium of the model turns out to be very different. In the decentralized monopsonistic setting, the household enjoys full marginal returns when she works with the inferior technology. The household allocation of labor between the two sectors is obtained by a "market hours arbitrage condition" that implies an upward positive relationship between the wage paid in the sector that use the superior technology and the quantity of labor hired. Such arbitrage condition is the key relationship that the representative monopsonistic firm exploits in her profit maximization problem. While the representative worker takes the firm problem as given, in equilibrium the profits of the monopsony are ex-post rebated to the consumers.

The decentralized equilibrium with monopsonistic power implies either "growth" or "level" effects in the determination of the labor share. If the exogenous TFP growth in the two sectors is identical (so that $g^s = g^i$), the model features a balanced growth path and a constant labor share, but the "level" of the labor share is always lower than the corresponding value in an efficient setting with no monopsony and no use of the inferior technology. If the superior sector features a larger productivity growth than the inferior sector (so that $g^s > g^i$), the equilibrium of the model takes the form of an asymptotic balanced growth path at rate g^s with full asymptotic absorption of labor in the more efficient sector. In such "unbalanced" equilibrium, the labor share asymptotically declines.

Since the model is fairly parsimonious, we can quantitatively assess the misallocative effects of monopsony. Our exercise shows that the welfare cost of monopsony can be large and easily reach output loss of 20 percent if the economy is on balanced growth. In the case of unbalanced growth there are is no asymptotic welfare loss, yet there is sizeable transitional output and consumption loss that in our simulations accounts for an average value of approximately 9 percent of output and more than 5 percent of consumption.

The model can also be generalized to account for balanced growth with endogenous labor supply and declining hours worked, in line with the recent contribution of Boppart and Krusell (2020). We show that the misallocation generated by monopsony is larger when the model accounts for endogenous labor supply, as the redistribution from wages to profits induces also a reduction in labor supply. In the extended model with endogenous choice of labor, the reduction in labor supply induced by monopsony is very persistent, and survive also when the productivity differential between the two sectors becomes infinitely large.

The paper is thus related to at least five strands of the macroeconomic and growth literature. First, the results appear coherent with the recent evidence of historical decline in the labor share (Karabarbounis and Neiman, 2014; Bergholt et al., 2022; Karabarbounis, 2024), and by papers that study wage stagnation. Second, the paper contributes to the literature that aim at incorporating monopsony into aggregate growth models (Deb et al., 2022; Barr and Udayan, 2008) and the general macroeconomic effcts of monopsony (Manning, 2021). Third, the model presented is also coherent with the growth literature on structural change and employment changes across sectors (Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). Specifically, the asymptotic equilibrium in case of unbalanced growth has various similarities with the model of Acemoglu and Guerrieri (2008). Fourth, the paper relates to the literature that discusses balanced growth with long run labor supply (Boppart and Krusell, 2020). Fifth, the paper contributes to the growing field of factor misallocation in the process of economic growth (Hsieh and Klenow, 2009)

The paper proceeds as follows. Section 2 reviews the relevant literature in more details. Section 3 presents our model and shows also how it compares to a simple optimal growth model with one sector. Section 4 derives the balanced and unbalanced equilibrium with monopsonistic power. Section 5 discusses the broader implications of the growth model for the dynamics of the labor share and the misallocative effects induced by monopsony. Section 6 presents the extended model with endogenous labor supply. Section 7 summarizes and concludes.

2 Literature Review

Macroeconomics of Declining Labor Share

The secular decline in the labor share has been first recognized in the seminal paper by Karabarbounis and Neiman (2014). Since the latter paper was published, the empirical literature on the measure of the labor share has been very vivid across sides of the Atlantic, and in most developing countries (Gilbert Cette et al., 2019; Brooks et al., 2021). At the empirical methodological level, there is strong debate about measurement issues linked to the role of housing, self employment and the role of Government. Atkenson (2020)- for example- argues that the estimates of the falling labor share are largely due to changes in measurement details by the Bureau of Labor Statistics. More recently Karabarbounis (2024) reviews the large work on the decline in the labor share and discusses the five possible drivers of the secular decline observed not only in the United States, but in most advanced economies. The possible explanations for the secular fall are different, and they relate to technology (Acemoglu and Rastrepo, 2022), cost of capital (Kaymak and Schott, 2023), market power by firms (Autor et al., 2020; Deb et al., 2022), changes in labor market institutions and firms' market power (Yeh et al., 2022) and globalization. While it is difficult to give weights to the possible concurrent institutions, this paper examines the relationship between balanced growth in consumption and GDP with a declining labor share and monopsonistic power by firms. In a companion paper, Garibaldi and Turri (2024) study the effects of mark-up by monopolistic firms on the labor share in a growth model, in the spirit of Autor et al. (2020) and Deb et al. (2022)

Long Run Hours Worked

Traditional growth theory used to take the long run stability of total labor as a key stylized fact. In other words, per capita leisure has been traditionally considered constant. Prescott (1986) argues that leisure shows no secular trend while real wage has grown steadily. In terms of preferences, the stability of leisure has been obtained by representative models in which the income and substitution effects of wage increase cancel each other out. Ramey and Francis (2009) provide new empirical evidence on long run data on work, home production and leisure and partly challenge the traditional result of Prescott (1986). While the results on total hours is almost confirmed, the evidence on hours of work provided by Ramey and Francis (2009) is fairly complex and relevant for the results and topic of this paper. Over the entire 20th century, the amount of leisure per capita remained roughly constant, while the amount of work of the representative agent has apparently declined.[1] More recently, Boppart and Krusell (2020) argue that the decline in average hours is robust and show that the income effect slightly dominate the substitution effect. Boppart and Krusell (2020) provide also a set of preferences that is coherent with balanced growth path and declining hours worked. In the paper we do rely to such set of preferences for studying the model with endogenous labor supply.

Growth and Structural Change

The model we present has two sectors that feature a labor reallocation across sectors driven by TFP growth differential. In this sense, our theory borrows some of the ideas

[1] Ramey and Francis (2009) argues that the decline in hours worked per person are mainly accounted for by an increase in schooling by the young workers. Conversely, prime age individuals between the ages of 25 and 54 are working the same number of hours now as in 1900, as a combination of a rise in female hours worked and reduction in male hours. Naturally, home production by female workers declined substantially while it increased by male workers.

from the structural change literature (Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008). The long run asymptotic balanced growth path that we obtain when productivity growth in the superior sector is larger than productivity growth in the inferior sector is similar to the long run equilibrium obtained by Acemoglu and Guerrieri (2008). Differently from the demand factors emphasized by Ngai and Pissarides (2007), the technologies in the two sectors produce the same final consumption good. Further, the key labor market relationship of our paper- the "market hours arbitrage condition" - crucially depends on the differential TFP productivity among the two sectors typical of the structural change literature.

Ngai and Pissarides (2008) study the long run effects of shifts from home production to market production as well as the secular trends in leisure in a model with structural change. With respect to the research in this paper, Ngai and Pissarides (2008) ignore the effects on labor share and do not deal with monopsonistic power in the labor market.

Bridgman (2016) estimates TFP differential between the home and the market. He shows that the level of TFP at home is approximately 70 percent of the level of TFP in the market, and further TFP at home slowdown significantly from the 70s. Home productivity grew at a rate similar to that of the market economy in the postwar period until the 1970s. Home labor productivity grew an average of 2.0 percent a year during the period 1948-1977, very similar to the 2.1 percent in market. There is a severe slowdown in home productivity in the late 1970s. Labor productivity was nearly flat, growing an average of only 0.02 percent from 1978 to 2010. In contrast, market labor productivity grew 1.6 percent annually. TFP growth at home followed a similar path to home productivity, and thus such productivity differential between market and home production is in line with one of the key mechanism outlined in the paper.

Monopsony in Growth and Aggregate Labor Markets

The theory of economic growth has paid little attention to the effect of labor market imperfections on growth, even though market failures and monopolistic power has played an important role in the theory of endogenous growth (Aghion and Howitt, 2009; Romer, 1990). There has been some work on the effect of growth on unemployment (Mortensen and Pissarides, 1998), and more recently on how to integrate constant unemployment in a balanced growth perspective with declining search friction (Menzio and Martellini, 2020). The research on the effects of monopsony in aggregate labor market is vast and widespread both theoretically and empirically, as documented- among others- by the the recent survey by Manning (2021). Much less attention has been devoted to the long run effect of monopsony in a long run growth perspective. One exception is the endogenous growth model proposed by Barr and Roy (2008). In their paper, the supply of labor is driven by spatial labor mobility costs between an informal sector and a urban production, and the low wage in equilibrium leads to low human capital accumulation and slow growth. The resulting endogenous growth is suboptimal. With respect to this paper, the research by Barr and Roy (2008) is more a development model in spirit and there is no effort to compare it to the standard neoclassical growth model, as we do in most of this research. In addition, the model by Barr and Roy (2008) does not link monoposny to the dynamics of the labor share.

Misallocation

Jones (2022) in his recent survey on the future of economic growth suggests that the literature on the misallocation of factors in production will be a key research avenue in the years to come. The insights of the effect of misallocation of inputs into empirical TFP growth are due in large part to the research of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The intuition is that misallocation at the micro level aggregate up into TFP differences. For example, Hsieh et al. (2022) argues that reduced misallocation of talent in the US due to a fall in labor market discrimination accounts for 0.3 of TFP growth in the last 20 years. The research in this paper on labor market imperfections suggests that the widespread presence of monospony in labor market leads to misallocation of capital across sectors, since less efficient capital is used into aggregate production. The theory of this paper is linked to the growing field of misallocation.

3 A Model of Monopsony and Growth

3.1 The Environment

The environment is coherent with the decentralized setting of the Neoclassical Growth Model (NGM) *à-la* Ramsey or Cass-Koopmans, to which we add two important features: first, in the economy there are both a superior and an inferior technology and - second - the representative firm that has access to the superior technology has monopsonistic power in the hiring of labor. The model is also coherent with recent evidence on balanced growth with long term trend in labor supply proposed by Boppart and Krusell (2020). The environment is an infinite-horizon economy in continuous time.

We being with the description of technology. In the economy there is a single good that is potentially produced with two different technologies in two different sectors. We call them technology s and technology i, where technology s refers to superior and technology i refers to inferior in a way that we describe below. The superior technology is indicated with the capital letter F, so that

$$Y_t^s = F(K_t^s, N_t^s, A_t^s)$$

where K_t^s and N_t^s are aggregate capital and labor in the superior sector, and A_t^s is exogenous labor augmenting TFP growth, while Y_t^s is the amount of good produced using technology s. F has standard neoclassical features, so that it has positive and diminishing marginal products in K and N and constant returns to scale in (K_t^s, N_t^s). Formally, this implies $F_K > 0$, $F_N > 0$ and $F_{KK} < 0, F_{NN} < 0$. We also assume that F satisfies standard Inada conditions. F features labor-augmenting technological progress in line with Uzawa's theorem for balanced growth, and this is equivalent to F being homogeneous of degree one in the pair (K_t^s, A_t^s) as well.[2] TFP in sector s grows at rate g^s, so that

$$A_t^s = A^s e^{g^s t} \ ; \quad g^s \geq 0 \ , \ A^s > 0 \ .$$

Capital allocated to production in the sector that uses the superior technology depreciates at an instantaneous rate of $\delta^s \in [0,1]$.

The inferior technology is indicated with superscript i, and produces Y_t^i according to the following neoclassical technology

$$Y_t^i = G(K_t^i, N_t^i, A_t^i)$$

where K_t^i and N_t^i are aggregate capital and labor used with the inferior technology, and A_t^i is exogenous labor augmenting TFP growth. Similar to F, also the technology G has standard neoclassical features, so that it has positive and diminishing marginal products in K^i and N^i and constant returns to scale. Formally, this implies $G_K > 0, G_N > 0$ and $G_{KK} < 0, G_{NN} < 0$, as well as the Inada conditions. G is homogeneous of degree one in (K_t^i, A_t^i) as well. Capital used by the inferior sector depreciates at rate $\delta^i \in [0,1]$. The exogenous labor augmenting TFP growth in sector i is such that

$$A_t^i = A^i e^{g^i t} \ ; \quad g^i \geq 0 \ , \ A^i > 0 \ .$$

We assume that sector s is superior in the following sense.

Assumption 1. *Initial values of technology A^s and A^i and rates of growth g^s and g^i are such*

$$A^s \geq \kappa A^i \quad \text{and} \quad g^s \geq g^i \ .$$

where $\kappa > 0$ depends on the parameters of the model and it is specified in Appendix.

The economy is populated by a measure one of identical, infinitely-living (or perfectly altruistic) agents whose preferences are coherent with the NGM and potentially also with long run labor supply, and we abstract from population growth. The individual is endowed with 1 unit of time and we shall indicated with $h_t \leq 1$ the total hours supplied by the worker, while we denote consumption by C_t. The instantaneous utility function reads

$$U(C_t, h_t) = u(C_t) - v(h_t).$$

$U(C_t, h_t)$ is additive separable in consumption and total hours and features standard assumption. Thus $u(c)$ is defined on $\Re_+$, it is strictly increasing, concave and twice differentiable, with derivatives $u' > 0$ and $u'' < 0$ inside of the domain. $v(h_t)$ represents the dis-utility of labor and is defined on $\Re_+$, it is strictly increasing, concave and twice differentiable, with derivatives $v' > 0$ and $v'' > 0$ in its domain.

[2] Even though thanks to Uzawa's theorem we could write our production functions as functions of capital and *efficient units of labor* (in this case, $A_t^s N_t^s$) only, we prefer the more abstract notation provided in the text since the extensive use of second derivatives in the paper would lead to some ambiguities in the notation.

In light of the recent contribution of Boppart and Krusell (2020) on long term trend in hours worked, we will consider the instantaneous utility used by the canonical NGM that belongs to the CRRA (Constant Relative Risk Aversion) class and was proposed originally by MaCurdy (1981) .

Assumption 2. *The utility function is CRRA in consumption with coefficient* $\gamma > 0$ *and features a constant Frisch elasticity* θ*, so that*

$$u(C_t) - v(h_t) = \begin{cases} \frac{C_t^{1-\gamma}-1}{1-\gamma} - \phi\frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} & \text{if } \gamma \neq 1, \\ \\ \log C_t - \phi\frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} & \text{if } \gamma = 1. \end{cases} \tag{1}$$

The parameter θ is thus the percentage change in hours when the wage is changed by 1%, keeping the marginal utility of consumption constant. The parameter γ is the inverse of the elasticity of the marginal utility of consumption. To simplify the analysis, we first present the model without cost of labor (thus $\phi = 0$) and we postpone the full specification of the preferences to Section 6.

The present discounted utility value of a stream $\{C_t, h_t\}_{t\geq 0}$ to a household is

$$\int_0^\infty e^{-\rho t}\left[u(C_t) - v(h_t)\right] dt$$

where $\rho > 0$ is the subjective discounting.

Households own all factors of production. Given their time endowment, they supply labor in quantities n_t^s and n_t^i to the firms in both sectors so that

$$\begin{aligned} n_t^i + n_t^s &= h_t, \\ h_t &\leq 1. \end{aligned}$$

Until Section 6, where we develop the model with endogenous hours, total hours worked equal 1. In sector s factors are organized with formal markets, so that agents save in safe asset $\mathbb{A}_t$ which gives them a one to one claim to capital used by the superior sector, while the hiring of labor is monopsonistic. In sector i households have access to the marginal values of production, possibly without relying on the institutional set up of a formal market. The generality of this modelling choice allows the inferior sector to be interpreted as either home production or a second formal sector throughout the paper.

3.2 Firm Intratemporal Problem

The monopsonist produces with the superior technology F and faces a positive relationship between the wage paid and the supply of labor, which we denote by $W_t(N_t^s)$. The relationship between the wage paid and the quantity of labor hired comes from the monopsonist access to three key pieces of information. First, the firm knows that she is the only buyer of labor N_t^s for using it with the superior technology. Second, the firm knows that individuals have access to an alternative technology in quantity $N_t^i = \overline{H}_t - N_t^s$, where N_t^s is the aggregate amount of labor she hires for use with technology and $\overline{H}_t$ is taken as given by the firm. Third, the firm knows that the marginal product of the individuals in their alternative use of time is exactly the marginal product $G_N()$ of the inferior technology. This implies that the wage schedule faced by the monopsonistic firm reads

$$W_t(N_t^s) = G_N(K_t^i, \overline{H}_t - N_t^s, A_t^i) \ , \tag{4}$$

where K_t^i is taken as given by the monopsonist.[3] These three pieces of information and the wage schedule (4) imply that the total labor costs to the monopsonist are

$$W_t(N_t^s)N_t^s = G_N(K_t^i, \overline{H}_t - N_t^s, A_t^i)N_t^s \ .$$

The representative firm problem in extensive form is intratemporal and reads

$$\max_{K^s, N^s} \Pi(N^s, K; G(\cdot), R_t, A_t^s, A_t^i, \overline{H}_t, K_t^i) = F(K^s, N^s, A_t^s) - G_N(K_t^i, \overline{H}_t - N^s, A_t^i)N^s - R_t K^s \ . \tag{5}$$

The choice of quantity of labor[4] is a *classical* monopsonistic problem in the spirit of Robinson (1969), while the choice of capital is a standard capital demand in a competitive markets. The first order conditions of the firm are

$$F_K(K_t^s, N_t^s, A_t^s) = R_t \ , \tag{6}$$
$$F_N(K_t^s, N_t^s, A_t^s) = G_N(K_t^i, \overline{H}_t - N_t^s, A_t^i) - G_{NN}(K_t^i, \overline{H}_t - N_t^s, A_t^i)N_t^s \tag{7}$$

While equation (6) is completely standard, the condition in equation (7) is novel in the NGM, yet it is standard in introductory textbooks of microeconomics. Figure 1 describes the behavior of the monopsonistic firm in a simple static diagram. The dotted downward sloping line is the marginal product in using the superior technology, or the left hand side of equation (7). The upward dotted sloping line is the marginal cost of labor to the firm, or the right hand side of equation (7). The firm hires at the point where the marginal cost of labor is equal to the marginal revenue in point E_1. Yet, the wage obtained by the worker is determined along the continuous line in Figure 1, and is indicated by the point E_2 in the Figure. The x-value at E_1 and E_2 is thus the amount of labor employed by the sector with superior technology. The y-value at E_2 is the equilibrium wage, while the y-value at E_1 is the marginal productivity of labor employed in the monopsonistic sector. In general equilibrium, the remaining amount of labor will be supplied and used to firms that use the inferior technology.

Firm optimization has important implications in terms of profits. Since the production function has a standard CRS technology and is homogeneous of degree zero in

[3]The wage schedule - or the fundamental arbitrage condition of the individual- is derived endogenously in the individual maximization problem solved in Section 3.3.

[4]Notice that differently from standard NGM in which the size of the firm is undetermined, the size of the monopsonist, hence of sector s, is. The inferior sector has instead undetermined size.

(K, N), Euler theorem applies and combined with the first order condition of the firm implies

$$F(K_t^s, N_t^s, A_t^s) = R_t K_t^s + \left[G_N(K_t^i, \overline{H}_t - N_t^s, A_t^i) - G_{NN}(K_t^i, \overline{H}_t - N_t^s, A_t^i)N_t^s\right] N_t^s$$

so that optimal profits of the superior sector are

$$\Pi_t = -G_{NN}(K_t^i, \overline{H}_t - N_t^s, A_t^i)(N_t^s)^2 > 0 \ .$$

Since individuals own all factor productions, profits are fully distributed period by period in equal amount to all agents.

(Insert Figure 1 here)

3.3 Household Problem and Budget Constraint

As anticipated, we solve the model first in assuming that agents have a fixed labor supply of measure one, and utility depends only on consumption. Agents can save in a safe asset that yields rights to capital in sector s, they own all firms in the economy and are thus entitled to a rebate of profits from the monopsonistic firm. As anticipated, in sector i agents have access to the return to capital, so we directly include it in the budget constraint. Agents can obtain a wage in sector s that we label as w_t^s and have direct access to the marginal product of labor in the inferior sector that is indicated by ω_t^i. These amounts, together with rate of returns on capital, are time varying and taken as given in the maximization problem. We denote the household supply of labor in sector s at time t by n_t^s. Since households are endowed with one unit of time, and supply all their labor inelastically, this implies $n_t^i = 1 - n_t^s$. The total asset holdings of the representative household is the sum of a safe asset $\mathbb{A}_t$, that yields the instantaneous rate of return r_t, plus the capital in sector i is K_t^i that yields the marginal product ξ_t^i.

The household solves

$$\begin{aligned} \max_{[C_t, \mathbb{A}_t, K_t^i, n_t^s, n_t^i]_{t\geq 0}} \quad & \int_0^\infty e^{-\rho t} u(C_t) \mathrm{d}t \\ s.t. \quad & C_t + \dot{\mathbb{A}}_t + \dot{K}_t^i = w_t^s n_t^s + r_t \mathbb{A}_t + \Pi_t + \omega_t^i n_t^i + \xi_t^i K_t^i \\ & n_t^s + n_t^i = 1 \ , \ n_t^s, n_t^i \geq 0 \ , \ \mathbb{A}_t \geq 0 \ , \ K_t^i \geq 0, \end{aligned} \tag{8}$$

where Π_t denotes the profit rebate from the sector s where the monopsonistic firm operates. The budget constraint in (8) deserves some comments. Factors allocated to the sector s yield wage income $w_t^s n_t$ and capital income $r_t A_t$. Indirectly, renting labor and capital to the formal sector yields also profits Π_t. Conversely, factors allocated to the inferior sector yield the marginal return to labor (ω_t^i) and capital (ξ_t^i) of total production $G(K_t^i, (\overline{h} - n_t^s), A_t^i)$ that can be used to finance consumption and capital accumulation. The marginal returns are taken as given in the right hand side of the budget constraint and are indicated with ω_t^i and ξ_t^i. Note also that the budget constraint implies that the safe assets and the capital in sector i can be converted one to one in consumption. In order to have finite utility at equilibrium, we need to impose the following parametric restriction.

Assumption 3. *Parameters are such that* $-\rho + (1-\gamma)g^s < 0$.

In order to write the Hamiltonian, define $\dot{K}^i_t = q^i_t$, so that we have the states $x_t = (\mathbb{A}_t, K^i_t)'$ and the controls $z_t = (C_t, n^s_t, q^i_t)'$. The current value Hamiltonian reads

$$\hat{H}(\mathbb{A}_t, K^i_t, C_t, n^s_t, q^i_t) = u(C_t) + \mu^{\mathbb{A}}_t(-C_t - q^i_t + w^s_t n^s_t + r_t\mathbb{A}_t + \Pi_t + \omega^i_t(1-n^s_t) + \xi^i_t K^i_t) + \mu^{K^i}_t q^i_t$$

So that for an interior solution with $n^s_t \in (0,1)$[5] we obtain the system

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma} \tag{9}$$

$$w^s_t = \omega^i_t \tag{10}$$

$$r_t = \xi^i_t \tag{11}$$

$$C_t + \dot{\mathbb{A}}_t + \dot{K}^i_t = w^1_t n_t + r_t\mathbb{A}_t + \Pi_t + \omega^i_t(1-n^s_t) + \xi^i_t K^i_t$$

$$\lim_{t\to\infty} e^{-\rho t}u'(C_t)\mathbb{A}_t = \lim_{t\to\infty} e^{-\rho t}u'(C_t)K^i_t = 0$$

These first order conditions deliver some key features and properties of the monopsonistic market. Equation (9) is a standard Euler condition for consumption, as in the NGM literature. The second condition is a key condition for the existence of monopsony in equilibrium and it implies endogenously the function $w^s_t(n^s_t)$ assumed in the firm problem of equation (7). Condition (10) is thus a *fundamental arbitrage condition in the labor market*. It says that along an equilibrium path the wage obtained in the formal labor market and paid by the monopsonist must coincide with the marginal productivity of labor in the second sector that the consumer takes as given. Equation (11) is a similar condition for capital and can be labelled as *fundamental arbitrage condition in the capital market*. The last two conditions are just the budget constraint and the transversality conditions.

3.4 Characterization of Equilibrium

As we anticipated, the marginal returns to sector i that the individual takes as given must coincide with returns to using the inferior technology i so that at equilibrium

$$\omega^i_t = G_N(K^i_t, \overline{H}_t - N^s_t, A^i_t) \tag{12}$$

$$\xi^i_t = G_K(K^i_t, \overline{H}_t - N^s_t, A^i_t) - \delta^i \tag{13}$$

Equation (12) is the right hand side of the fundamental arbitrage condition and produces the upward sloping relationship that the representative monopsonistic firm takes as given in her maximization. In equilibrium, equation (12) plays the same role of the upward sloping labor supply in traditional monopsonistic models *à-la* Manning (2021). Equation (13) implies that the marginal rate of return to capital in the inferior sector must be the same as the rate return of the investment asset.

[5] This is true at equilibrium under Assumption 1.

In a market clearing equilibrium the total claims held by the individual must be equal to the amount of formal capital. In addition, aggregate employment by the monopsonistic firm must be equal to household labor allocated to the formal sector. Further, since household assets are the same as the capital stock and capital used in the superior sector depreciates at rate δ^s, the market rate or return must be equal to the cost of capital net of depreciation. Finally, the wage function of the monopsonistic firm must be coherent with the wage arbitrage condition of the household. This implies that at equilibrium of the asset and labor markets the following conditions to prices

$$r_t = R_t - \delta^s$$
$$w_t^s(\cdot) = G_N(K_t^i, \overline{H}_t - (\cdot), A_t^i)$$

as well as quantities

$$\mathbb{A}_t = K_t^s$$
$$n_t^s = N_t^s$$
$$1 = \overline{H}_t$$

apply.

3.4.1 Dynamic Monopsonistic Equilibrium

We are now in a position to define a dynamic monopsonistic equilibrium. We can also derive the the equilibrium conditions defined above in a compact form.

Definition 1. *Given paths for productivity in the two sectors* $\{A_t^s, A_t^i\}_{t\geq 0}$ *a dynamic monopsonistic equilibrium (with no disutility from labor) is a set of paths for quantities* $\{C_t, K_t^s, K_t^i, n_t^s, \}_{t\geq 0}$*, factor prices* $\{w_t^s(\cdot), R_t\}_{t\geq 0}$*, profits* $\{\Pi_t\}_{t\geq 0}$ *and marginal returns* $\{\omega_t^i, \xi_t^i\}_{t\geq 0}$ *such that for given initial capital stocks in the two sectors* K_0^s *and* K_0^i*,*

1. $\{C_t, K_t^s, K_t^i, n_t^s\}_{t\geq 0}$ *solve the consumer problem* (8) *given* $\{w_t^s, R_t - \delta^s, \xi_t^i, \omega_t^i\}_{t\geq 0}$ *and* $\{\Pi_t\}_{t\geq 0}$*;*

2. $\{K_t^s, n_t^s\}_{t\geq 0}$ *solve the monopsonistic firm problem* (5) *given* $\{R_t, A_t^s, A_t^i, \overline{H}_t = 1, K_t^i\}_{t\geq 0}$ *and determine* $\{\Pi_t\}_{t\geq 0}$*,*

3. $\{\omega_t^i, \xi_t^i\}$ *are the marginal productivities of labor and capital in the inferior sector, i.e. Equations* (12) *and* (13) *hold.*

3.4.2 The Fundamental System

Combining firm and household optimality conditions, the equilibrium is described by the following system of differential equations that - once solved - fully characterizes the dynamic equilibrium in terms of $\{K_t^s, n_t^s, K_t^i, C_t\}_{t\geq 0}$ given the paths of A_t^s and A_t^i.

$$\frac{\dot{C}_t}{C_t} = \frac{F_K(K_t^s, n_t^s, A^s) - \delta^s - \rho}{\gamma} \tag{14}$$

$$F_N(K_t^s, n_t^s, A_t^s) = G_N(K_t^i, 1-n_t^s, A_t^i) - G_{NN}(K_t^i, 1-n_t^s, A_t^i)n_t^s$$

$$F_K(K_t^s, n_t^s, A_t^s) - \delta^s = G_K(K_t^i, 1-n_t^s, A_t^s) - \delta^i \tag{15}$$

$$C_t + \dot{K}_t^s + \dot{K}_t^i = F(K_t^s, n_t^s, A_t^s) - \delta^s K_t^s + G(K_t^i, 1-n_t^s, A_t^i) - \delta^i K_t^i$$

$$\lim_{t\to\infty} e^{-\rho t}u'(C_t)\mathbb{A}_t = \lim_{t\to\infty} e^{-\rho t}u'(C_t)K_t^i = 0$$

where we use Euler theorem in sectors s and i to introduce F and G in the budget constraint.

3.4.3 Equilibrium in Efficiency Units

In line with traditional NGM, it is useful to characterize the equilibrium in terms of variables in efficiency units. The subtle issue is that in the model there two different TFP factors and that labor in the superior and inferior sectors (thus n_t^s and $n_t^i = 1 - n_t^s$ respectively) are equilibrium quantities. The capital labor ratio in efficiency units are defined as

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s}; \quad \text{and} \quad x_t^i = \frac{K_t^i}{A_t^i(1-n_t^s)} \quad \text{and} \quad c_t = \frac{\tilde{C}_t}{A_t^s n_t^s},$$

where x_t^s and x_t^i are capital labor ratio in efficiency unit in the superior and inferior sectors and c_t is consumption. We also define the instantaneous rates of growth $g_t^{n^s} = \frac{\dot{n}_t^s}{n_t^s}$ and $g_t^{n^i} = \frac{\dot{n}_t^i}{n_t^i} = -\frac{\dot{n}_t^s}{1-n_t^s}$. The equilibrium system in efficiency units is

$$\frac{\dot{c}_t}{c_t} = \frac{F_K(x_t^s, 1, 1) - \delta - \rho}{\gamma} - g^s - g_t^{n^s} \tag{16}$$

$$A_t^s F_N(x_t^s, 1, 1) = A_t^i G_N(x_t^i, 1, 1) - A_t^i G_{NN}(x_t^i, 1, 1)\frac{n_t^s}{1-n_t^s} \tag{17}$$

$$F_K(x_t^s, 1, 1) - \delta^s = G_K(x_t^i, 1, 1) - \delta^i \tag{18}$$

$$c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i}{A_t^s}\frac{1-n_t^s}{n_t^s} = F(x_t^s, 1, 1) - (\delta^s + g^s + g_t^{n^s})x_t^s + \tag{19}$$

$$+ \left[G(x_t^i, 1, 1) - (\delta^i + g^i + g_t^{n^i})x_t^i\right] \frac{A_t^i}{A_t^s}\frac{1-n_t^s}{n_t^s},$$

were we made extensive use of the homogeneity properties of F and G. As in traditional NGT it is useful to introduce the concept of balanced growth as a pattern of growth consistent with constant rate of output growth and constant capital-output ratio. Further, in balanced growth the rental rate of return R_t and the interest r_t are also constant, so as to imply a constant consumption growth. As it turns out, existence and properties of paths of balanced growth depends on whether the TFP of the two sectors grow at the same rate. In the paper we thus speak of *balanced growth* when $g^s = g^i$ and of *unbalanced growth* when $g^s > g^i$.

3.5 Efficient Optimal Growth

Before further characterizing the details of the dynamic monopsonistic equilibrium in case of balanced and unbalanced growth, we solve for the efficient optimal growth problem. In a representative framework the optimal growth problem is obtained by maximizing the utility of the representative agent with respect to consumption and the two capital subject to an aggregate budget constraint. Since one technology is superior to the other, it is obvious that the optimal growth problem involves using only the superior technology, apart from the knife-edge (and trivial case) in which the two technologies can be used indifferently.

Lemma 1. *Under Assumption 1, sector s operates in the optimal growth problem.*

This justifies our calling sector s the superior sector. Notice that the optimal growth model correspond to the balanced path consumption in a traditional NGM *à-la* Cass (1965) and Koopmans (1965), and the model boils down to Ramsey when only one sector operates with inelastic labor supply, exactly as in specialized textbooks (Acemoglu, 2009). The optimal growth model is thus much simpler than the monopsonistic equilibrium. Since only the superior technology operates with employment equal to one, the system of equations governing the model is

$$\frac{\dot{c}_t}{c_t} = \frac{F_K(x_t^e, 1, 1) - \delta^s - \rho}{\gamma} - g^s$$
$$c_t + \dot{x}_t^e = F(x_t^e, 1, 1) - (\delta^s + g^s)x_t^e$$

where x_t^e is the efficient capital allocated to the superior sector, and $n_t^e = 1$ is employment allocated to the superior sector, since for now there is no disutility from labor. In an Efficient Balanced Growth Path (EBGP) $x^e = k^e = \frac{K_t^e}{A_t^s}$ is pinned down by the Euler equation when $\frac{\dot{c}_t}{c_t} = 0$ and consumption for the individual reads

$$C_t^e = A_t^s[F(x^e, 1, 1) - \delta^s x^e] = A_t^s c^e = A^s c^e e^{g^s t} \ ,$$

where $c^e = F(x^e, 1.1) - \delta^s x^e$ is consumption in efficiency units along the EBGP. Since the welfare theorems apply to the NGM, the efficient decentralized equilibrium can be supported by a competitive equilibrium.

Finally, one can also the derive the constant value of the labor share along an EBGP in which the superior technology is operating as

$$\alpha_L^e = \frac{F_N(x^e, 1, 1)}{F(x^e, 1, 1)}$$

As usual, in the case in which F is Cobb-Douglas with capital intensity α, the labor share reads $\alpha_L^e = 1 - \alpha$.

4 Equilibrium Dynamics

We turn to studying the dynamic monopsonistic equilibrium and use the optimal outcome as a benchmark. In monopsonistic equilibrium also the inferior technology is exploited. In light of the results from the optimal growth problem there will be unambiguous welfare and misallocative effects. We look for a balanced growth path solution to the model.

Definition 2. *A monopsonistic balanced growth path (MBGP) is a solution of the model with monopsony along which the rate of growth of consumption is constant.*

As it turns out, a BGP exists in our model if and only if there is balanced growth. If there is unbalanced growth. there is no exact balanced growth path, but only an asymptotic balanced growth path.

Definition 3. *A monopsonistic asymptotic balanced growth path, or monopsonistic unbalanced growth path (MUGP) is a solution of the model along which the rate of growth of consumption is asymptotically constant.*

4.1 Balanced Growth

In balanced growth case both technologies grow at the same rate so that $g^s = g^i = g$. A balanced growth solution exists, as the following theorem formally establishes. The theorem highlights the interaction of the two sectors with the mechanics of balanced growth paths, and also how and why balanced growth is required for the existence of MBGP.

Theorem 1. *Under balanced growth, there exists a unique MBGP. Along this solution consumption grows at rate g,*

$$\frac{\dot{C}_t}{C_t} = g \ .$$

Both sectors operate in equilibrium, x_t^s and x_t^i are constant and pinned down by

$$F_K(x^s, 1, 1) = \rho + \delta^s + \gamma g \ , \quad G_K(x^i, 1, 1) = \rho + \delta^i + \gamma g.$$

Labor in the two sectors is constant, and in particular

$$n^s = \frac{A^s F_N(x^s, 1, 1) - A^i G_N(x^i, 1, 1)}{A^s F_N(x^s, 1, 1) - A^i G_N(x^i, 1, 1) - A^i G_{NN}(x^i, 1, 1)} \in (0, 1) \ , \quad n^i = 1 - n^s \ .$$

This in turn implies that capital in the two sectors grows at rate g as well,

$$\frac{\dot{K}_t^s}{K_t^s} = \frac{\dot{K}_t^i}{K_t^i} = g \ .$$

Finally, the fraction produced by each sector is constant, as is the risk free rate of return, wages grow at rate g, and there are positive profits that grow at rate g as well.

Proof. First of all, notice that under balanced growth the fundamental system describing the evolution of the model written in intensive form is time stationary, as A_t^s and A_t^i only appear in ratios. The Euler equation (14) implies that if the rate of growth of consumption is constant, then x_t^s must be constant. Then equation (18) implies that x_t^i is constant as well, which in turn implies through equation (17) that n_t^s is constant and positive under Assumption 1. This implies $g_t^{n^s} = g_t^{n^i} = 0$. But then equation (19) pins down the rate of growth of consumption to g. Since x_t^s and x_t^i are constant and labor in the two sectors is constant as well, it must be that capital grows at rate g as well. Finally, the share of output produced by each sector depends on x_t^s, x_t^i, $\frac{n_t^s}{n_t^i}$ and $\frac{A_t^s}{A_t^i}$ only, which are all constant along a MBGP; furthermore the rate of return depends only on x_t^s, while wages and profits, once x_t^s and x_t^i are constant, grow with A_t^i and A_t^s. □

Because of monopsonistic power in the superior sector, at equilibrium both technologies are up and running. The arbitrage condition in the labor market is binding and at the marginal amount of labor n^s the individual is indifferent between working in the market with the superior technology or working with the inferior technology. The monopsonistic firm - in turn - makes positive profits driven by the wedge between the marginal product of labor and wage paid to the worker. Even though both technologies are operated at the MBGP, the more superior (in terms of $A^s > A^i$) sector s is, the bigger the share of labor it employs, as the following corollary states.

Corollary 1. *The following asymptotic result holds.*

$$\lim_{\frac{A^s}{A^i} \to \infty} n^s = 1 \ .$$

The study of convergence of equilibrium paths toward the balanced growth path is traditional in growth theory. The economy starts with a capital allocation $K_0^s > 0$ and $K_0^i > 0$. Yet, capital is perfectly mobile across sectors and the marginal condition along the equilibrium path is coherent with an instantaneous reallocation of factors across sectors. The question is whether such initial allocation converges toward the balanced growth path characterized in Theorem 1. We are able to establish first of all that this is the case if the technological ratio is sufficiently high. Then, we derive a sufficient condition under which the MBGP is saddle path stable under Assumption 1 if the two sectors operate with Cobb-Douglas production with coefficient α on capital in the superior sector and ϵ in the inferior sector, with obviously $\alpha, \epsilon \in (0,1)$

Theorem 2. *If the technological ratio $\frac{A^s}{A^i}$ is sufficiently high, then the MBGP is locally saddle path stable. Under the assumption that*

$$F(K_t^s, N_t^s, A_t^s) = (K_t^s)^\alpha (A_t^s N_t^s)^{1-\alpha}$$

and

$$G(K_t^i, N_t^i, A_t^i) = (K_t^i)^{1-\epsilon} (A_t^i N_t^i)^\epsilon \ ,$$

if

$$\alpha\epsilon(\delta^i - \delta^s) - (1 - \alpha - \epsilon)(\rho + \delta^s + \gamma g)$$

then the MBGP is locally saddle path stable.

The proof is in appendix.

4.2 Unbalanced Growth

We now study a situation of unbalanced growth, in which productivity in the superior sector grows permanently faster than productivity in the inferior sector. In this case the equilibrium allocation is called asymmetric balanced growth, in line with the structural change model of Acemoglu and Guerrieri (2008). In asymptotic balanced growth there is a permanent transition that converges to a well defined and fully characterized equilibrium.

Theorem 3. *Under unbalanced growth, there exists a unique MUBP. Along this solution, consumption asymptotically grows at rate g^s,*

$$\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = g^s \ .$$

Both sectors operate at all times, and the limiting values $\lim_{t\to\infty} x_t^s = x^s$ and $\lim_{t\to\infty} x_t^i = x^i$ are defined by

$$F_K(x^s, 1, 1) = \rho + \delta^s + \gamma g^s \ , \quad G_K(x^i, 1, 1) = \rho + \delta^i + \gamma g^s \ .$$

Labor in the superior sector converges to one, and labor in the inferior sector converges to zero, where

$$\lim_{t\to\infty} \frac{\dot{n}_t^i}{n_t^i} = g^i - g^s \ .$$

This in turn implies that

$$\lim_{t\to\infty} \frac{\dot{K}_t^s}{K_t^s} = g^s \ , \quad \lim_{t\to\infty} \frac{\dot{K}_t^s}{K_t^s} = 2g^i - g^s \ .$$

Finally, the share of total production by sector s converges to one, while the risk free rate is asymptotically constant, wages asymptotically grow at rate g^i, while profits asymptotically grow at rate g^s.

The proof is in appendix.

Notice that the rate of growth of capital in the inferior sector, $2g^i - g^s$ can be either positive or negative depending on model parameters. In the former case, even though labour in the inferior sector decreases to zero, it is optimal to accumulate capital as the unbalance between the two sectors is not too big. In the latter case, in the end all capital is allocated to the superior sector. However this is only a limit result, and both capital and labor in the inferior sector remain strictly positive at all times.

Despite a structural productivity gap between the two sectors, the presence of monopsonistic power in the superior sector implies that is optimally to keep operating the inferior sector along a permanent transition, in which only at the limit the superior sector produces all output. The unbalanced equilibrium has also important implications for wage dynamics, with a permanent difference between the growth of wages and productivity in the superior sector. This - in turn - has dramatic implications for the distribution of income between factors of production. There is a permanent fall in the labor share that tends to zero.

We also study the process of convergence towards the asymptotic path. We establish that there is saddle path stability for given initial conditions K_0^s and K_0^i regardless of the shape of the production functions.

Theorem 4. *The unbalanced growth system is locally saddle path stable.*

The proof in in Appendix.

5 The Consequences of Monopsony on Growth

This section focuses on the the relevance of monopsony for economic growth. We highlight how the macroeconomic predictions of our (asymptotic) balanced growth solutions differ from those of the standard NGM. We first highlight the misallocative effects of monoposny, linked to the use of the inferior technology in equilibrium. We then show that monopsony has an impact on the labor share consistent with recent empirical findings (Karabarbounis, 2024). In section 6 we highlight a further misallocative effects of monopsony when the model account for endogenous labor supply.

5.1 Welfare Loss and Misallocative Effects

The different structure of the economy between the optimal path and the monopolistic equilibria (namely, that the inferior sector operates only in the monopsonistic case) suggests the existence of potentially sizeable allocative effect in general equilibrium, and most likely a welfare loss. Indeed, standard economic theory predicts that in a single factor market, monopsony involves a static welfare loss in the spirit of Robinson (1969). Our model extends those prediction to general equilibrium with a representative agent. Figure 2 provides a graphical visualization of the size of misallocation in a general equilibrium setting. The figure plots the marginal product of the superior and inferior sectors (respectively $F_N(.)$ and $G_N(.)$ with respect to total employment in the economy. Since the marginal product F_N is larger than G_N for any employment level, the Figure shows that a measure of the efficient allocation is given by the integral below the marginal product F_N over the entire labor force, since only one sector is exploited in equilibrium. In a monopsonistic equilibrium the size of the superior sector is $n_s < 1$, and thus the general equilibrium involves also the production obtained with the inferior technology given by the area below the inferior productivity G_N. As a result, the Figure naturally shows that the misallocation in the trapezoid plotted with horizontal and oblique lines.

The first effect we derive analytically concerns the level of consumption in the balanced equilibrium, as this simple proposition shows.

Proposition 1. *Given an initial stock of capital, the level of consumption C_t^m in monopsonistic balanced equilibrium (MBGP) is lower than the efficient level C_t^e at any t, while their ratio is constant. When $\frac{A^s}{A^i} \to \infty$, the level of consumption C_t^m tends to the efficient level C_t^e. A monopsonistic unbalanced equilibrium (MUGP) asymptotically features no consumption loss, as $\frac{C_t^m}{C_t^e} \to 1$.*

The proof is in Appendix. While welfare depends only on consumption when hours worked are fixed, we also study analytically the effect of monopsony on the level of GDP. The appendix shows that -in balanced growth- the ratio of production with monopsony to production in optimal growth is constant and that when the fraction of the initial productivity tends to infinity, the fraction of the two GDP tends to one, so that $\frac{A^s}{A^i} \to \infty$ implies that $\frac{Y_t^m}{Y_t^e} = n^s + \frac{A^i}{A^s}\frac{n^i}{n^s}\frac{G(x^i,1,1)}{F(x^s,1,1)} \to 1$. Yet, for low values of the initial productivity ratio, the fraction of GDP can also be greater than one, because of the curvature of marginal productivities of capital. Yet, this "production surplus" is depleted in depreciation, leading to the presence of consumption loss as above. In unbalanced growth, again along any path $\frac{Y_t^m}{Y_t^e} \to 1$.

While the welfare loss may depends on the actual shape of the utility function, we have a proposition for the case of CES with parameter γ.

Proposition 2. *Under Assumption 1 and CES utility with parameter γ there is a welfare loss for any finite ratio $\frac{A^s}{A^i}$, so that $\mathcal{U}^m < \mathcal{U}^e$. However, the welfare loss disappears as sector s increases its superiority, so that $\lim_{\frac{A^s}{A^i}\to\infty} \mathcal{U}^m = \mathcal{U}^e$.*

This follows immediately from utility depending on consumption only.

While the two propositions on consumption and welfare provide analytical results, the question of their relevance is a quantitative one. Since the model is parsimonious, it is possible to provide some basic numerical simulations, using the parameters described in Table 1. The results of the simulation are reported in Figure 3. The first and the third panel plot the balanced growth loss of GDP and consumption with respect to the initial productivity ratio A^s/A^i. While the results of Proposition 1 suggests that the consumption loss fall with respect to the initial fraction $\frac{A^s}{A^i}$, the quantitative effect is still large. In a balanced growth path consumption is approximately 10 percent lower than the corresponding value in a Ramsey economy. The loss in GDP is even larger (first top panel). The top right panel shows also that the loss depends on the employment share in the superior sector in balanced growth, and the loss is larger the smaller the share of workers in the monopsonistic sector. The right picture in the bottom panel of Figure 3, confirms that utility is indeed lower, as suggested by Proposition 2.

While propositions 1 and 2 provide the analytical results with respect to balanced growth case, it is not possible to derive a closed form expression for welfare with unbalanced growth. We can instead simulate the dynamic path of a monopsonistic equilibrium versus an efficient Ramsey. Some effects are worth mentioning, especially the dynamics of n_t^s toward 1 and the absence of the inferior technology. The simulation use the

Parameter/Function	Symbol	Fixed Labor Supply		End. Labor Supply	
		Bal.	Unb.	Bal.	Unb.
		Technology			
Sup. Technology F	$(K^s)^\alpha(A^sL^s)^{1-\alpha}$				
Capital share	α	0.3	0.3	0.3	0.3
Initial Tech Level	A^s	2	2	2	2
TFP growth	g^s	0.02	0.02	0.02	0.02
Depreciation Rate	δ^s	0.1	0.1	0.1	0.1
Inf. Technology G	$(K^i)^{1-\epsilon}(A^iL^i)^\epsilon$				
Capital Share	$1-\epsilon$	0.2	0.2	0.2	0.2
Initial Tech Level	A^i	0.6	0.6	0.6	0.6
TFP growth	g^i	0.02	.005	0.2	0.005
Depreciation Rate	δ^i	0.1	0.1	0.1	0.1
		Preferences			
Discount Rate	ρ	0.02	0.02	0.02	0.02
CES Parameter	γ	2	2	2	2
Cost of Labor	ψ	0	0	1.5	1.5
Frisch Elasticity	θ	/	/	2	2

Table 1: Parameter Values Used in Simulations

shooting algorithm proposed by Sargent and Stachursky (2024) applied to our model as indicated in Appendix. Figure 6 visualizes the distortion in consumption and output in unbalanced growth and compares it to the balanced growth case. In the case of unbalanced growth, the average fall in consumption is around 5 percent while the average fall in GDP is around 9 percent.

(Insert Figure 2 here)

(Insert Figure 3 here)

(Insert Figure 4 here)

(Insert Figure 5 here)

(Insert Figure 6 here)

5.2 Growth and Level Effects on the Aggregate Labor Share

The presence of monopsonistic power at equilibrium has also important implications for the distribution of income between factors of production. We begin our analysis by characterizing the behavior of the sectoral labor share of the superior sector.

Proposition 3. *At time t, the share of production of the superior sector that is paid in wages (or the sectoral labor share of sector s) is*

$$\alpha^s_{L,t} = \frac{w^s_t n^s_t}{Y^s_t} = \frac{A^i_t G_N(x^i_t,1,1)}{A^s_t F_N(x^s_t,1,1)} \ .$$

The sectoral labor share is constant along a MBGP in balanced growth, and equal to

$$\alpha_L^s = \frac{A^i G_N(x^i, 1, 1)}{A^s F(x^s, 1, 1)} \ ,$$

which is strictly less than the labor share in the efficient case in which only the superior sector operates, since

$$\alpha_L^s = \frac{A^i}{A^s} \frac{G_h(x^i, 1, 1)}{F(x^s, 1, 1)} < \frac{F_N(x^e, 1, 1)}{F(x^e, 1, 1)} = \alpha_L^e \ .$$

If there is unbalanced growth, the sectoral labor share along a MUBP asymptotically decreases to zero and

$$\lim_{t \to \infty} \frac{\dot{\alpha}_{L,t}^s}{\alpha_{L,t}^s} \to g^i - g^s \ .$$

Thus, along the MBGP of the model with balanced growth wages grow at rate g, exactly as (exogenously growing) TFP. Yet, the presence of positive profits in the monopsonistic market implies a permanent change in the allocation of income between labor and capital with respect to the efficient equilibrium. This is linked to the fact that part of the productivity of labor is now accrued to capital in forms of profits. This downward shift in the labor income is the *level effect* of monopsony on the labor share. With unbalanced growth, the effect becomes even stronger, and the wage share declines to zero as profits increase to absorb the total value of the marginal productivity of labor. This is the *growth effect* of monopsony on the labor share.

The labor share has received great attention in macroeconomics in recent years, notably because of the empirically robust finding of a downward trend in its value (Karabarbounis, 2024). Proposition 3 can clearly rationalize the recent evidence. Before proceeding however, we need to acknowledge that while the concept of the labor share of the sector that use the superior technology is well defined, we still miss a precise definition of the aggregate labor share. As a result, we need to be precise about what is a measure of GDP in the model and what is the empirical counterpart of the inferior sector.

There are two possible interpretations: the inferior technology can either be home production or a formal producing sector. In presenting the model, we argued that when the individual operates with the alternative technology, she has direct access to the marginal products of labor and capital. The home production interpretation appears thus natural, and in this case the access to the inferior technology does not require formal markets. In the alternative interpretation, the use of the inferior technology is organized as a formal sector in which both the labor market and the capital market are competitive. Also in this case she earns the marginal productivity of production. According to this alternative interpretation, the aggregate labor share measured in the data should include also the labor share of the inferior sector. Yet, as we discuss and show in this section, the growth and level effects results are robust to both interpretations.

Formally, we can always define the notion of a abor share in the inferior sector as

$$\alpha^i_{L,t} = \frac{G_N(x^i_t, 1, 1)}{G(x^i_t, 1, 1)} \ .$$

Using the definition of $\alpha^i_{L,t}$, alongside the concept of $\alpha^s_{L,t}$ and of the efficient labor share from the optimal growth problem α^e_L (which is always time independent), and defining

$$\lambda^s_t = \frac{Y^s_t}{Y^s_t + Y^i_t}$$

to be the share of total production produced by the sector operating the superior technology at time t, we have

$$\alpha^m_{L,t} = \begin{cases} \alpha^s_{L,t} & \text{if sector } i \text{ is home production} \\ \lambda^s_t \alpha^s_{L,t} + (1-\lambda^s_t)\alpha^i_{L,t} & \text{if sector } i \text{ is a pure competitive labor market} \end{cases}$$

The following proposition follows.

Proposition 4. *In the balanced growth case, the labor share is constant along any MBGP. If sector i is home production, then the labor share is lower than the share in the optimal growth problem. If, conversely, sector i is organized as a formal market, than the labor share is less than the optimal case if the two sectors produce with the same production function. In general, the labor share is higher as long as the production share of the superior sector is sufficiently large.*

In the unbalanced growth case instead, $\lim_{t\to\infty} \lambda^s_t = 1$, so that we have the following proposition.

Proposition 5. *In the unbalanced growth case, the wage share of the economy converges to zero regardless of the organization of sector i.*

The results in Propositions 4 and 5 can be illustrated with the help of a simple diagram, as we do in Figure 7. The top panel plots the evolution of the labor share along the dynamic equilibrium path in the case of balanced growth. With a simple Cobb-Douglas production function with a coefficient $\alpha = .3$ on capital, the optimal growth implies a constant labor share at the level $\alpha^e = .7$ along the entire path. In the case of monopsonistic equilibrium, the labor share falls during the transitional dynamics, but eventually stabilise at a level that is significantly lower than in the efficient. As argued above, the actual value of the labor share depends on the interpretation to the second sector. In the Figure, it is clear that in the interpretation of home production for the technology i, the level is lower than in the interpretation of the competitive inferior sector level, but in any case the labor share is simulated to be approximately 50 percent lower than the efficient case. The bottom chart of Figure 7 plots the dynamics of the labor share in the unbalanced growth equilibrium. The labor share monotonically tends to zero both in the case of home production as in the case of a competitive inferior sector.[6]

Note also that the unbalanced growth equilibrium is somewhat consistent with the existing evidence of a TFP differential growth between home production and home production, as documented by Bridgman (2016). In this respect, the empirical findings of a negative time trend in the labor share- as documented by Karabarbounis (2024)- appears consistent with our unbalanced monopsonistic equilibrium.

(Insert Figure 7 here)

6 Growth with Endogenous Labor Supply and Monopsony

We now study the full model with endogenous labor supply. This extension is important along three dimensions: first, we show that the monopsonistic model can easily account for the recent empirical evidence on the long run decline of hours worked; second, we show that as a consequence of the downward pressure on wages caused by the existence of monopsony, the model with unbalanced growth can generate declining hours worked even when income and substitution effects cancel each other in the class of preferences in Boppart and Krusell (2020); third, we highlight a further amplification effect in the missallocative effect of monopsony, since we show that over and beyond the distortionary effects on factor allocation outlined above, the presence of monopsony induces also a reduction of labor supply with respect to an optimal growth model.

In the rest of this section we highlight the main innovation in the model when labor supply is endogenous, and we also look at optimal growth with endogenous labor supply and declining hours worked as a natural benchmark. The details of the derivation are left to the appendix.

6.1 Generalized Model

The innovation of the generalized model is that we now work with the intraperiod utility function specified in equation (1) with $\phi > 0$, and then the agent solves

$$\begin{aligned} \max_{\{C_t,\mathbb{A}_t,K_t^i,h_t,n_t^s,n_t^i\}_{t\geq 0}} \quad & \int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma}-\phi\frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right]dt \\ s.t. \quad & C_t+\dot{\mathbb{A}}_t+\dot{K}_t^i = w_t n_t + r_t\mathbb{A}_t+\Pi_t+\omega_t n_t^i+\xi_t K_t^i \\ & n_t^s+n_t^i = h_t\ ,\ n_t^s,n_t^i\geq 0\ ,\ h_t\leq 1\ ,\ \mathbb{A}_t\geq 0\ ,\ K_t^i\geq 0 \end{aligned}$$

[6]The non-monotonic dynamics close to the end of the time horizon of the simulation is due to the fact that we perform turnpike simulations at a finite horizon.

The firm problem is identical to the fixed labor supply case. As before, we define $\dot{K}_t^i = q_t^i$ in the current value Hamiltonian. The first order condition reads (substituting out $n_t^i = h_t - n_t^s$).

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \frac{r_t - \rho}{\gamma} \\
\psi(h_t)^{\frac{1}{\theta}} &= C_t^{-\gamma} w_t && (20) \\
\psi(h_t)^{\frac{1}{\theta}} &= C_t^{-\gamma} \omega_t && (21) \\
r_t &= \xi_t \\
C_t + \dot{\mathbb{A}}_t + \dot{K}_t^i &= w_t n_t^s + r_t \mathbb{A}_t + \Pi_t + (h_t - n_t^s)\omega_t + \xi_t K_t^i \\
\lim_{t\to\infty} e^{-\rho t} u'(C_t)\mathbb{A}_t &= \lim_{t\to\infty} e^{-\rho t} u'(C_t) K_t^i = 0
\end{aligned}$$

where the new relevant first order conditions are (20) and (21). Along the optimal path, the agent solves an intratemporal labor supply problem along which the marginal disutility of labor $\psi(h_t)^{\frac{1}{\theta}}$ is equal to the wage in the superior and in the inferior sector. From equations (20) and (21) it immediately follows that $w_t = \omega_t$, so the fundamental arbitrage condition in the labor market continues to hold, as well as the condition on the capital market.

Imposing equilibrium exactly as before, we find the following fundamental system

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \frac{F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho}{\gamma} \\
F_N(K_t^s, n_t^s, A_t^s) &= G_N(K_t^i, h_t - n_t^i, A_t^i) - G_{NN}(K_t^i, h_t - n_t^i, A_t^i) n_t^s \\
\psi(h_t)^{\frac{1}{\theta}} &= C_t^{-\gamma} A_t^i G_N(K_t^i, h_t - n_t^i, A_t^i) && (22) \\
F_K(K_t^s, n_t^s, A_t^s) - \delta^s &= G_K(K_t^i, h_t - n_t^i, A_t^i) - \delta^i \\
C_t + \dot{K}_t^s + \dot{K}_t^i &= F(K_t^s, n_t^s, A_t^s) - \delta^s K_t^s + G(K_t^i, h_t - n_t^i, A_t^i) - \delta^i K_t^i
\end{aligned}$$

where (22) is the equilibrium intratemporal labor supply problem. Indeed, the model has now an additional endogenous variable h_t. We can rewrite the system in efficiency units as before and find

$$\frac{\dot{c}_t}{c_t} = \frac{F_K(x_t^s, 1, 1) - \delta^s - \rho}{\gamma} - g^s - g_t^{n^s}$$

$$A_t^s F_N(x_t^s, 1, 1) = A_t^i G_N(x_t^i, 1, 1) - A_t^i \frac{n_t^s}{h_t - n_t^s} G_{NN}(x_t^i, 1, 1) \tag{23}$$

$$\psi(h_t)^{\frac{1}{\theta}} = c_t^{-\gamma} \frac{A_t^i}{(A_t^s n_t^s)^\gamma} G_N(x_t^i, 1, 1) \tag{24}$$

$$F_K(x_t^s, 1, 1) - \delta^s = G_K(x_t^i, 1, 1) - \delta^i \tag{25}$$

$$c_t + \dot{x}_t^s + \dot{x}_t^i \frac{A_t^i}{A_t^s} \frac{h_t - n_t^s}{n_t^s} = F(x_t^s, 1, 1) - (\delta^s + g^s + g_t^{n^s}) x_t^s + \tag{26}$$

$$+ \left[G(x_t^i, 1, 1) - (\delta^i + g^i + g_t^{n^i}) x_t^i \right] \frac{A_t^i}{A_t^s} \frac{h_t - n_t^s}{n_t^s}$$

Before characterizing the equilibrium, we look again at the optimal growth problem.

6.2 Optimal Growth with Endogenous labor Supply

In the optimal growth problem the planner chooses to operate just with the superior sector.

Lemma 2. *Under Assumption 1, sector s operates in the optimal growth solution with endogenous labor supply.*

The system of differential equations equations governing the evolution of the model in efficiency units simplifies to

$$\frac{\dot{c}_t}{c_t} = \frac{F_K(x_t^e, 1, 1) - \delta^s - \rho}{\gamma} - g^s - g_t^h \tag{27}$$

$$\psi(h_t)^{\frac{1}{\theta}+\gamma} = c_t^{-\gamma} (A_t^s)^{1-\gamma} F_N(x_t^e, 1, 1) \tag{28}$$

$$c_t + \dot{x}_t^e = F(x_t^e, 1, 1) - (\delta^s + g^s + g_t^h) x_t^e \tag{29}$$

Boppart and Krusell (2020) show that this system displays an EBGP solution in which both consumption and hours worked to grow at constant rates, which are equal to[7]

$$g^h = \frac{1-\gamma}{\frac{1}{\theta} + \gamma} g^s \ , \quad g^C = g^K = \frac{\frac{1}{\theta} + 1}{\frac{1}{\theta} + \gamma} g^s \ .$$

As Boppart and Krusell (2020) show the EBG requires the coefficient γ be greater than one, if hours fall in equilibrium.

[7]To derive this results, notice that the Euler equation (27) implies x_t^e constant (and determines its value) when C_t grows at a constant rate. Then the budget constraint (29) implies that c_t is constant. Now differential with respect to time (28) to find the rate of growth of hours worked. The crucial step is that c_t being constant implies $g^C = g^s + g^h$, from which we can recover the rate of growth of C_t. Notice that once x^e is pinned down by $F_K(x^e, 1, 1) = \rho + \delta^s + \gamma g^C$, $c^e = F(x^e, 1, 1) - (\delta^s + g^C) x^e$. In turn this implies a well defined initial value for h_t, $h_0^e = (c_0^e)^{-\gamma} (A^s)^{1-\gamma} F_N(x^s, 1, 1)$.

6.3 Equilibrium Dynamics

We now extend the definitions of balanced growth path and asymptotic balanced growth path to the case with endogenous labor supply.

Definition 4. *A monopsonistic balanced growth path (MBGP) with endogenous labor supply has constant rates of growth of consumption and hours worked. A monopsonistic unbalanced growth path (MUGP) with endogenous labor supply has asymptotically constant growth of consumption and hours worked.*

As in the model solved in section 3, a MBGP exists if an only if there is balanced growth, while in the case of unbalanced growth we can characterize a MUGP. In balanced growth, we again denote $g^s = g^i = g$.

Theorem 5. *Under balanced growth, there exist a unique MBGP. Along this solution, the growth rates of consumption and hours worked are*

$$\frac{\dot{C}_t}{C_t} = g^C = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma}g \ , \quad \frac{\dot{h}_t}{h_t} = g^h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}g \ .$$

Both sectors operate: x_t^s *and* x_t^i *are constant and pinned down by*

$$F_K(x^s,1,1) = \rho + \delta^s + \gamma g \ , \quad G_K(x^i,1,1) = \rho + \delta^i + \gamma g \ ,$$

while share of labor employed by the two sectors is constant, and we define

$$\nu = \frac{n_t^s}{n_t^i} = \frac{G_N(x^i,1,1) - \frac{A^s}{A^i}F_N(x^s,1,1)}{G_{NN}(x^i,1,1)}$$

and $\nu > 0$*, so that*

$$n_t^s = \frac{\nu}{1+\nu}h_t \ , \quad n_t^i = \frac{1}{1+\nu}h_t \ ,$$

which implies that the growth rate of labor in the two sectors is constant and equal to that of total hours worked,

$$g^{n^s} = g^h \ , \quad g^{n^i} = g^h \ .$$

This in turn implies that capital in the two sectors grows at rate g^C *as well,*

$$\frac{\dot{K}_t^s}{K_t^s} = \frac{\dot{K}_t^i}{K_t^i} = g^C \ .$$

Finally, the share of total production produced by each sector is constant, as is the risk free rate of return, wages grow at rate g*, and there are positive profits that grow at rate* g^C*.*

We illustrate the growth rates in the economy and the dynamics of hours worked in the two sectors under balanced growth in Figure 8.

(Insert Figure 8 here)

Theorem 6. *Under unbalanced growth, there exists a unique MUBP. Along this solution, the asymptotical growth rates of consumption and hours worked are*

$$\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = g_\infty^C = \frac{\frac{1}{\theta} g^s + g^i}{\frac{1}{\theta} + \gamma} \ , \quad \lim_{t\to\infty} \frac{\dot{h}_t}{h_t} = g_\infty^h = \frac{g^i - \gamma g^s}{\frac{1}{\theta} + \gamma} \ .$$

Both sectors operate at all times, and the limiting values $\lim_{t\to\infty} x_t^s = x^s$ *and* $\lim_{t\to\infty} x_t^i = x^i$ *are defined by*

$$F_K(x^s, 1, 1) = \rho + \delta^s + \gamma g^s \ , \quad G_K(x^i, 1, 1) = \rho + \delta^i + \gamma g^s \ .$$

The ratio of labor in the superior sector to labor in the inferior one goes to infinity, $\nu_t \to \infty$*, so that*

$$\frac{n_t^s}{h_t} \to 1 \ , \quad \frac{n_t^i}{h_t} \to 0$$

and

$$\lim_{t\to\infty} \frac{\dot{n}_t^s}{n_t^s} \to g_\infty^h \ , \quad \lim_{t\to\infty} \frac{\dot{n}_t^i}{n_t^i} = g_\infty^h + g^i - g^s \ .$$

This in turn implies that

$$\lim_{t\to\infty} \frac{\dot{K}_t^s}{K_t^s} = g_\infty^C \ , \quad \lim_{t\to\infty} \frac{\dot{K}_t^s}{K_t^s} = g_\infty^C + 2(g^i - g^s) \ ,$$

so that the rate of accumulation of capital in the inferior sector can be either positive or negative. Finally, the share of total production produced by sector s *converges to one, while the risk free rate is asymptotically constant, wages asymptotically grow at rate* g^i*, while profits asymptotically grow at rate* g_∞^C*.*

Remark 1. *Notice that in the model with monopsony and unbalanced growth, hours worked decline if and only if* $g^i - \gamma g^s < 0$*, which is a requirement less strict than the optimal growth case* $\gamma > 1$*, so that it is possible that, while it would be optimal to keep hours worked constant (*$\gamma = 1$*) the market equilibrium outcome implies that hours decline at a constant rate. This is due to the fact that monopsony power keeps wages low, while the marginal utility of consumption decreases to the presence of profits, generating an income effect that overweights the substitution effect.*

The results on the labor share derived in the model with fixed labor supply generalized straightforwardly to the model with endogenous supply, so that is there is balanced growth, the labor share is constant and depends on the organization of the inferior sector, be it home production or a formally organized sector. With unbalanced growth, the labor share declines to zero.

6.4 Misallocative Effects on Consumption and Hours with Endogenous Labor Supply

Similarly to the analysis carried out for the model with exogenous labor supply, we can study the misallocative effects of monopsony when labor supply is endogenously determined. In balanced growth the growth rates are the same in the monoposonistic and in the optimal equilibrium, yet the loss in the levels of GDP and consumption increases in the ratio of the productivity levels, as the following proposition shows.

Proposition 6. *In the balanced growth path with falling hours, the output and consumption losses are constant over time, the consumption loss is always positive, and both losses tend to one when* $\frac{A^s}{A^i} \to \infty$*. In unbalanced growth, along any path the instantaneous losses tend to one.*

The proof is in Appendix. The result of Proposition 6 suggests that in the case of endogenous labor there is an amplification effect with respect to the fixed labor supply case. This is linked to the effect of monopsony on the long run choice of hours, as we now illustrate.

Proposition 7. *Hours worked in the monopsonistic balanced growth path are always different than the efficient level, and the distortion increases as the technological ratio increases. When the utility function is given by equation* (1)*, the welfare loss is increasing in the productivity ratio* A^s/A^i

The proof is illustrate it here. Along an EBGP only sector s operates and we have

$$h_0^e = \left(\frac{(c^e)^{-\gamma}}{\psi} (A^s)^{1-\gamma} F_N(x^e, 1, 1) \right)^{\frac{1}{\frac{1}{\theta}+\gamma}}$$

and total consumption as

$$C_0^e = c^e A^s h_0^e$$

where x^e is such that $F_K(x^e, 1, 1) = \rho + \delta^s + \gamma g^C$ and $c^e = F(x^e, 1, 1) - (\delta^s + g^C)x^e$. Along a MBGP instead

$$h_0^m = \left(\frac{(c^m)^{-\gamma}}{\psi} \frac{A^i}{(A^s)^\gamma} \left(\frac{1+\nu}{\nu} \right)^\gamma G_N(x^i, 1, 1) \right)^{\frac{1}{\frac{1}{\theta}+\gamma}}$$

and

$$C_0^m = c^m h_0^m \frac{\nu}{1+\nu} A^s$$

as usual and

$$c^m = F(x^s, 1, 1) - (\delta^s + g^C)x^s + [G(x^i, 1, 1) - (\delta^i + g^C)x^i] \frac{A^i}{A^s \nu} ,$$

where x^s and x^i are pinned down by the Euler equation. As $\frac{A^s}{A^i} \to \infty$, we know that $\nu \to \infty$. This implies that $c^m \to c^e$, which was is exactly the force behind the disappearance consumption loss in the fixed supply case. It is also true then that $\lim_{\frac{A^s}{A^i}\to\infty} C_0^m = c^e A^s \lim_{\frac{A^s}{A^i}\to\infty} h_0^m$. However, hours worked do differ between the optimal and monopsonistic case, and this distortion gets worse as the technological ratio increases. To see this, consider fixing A^s and let A^i increase. This case is particularly sharp in that h_0^e remains constant while h_0^m shrinks to zero. Alternatively, we can fix A^i and let A^s increase, and notice that h_0^e changes with $(A^s)^{\frac{1-\gamma}{\frac{1}{\theta}+1}}$, while h_0^m changes with $(A^s)^{-\frac{\gamma}{\frac{1}{\theta}+1}}$. The details on the proof on the utlity are left to the Appendix.

The effects discussed in this section are illustrated quantitatively in the simulations performed in Figure 9, which relies on the parameters specified in Table 1. The charts in the left panels illustrate the results of Proposition 6, and show the sizeable GDP and consumption loss that increases with the productivty ratio A^s/A^i. Interesting, the second chart in the left shows that the consumption in efficiency units does converge to the corresponding efficient level, excatly as in the case of fixed labor supply. This clearly indicates that the total effect in the level of GDP and consumption must be associated to the change in total hours worked. This is illustrated in the right panel of Figure 9. The total hours in balanced growth fall relative to the Ramsey outcome. The reason for this effect is an income effect due to the presence of monoposny and its effect on wages. It is not possible to obtain closed form expressions for welfare with unbalanced growth. However, the intuition described above carries through, and welfare loss is greater in the case of unbalanced growth with endogenous labor supply.

(Insert Figure 9 here)

7 Discussion and Conclusions

The Neoclassical Growth Model is the key workhorse model in macroeconomics and it is the basic model of the contemporary economics of growth. As a natural starting assumption, the model assumes that all products and factor markets are competitive. In the economics of growth, since the seminal contributions of Romer (1990) and Aghion and Howitt (1992), the introduction of monopolistic power in the intermediate markets turned out to be a key factor for understanding endogenous technological progress and incorporating increasing returns to scale in standard models. In recent years, interest in long run labor supply has been surging, mostly thanks to the contribution of Boppart and Krusell (2020) and to the observation that models of balanced growth need to incorporate declining hours worked in the long run. Despite this interest in the labor market, standard growth theory has devoted very little effort on labor market imperfections in the standard growth model.

This paper argued that ignoring monopsonistic power in the labor market is not a harmless assumption. The secular decline of the labor share -observed, among others, by Karabarbounis (2024)- is not easily accounted for by standard models of growth. In the theory presented, the representative firm hires labor in a classical monopsonistic market a la Robinson (1969), and set wages to match the marginal productivity in an alternative use of time available to the representative individual. Even though profits are ex post rebated to the individuals, the resulting equilibrium allocation has important positive and normative effects. From the positive standpoint, the model can account for both "level" and "growth" effects in the labor share. If productivity growth in the main superior sector and in the alternative sector are identical, the labor share is still constant in balanced growth, yet it is permanently lower than the corresponding labor share level obtained by optimal growth models *à la* Cass (1965) and Koopmans (1965). Further, if the productivity of the superior sector grows permanently faster than the productivity of the inferior sector, the labor share features permanent downward trend and even tends to zero in the asymptotic balanced equilibrium. These results certainly depend on the assumption that monopsony operates in the superior sector. Indeed, if the monoposnistic market were in the inferior technology, our optimal growth problem suggests that in such case there would not be any monopsony in the economy, as the superior sector would hire the entire labor force.

From the normative standpoint, the equilibrium of the monopsonistic long run equilibrium implies a significant misallocation of factors of production and welfare loss, over and beyond the standard short run welfare loss emphasized by the classical textbook monopsony introduced by Robinson (1969). In response to the presence of monopsony in the labor market, the economy operates with an inferior technology that would not be exploited by an optimal growth context. As a consequence, the representative individuals enjoys lower long run consumption level and lower capital accumulation than the corresponding individual in an optimal growth model. In simple back of the envelope simulations, the steady state welfare loss and the GDP loss can easily be of the order of 20 percent with respect to a NGM.

Allowing for endogenous labor supply, in line with the long run labor supply regularities pointed out by Boppart and Krusell (2020), opens up further long run effects of monopsony and the basic misallocative results outlined above turns out to be amplified. Indeed, when monopsony is persistent and long standing, the representative individual reduces her endogenous work effort and her long run labor supply. The effects of welfare become thus even stronger.

While the paper has potentially opened up a new wave of studying the interaction between imperfect labor markets, capital accumulation and long run growth, much remains to be done. First, it is necessary to provide a quantitative assessment of the long run model presented. This require tacking a strong stance on what the inferior technology stands for in real life economies, and whether it represent home production or a competitive fringe in the formal sector. That was not the goal of this paper. Our stylized theory is compatible with both intepretations. Further, the model can incorporate many of the insights of modern monopsony, particularly in the age of wage inequality, as recently surveyed by Manning (2021). Finally, the link between monopoly in endogenous growth model and monopsony in imperfect labor market is likely to be important. As mentioned in the literature review, Garibaldi and Turri (2024) take up some of these issues.

Appendices

A Assumption 1

Let

$$\kappa^1 = \frac{G_N(G_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^i),1,1)}{F_N(F_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^s),1,1)}$$

and

$$\kappa^2 = \frac{G(G_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^i),1,1) - (\delta^i + g^s)G_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^i)}{F(F_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^s),1,1) - (\delta^s + g^s)F_K(\cdot,1,1)^{-1}(\gamma g^s + \rho + \delta^s)} > 0 \ .$$

We define $\kappa = \max\{\kappa^1, \kappa^2\}$.

The intuitive reason why we need this assumption is to ensure that the superior sector is operated at an equilibrium featuring monopsonistic power and that a central planner that can dispose of monopsony would want to operate it. In this sense, Assumption 1 ensures that s is indeed "superior" and justifies our labelling of the two sectors. In the special case in which $F = G$ and $\delta^s = \delta^i$, then simply $\kappa^1 = \kappa^2 = 1$. The reason why $A^s > A^i$ is sometimes needed, or we can at times allow for $A^s < A^i$, is that different depreciation rates or shapes of the production functions are intuitively important as well as the TFP level when determining which sector is superior.

In practice, the assumption above is not particularly restrictive apart from matching the superior sector to the one that enjoys monopsony power. If $g^s = g^i$, and $A^s < \max\{\kappa^1, \kappa^2\}A^i$ we can relabel the two sectors. If $g^s > g^i$, then it is eventually true that $A^s \geq \kappa A^i$ whatever the initial values.

B Model with Exogenous Supply in Efficiency Units

In line with traditional NGM it turns out to be simple to characterize the equilibrium in terms of variables in efficiency units. The subtle issue is that in the model there two different TFP factors and that labor in the superior and inferior sector (thus n_t^s and $n_t^i = 1 - n_t^s$ respectively) are equilibrium quantities. We thus define

$$k_t^s = \frac{K_t^s}{A_t^s} \ , \quad k_t^i = \frac{K_t^i}{A_t^i} \ , \quad \tilde{c}_t = \frac{C_t}{A_t^s} \ ,$$

where capital in the superior sector and total consumption are defined with respect to TFP in the superior sector, while capital in the inferior sector is defined with respect to TFP level in the inferior sector.

The system of equations then becomes

$$\begin{aligned}
\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} &= \frac{F_K(k_t^s, n_t^s, 1) - \delta - \rho}{\gamma} - g^s \\
A_t^s F_N(k_t^s, n_t^s, 1) &= A_t^i G_N(k_t^i, 1 - n_t^s, 1) - A_t^i G_{NN}(k_t^i, 1 - n_t^s, 1) n_t^s \\
F_K(k_t^s, n_t^s, 1) - \delta^s &= G_K(k_t^i, 1 - n_t^s, 1) - \delta^i \\
\tilde{c}_t + \dot{k}_t^s + \dot{k}_t^i \frac{A_t^i}{A_t^s} &= F(k_t^s, n_t^s, 1) - (\delta^s + g^s) k_t^s + [G(k_t^i, 1 - n_t^s, 1) - (\delta^i + g^i) k_t^i] \frac{A_t^i}{A_t^s}
\end{aligned} \tag{30}$$

were we made extensive use of the homogeneity properties of F and G.

C Proofs

C.1 Proof of Theorem 2

Proof. We first of all prove that the system is saddle path stable under general technology if the technological ratio is sufficiently high.

For sake of readability, we suppress the time index and let F and its derivatives be evaluated at $(x^s, 1, 1)$ and G and its derivatives be evaluated at $(x^i, 1, 1)$.

Using the intratemporal labor and capital arbitrage equations (17) and (18) (notice that (17) is independent of time as it depends on the technological ratio only), we can use the Implicit Function Theorem to express n^s and x^i as functions of x^s. In fact write

$$\Psi(x^s, n^s, x^i) = \begin{pmatrix} A^s F_N - A^i \left(G_N - \frac{n^s}{1-n^s} G_{NN} \right) \\ F_K - \delta^s - G_K + \delta^i \end{pmatrix}$$

Then the Jacobian with respect to (n^s, x^i) is

$$\tilde{J}_\Psi = \begin{pmatrix} \frac{1}{(n^s)^2} G_{NN} & -A^i \left(G_{NK} - \frac{n^s}{1-n^s} G_{NNK} \right) \\ 0 & -G_{KK} \end{pmatrix}$$

so that $\det \tilde{J}_\Psi \neq 0$ for all values of parameters and thus we can apply the Implicit Function Theorem. Then we get

$$\frac{\partial x^i}{\partial x^s} = \frac{F_{KK}}{G_{KK}}$$

and

$$\begin{aligned}
\frac{\partial n^s}{\partial x^s} &= \frac{1 - n^s}{G_{NN}} \left[(1 - n^s) \frac{A^s}{A^i} F_{NK} - \frac{F_{KK}}{G_{KK}} \left((1 - n^s) G_{NK} - n^s G_{NNK} \right) \right] = \\
&= \frac{1 - n^s}{G_{NN}} \left[(1 - n^s) G_N - n^s G_{NN} \right] \left[\frac{F_{NK}}{F_N} - \frac{F_{KK}}{G_{KK}} \right]
\end{aligned}$$

Now notice that $\frac{\partial n^s}{\partial x^s} \to 0$ as $n^s \to 1$, which is implied by $\frac{A^s}{A^i}$ increasing. Also notice that

$$\dot{x}^i = \frac{\partial x^i}{\partial x^s} \dot{x}^s \ , \quad \dot{n}^s = \frac{\partial n^s}{\partial x^s} \dot{x}^s \ ,$$

so that we can write the dynamical system

$$\begin{pmatrix}\dot{c}\\ \dot{x}^s\end{pmatrix} = \begin{pmatrix}\Phi_1(c,x^s)\\ \Phi_2(c,x^s)\end{pmatrix} = \begin{pmatrix} c\left[\frac{F_K-\delta^s-\rho}{\gamma} - g^s - \frac{1}{n^s}\frac{\partial n^s}{\partial x^s}(1+\mathcal{X}(x^s))^{-1}\right](-c+\phi(x^s)) \\ (1+\mathcal{X}(x^s))^{-1}(-c+\phi(x^s))\end{pmatrix}$$

where

$$\mathcal{X}(x^s) = \frac{1}{n^s}\frac{\partial n^s}{\partial x^s} + \frac{1-n^s}{n^s}\frac{A^i}{A^s}\left(\frac{\partial x^i}{\partial x^s} - \frac{1}{1-n^s}\frac{\partial n^s}{\partial x^s}\right) ,$$

and

$$\phi(x^s) = F - (\delta^s + g^s)x^s + \frac{A^i}{A^s}\frac{1-n^s}{n^s}[G - (\delta^i + g^i)x^i] .$$

Notice that $\mathcal{X}(x^s) \to 0$ as $n^s \to 1$. We now want to study stability around the point $(c^*, x^{s,*})$ where $x^{s,*} = F_K(\cdot,1,1)^{-1}(\gamma g^s + \delta^s + \rho)$ and $c^* = \phi(x^{s,*})$. Then the Jacobian of the system reads

$$J_\Phi(c^*, x^{s,*}) = \begin{pmatrix} -\frac{1}{n^s}\frac{\partial n^s}{\partial x^s}(1+\mathcal{X}(x^s))^{-1} & \frac{c^*}{\gamma}F_{KK} - \frac{c}{n^s}\frac{\partial n^s}{\partial x^s}(1+\mathcal{X}(x^s))^{-1}\phi'(x^s) \\ -(1+\mathcal{X}(x^s))^{-1} & (1+\mathcal{X}(x^s))^{-1}\phi'(x^s)\end{pmatrix}$$

so that

$$\det J_\Phi(c^*, x^{s,*}) = (1+\mathcal{X}(x^s))^{-1}\left(\frac{c^*}{\gamma}F_{KK} - (1+c^*)\frac{1}{n^s}\frac{\partial n^s}{\partial x^s}(1+\mathcal{X}(x^s))^{-1}\phi'(x^s)\right)$$

But clearly

$$\det J_\Phi(c^*, x^{s,*}) \to \frac{c^*}{\gamma}F_{KK} < 0$$

as $n^s \to 1$, so that if the technological ratio is high enough, the system is saddle path stable.

We now turn to saddle path stability under Cobb-Douglas production functions. In this case it is simpler to use the model equations in terms of k^s and k^i that we defined in Appendix B. Under the Cobb-Douglas assumption, from (15) we have

$$\frac{k^i}{1-n^s} = \left(\frac{1-\epsilon}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha} - \delta^s + \delta^i}\right)^{\frac{1}{\epsilon}} .$$

Then (30) can be rewritten as

$$(1-n^s)\frac{A^s}{A^i}(1-\alpha)\left(\frac{k^s}{n^s}\right)^{\alpha}\left(\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha} - \delta^s + \delta^i}{1-\epsilon}\right)^{\frac{1-\epsilon}{\epsilon}} - \epsilon(1-\epsilon n^s) = 0$$

Notice that the derivative with respect to n^s of the equation above is

$$\frac{\epsilon(1-\epsilon n^s)}{n^s}\left[-\alpha + \frac{(1-\alpha)(1-\epsilon)}{\epsilon}\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha} - \delta^s + \delta^i} - \frac{n^s}{1-n^s}\frac{1-\epsilon}{1-\epsilon n^s}\right] ,$$

and that by substituting that at a BGP $\alpha\left(\frac{n^{s,*}}{k^{s,*}}\right)^{1-\alpha} = \delta^s + \rho + \gamma g$ we have by the sufficient condition in the statement of the Theorem that this derivative is negative. Then by the Implicit Function Theorem we have

$$\frac{\partial n^s}{\partial k^s} = -\frac{\epsilon(1-\epsilon n^s)\left[\frac{\alpha}{k^s} - \frac{1}{k^s}\frac{(1-\alpha)(1-\epsilon)}{\epsilon}\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}-\delta^s+\delta^i}\right]}{\frac{\epsilon(1-\epsilon n^s)}{n}\left[-\alpha + \frac{(1-\alpha)(1-\epsilon)}{\epsilon}\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}-\delta^s+\delta^i} - \frac{n^s}{1-n^s}\frac{1-\epsilon}{1-\epsilon n^s}\right]} =$$

$$= \frac{n^s}{k^s}\frac{\alpha\epsilon - (1-\alpha)(1-\epsilon)\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}-\delta^s+\delta^i}}{\alpha\epsilon - (1-\alpha)(1-\epsilon)\frac{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}}{\alpha\left(\frac{n^s}{k^s}\right)^{1-\alpha}-\delta^s+\delta^i} + \epsilon\frac{n^s}{1-n^s}\frac{1-\epsilon}{1-\epsilon n^s}}$$

so that at BGP values

$$\frac{\partial n^s}{\partial k^s}(k^{s,*}, n^{s,*}) = \frac{n^{s,*}}{k^{s,*}}\frac{\alpha\epsilon(\delta^i - \delta^s) - (1-\alpha-\epsilon)(\rho + \delta^s + \gamma g)}{\alpha\epsilon(\delta^i - \delta^s) - (1-\alpha-\epsilon)(\rho + \delta^s + \gamma g) + (\rho + \delta^i + \gamma g)\epsilon\frac{n^{s,*}}{1-n^{s,*}}\frac{1-\epsilon}{1-\epsilon n^{s,*}}} .$$

Derive with respect to time the two intratemporal equations to get $\dot{k}^h = \mathcal{K}(k^s)\dot{k}^s$ for

$$\mathcal{K}(k^s) = \frac{\frac{F_{KK}}{G_{KK}} - \frac{F_{KN}+G_{KN}}{G_{KK}}\frac{F_{KN}}{F_N}(G_N - n^s G_{NN})}{1 - \frac{F_{KN}+G_{KN}}{G_{KK}}},$$

and notice that $\mathcal{K}(k^s) > 0$. Now write the dynamic system $x = (c, k)'$ with $\dot{x} = \Phi(c, k)$ where

$$\Phi(c,k) = \begin{pmatrix}\Phi_1(c,k)\\ \Phi_2(c,k)\end{pmatrix} =$$

$$= \begin{pmatrix} c\left[\frac{F_K(k^s, n^s(k^s),1) - \delta^s - \rho}{\gamma} - g\right] \\ \left(1 + \mathcal{K}(k^s)\frac{A^i}{A^s}\right)^{-1}\left[-c + F(k^s, n^s(k^s), 1) - (\delta^s + g)k^s + [G(k^i(k^s), 1 - n^s(k^s)) - (\delta^i + g)k^i(k^s)]\frac{A^i}{A^s}\right]\end{pmatrix}$$

which is an autonomous system. We can then write the Jacobian

$$J_\phi(c^*, k^{s,*}) = \begin{pmatrix} 0 & \frac{c^*}{\gamma}\left[F_{KK} + F_{KN}\frac{\partial n^s}{\partial k^s}\right] \\ -\left(1 + \mathcal{K}\frac{A^i}{A^s}\right)^{-1} & \frac{\partial \Phi_2}{\partial k^s}\end{pmatrix}$$

The system is saddle path stable if there are two eigenvectors of opposite sign, i.e. if the determinant of the Jacobian is negative. But

$$\det J_\Phi = \left(1 + \mathcal{K}\frac{A^i}{A^s}\right)^{-1}\frac{c^*}{\gamma}\left[F_{KK} + F_{KN}\frac{\partial n}{\partial k}\right]$$

which is negative if and only if

$$\frac{\partial n}{\partial k}(k^*) < \frac{-F_{KK}}{F_{KN}} = \frac{n^{s,*}}{k^{s,*}} ,$$

which is true under the sufficient condition in the statement of the Theorem. □

There is an interesting interplay between the sufficient condition for stability in the Cobb-Douglas case and Assumption 1. In Assumption 1, $A^s > \kappa^1 A^i$ ensures the existence of an interior solution to the monopsonistic problem in which the superior sector is operated. Then $A^s > \kappa^2 A^i$ ensures that it is indeed optimal to operate sector s in the optimal growth case. In general we need to require both, but it turn out that $A^s > \kappa^1 A^i$ and $\alpha\epsilon(\delta^i - \delta^s) - (1-\alpha-\epsilon)(\rho + \delta^s + \gamma g)$ imply $A^s > \kappa^2 A^i$, so that if there exists an interior solution that is stable, the interior solution delivers higher utility than operating the inferior sector alone. From numerically analysing our model, we observed that there are indeed cases in which there is a MBGP with $n^s \in (0,1)$, which is suboptimal compared to operating sector i only, but these cases turn out to not to satisfy stability and hence can be unstable, thus not solutions of the model we might wish to consider.

C.2 Proof of Theorem 3

Proof. Notice first of all that the definition of MUGP and the Euler equation (14) imply that x_t^s is asymptotically constant and determined by the equation in the statement and denote $\lim_{t\to\infty} x_t^s = x^s$. Then by (18) x_t^i converges to a constant as well and denote $\lim_{t\to\infty} x_t^i = x^i$, which is pinned down by the condition in the statement. Now notice that (17) depends on time through the ratio $\frac{A_t^s}{A_t^i}$ and that n_t^s must converge to one, implying $g_t^{n^s} \to 0$, while the growth rate of n_t^i, which converges to zero, is pinned by rewriting (16) as

$$n_t^i = 1 - n_t^s = \frac{-A_t^i G_{NN}(x_t^i,1,1)n_t^s}{A_t^s F_N(x_t^s,1,1) - A_t^i G_N(x_t^i,1,1)}$$

which implies

$$\frac{\dot{n}_t^i}{n_t^i} = g_t^{n^i} \to g^i - g^s \ .$$

Then (19) implies that c_t is constant where

$$c = F(x^s,1,1) - (\delta^s + g^s)x^s \ ,$$

which in turn implies that the unique MUGP is such that

$$\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = g^s \ .$$

Since $x_t^s = \frac{K_t^s}{A_t^s n_t^s}$ and $x^i = \frac{K_t^i}{A_t^i n_t^i}$, using the result on $g_t^{n^i}$ above we have

$$\lim_{t\to\infty} \frac{\dot{K}_t^s}{K_t^s} = g^s \ , \quad \lim_{t\to\infty} \frac{\dot{K}_t^i}{K_t^i} = g^i + \lim_{t\to\infty} g_t^{n^i} = 2g^i - g^s \ ,$$

which can be either positive or negative.

To conclude, note that

$$\frac{Y_t^s}{Y_t^s + Y_t^i} = \frac{A_t^s n_t^s F(x_t^s,1,1)}{A_t^s n_t^s F(x_t^s,1,1) + A_t^i(1-n_t^s)G(x_t^i,1,1)} \to 1 \ ,$$

that $r_t = F_K(x_t^s,1,1) - \delta^s \to \rho + \gamma g^s$. Also recall that $w_t = A_t^i G_N(x_t^i,1,1)$, so that

$$\lim_{t\to\infty} \frac{\dot{w}_t}{w_t} = g^i \ ,$$

while since $\Pi_t = -\frac{A_t^i (n_t^s)^2}{1-n_t^s} G_{NN}(x_t^i,1,1)$

$$\lim_{t\to\infty} \frac{\dot{\Pi}_t}{\Pi_t} = g^i - \lim_{t\to\infty} g_t^{n^i} = g^s \ .$$

□

C.3 Proof of Theorem 4

Proof. For sake of readability we suppress the time index and in the remainder of the proof we leave implicit that F and its derivatives be evaluated at $(x^s, 1, 1)$ and G and its derivatives are evaluated at $(x^i, 1, 1)$. To begin, notice that (18) defines implicitly $x^i = x^i(x^s)$ and we can differentiate it to obtain

$$\dot{x}^i = \frac{F_{KK}}{G_{KK}} \dot{x}^s \ .$$

Now differentiate with respect to time (17) and get

$$\dot{n}^s = (1 - n^s)\tilde{\mathcal{N}}(c, x^s, n^s, \dot{x}^s) \ ,$$

for

$$\begin{aligned} \tilde{\mathcal{N}}(c, x^s, n^s, \dot{x}^s) = \frac{1}{G_{NN}} \Big\{ & (g^i - g^s)((1-n^s)G_N - n^s G_{NN}) + \\ & + \dot{x}^s \left[\frac{F_{NK}}{F_N}((1-n^s)G_N - n^s G_{NN}) - \frac{F_{KK}}{G_{KK}}((1-n^s)G_{NK} - n^s G_{NNK}) \right] \Big\} \end{aligned}$$

Now use (19) to obtain

$$\dot{x}^s = (1 + \mathcal{X}(x^s, n^s))^{-1} \left[-c + F - (\delta^s + g^s)x^s + \frac{1-n^s}{n^s} \frac{F_N}{G_N - \frac{n^s}{1-n^s} G_{NN}} [G - (\delta^s + g^i)x^i] \right]$$

for

$$\begin{aligned} \mathcal{X}(x^s, n^s) = & \frac{F_{KK}}{G_{KK}} \frac{1-n^s}{n^s} \frac{F_N}{G_N - \frac{n^s}{1-n^s} G_{NN}} + \left(\frac{1-n^s}{n^s} \frac{1}{G_{NN}} + \frac{1-n^s}{n^s} \frac{1}{G_{NN}} \frac{F_N}{G_N - \frac{n^s}{1-n^s} G_{NN}} \right) \cdot \\ & \cdot \left(\frac{F_{NK}}{F_N}((1-n^s)G_N - n^s G_{NN}) - \frac{F_{KK}}{G_{KK}}((1-n^s)G_{NK} - n^s G_{NNK}) \right) \ , \end{aligned}$$

so that we can substitute back $\dot{x}^s = \dot{x}^s(c, x^s, n)^s$ and get

$$\dot{n}^s = (1 - n^s)\mathcal{N}(c, x^s, n^s) = (1 - n^s)\tilde{\mathcal{N}}(c, x^s, n^s, \dot{x}^s(c, x^s, n^s))$$

Now observe that we want to study stability around the point $(c^*, x^{s,*}, 1)$ where $x^{s,*} = F_K(\cdot, 1, 1)^{-1}(\gamma g^s + \delta^s + \rho)$ and $c^* = F(x^{s,*}, 1, 1) - (\delta^s + g^s)x^{s,*}$ and that

$$\mathcal{X}(c^*, x^{s,*}, 1) = 0 \ , \quad \mathcal{N}(c^*, x^{s,*}, 1) = g^s - g^i \ .$$

Write the dynamical system

$$\begin{pmatrix} \dot{c} \\ \dot{x}^s \\ \dot{n}^s \end{pmatrix} = \begin{pmatrix} \Phi_1(c, x^s, n^s) \\ \Phi_2(c, x^s, n^s) \\ \Phi_3(c, x^s, n^s) \end{pmatrix} = \begin{pmatrix} c \left[\frac{F_K + \delta^s + \rho}{\gamma} - g^s - \frac{1-n^s}{n^s} \mathcal{N}(c, x^s, n^s) \right] \\ (1 + \mathcal{X}(x^s, n^s))^{-1} \left[-c + F - (\delta^s + g^s)x^s + \frac{1-n^s}{n^s} \frac{F_N}{G_N - \frac{n^s}{1-n^s} G_{NN}} [G - (\delta^s + g^i)x^i] \right] \\ (1 - n^s)\mathcal{N}(c, x^s, n^s) \end{pmatrix}$$

so that the Jacobian is

$$J_\Phi(c^*, x^{s,*}, 1) = \begin{pmatrix} 0 & \frac{c^*}{\gamma} F_{KK}(x^{s,*}, 1, 1) & g^s - g^i \\ -1 & \frac{\partial \Phi_2}{\partial x^s}(c^*, x^{s,*}, 1) & \frac{\partial \Phi_2}{\partial n^s}(c^*, x^{s,*}, 1) \\ 0 & 0 & g^i - g^s \end{pmatrix}$$

Saddle path stability is ensured by the presence of two negative eigenvectors and one positive eigenvector (since we have to state variables, x^s and n^s). The determinant of the Jacobian is

$$\det J_\Phi(c^*, x^{s,*}, 1) = \frac{c^*}{\gamma} F_{KK}(x^{s,*}, 1, 1)(g^i - g^s) > 0 \ .$$

Then either all eigenvectors are positive or they are one positive and two negative. To see that the latter is the case, notice that the eigenvectors solve

$$0 = \det \begin{pmatrix} -\lambda & \frac{c^*}{\gamma} F_{KK}(x^{s,*}, 1, 1) & g^s - g^i \\ -1 & \frac{\partial \Phi_2}{\partial x^s}(c^*, x^{s,*}, 1) - \lambda & \frac{\partial \Phi_2}{\partial n^s}(c^*, x^{s,*}, 1) \\ 0 & 0 & g^i - g^s - \lambda \end{pmatrix} = (g^i - g^s - \lambda) \det \begin{pmatrix} -\lambda & \frac{c^*}{\gamma} F_{KK}(x^{s,*}, 1, 1) \\ -1 & \frac{\partial \Phi_2}{\partial x^s}(c^*, x^{s,*}, 1) - \lambda \end{pmatrix}$$

so that one of the eigenvectors equals $g^i - g^s < 0$ and thus saddle path stability is proved. □

D The Allocative Effects on GDP and Consumption

GDP loss We start our analysis from the balanced growth case $g^s = g^i = g$. In order to discuss the effects of monopsony on total production comparing to the optimal growth outcome, notice first of all that balanced growth values are such that $x^e = x^s$ where $F_K(x^e, 1, 1) = \gamma g + \delta^s + \rho$. Also consider as usual $G_K(x^i, 1, 1) = \gamma g + \delta^i + rho$. Production in the optimal growth problem (where the solution of interest is always an exact balanced growth path) is

$$Y_t^e = F(K_t^e, 1, A_t^s) = A_t^s F(x^e, 1, 1) \ ,$$

while for monopsony we have

$$Y_t^m = F(K_t^s, n_t^s, A_t^s) + G(K_t^i, n_t^i, A_t^i) = A_t^s n^s F(x^s, 1, 1) + A_t^i n^i G(x^i, 1, 1)$$

where labor employed by the two sectors is again constant along a MBGP. Then we have

$$\frac{Y_t^m}{Y_t^e} = n^s + \frac{A^i}{A^s} \frac{n^i}{n^s} \frac{G(x^i, 1, 1)}{F(x^s, 1, 1)} \ ,$$

so that the ratio of production in the monopsonistic equilibrium to production in optimal growth is constant along an exact balanced growth path. Notice that for low values of this ratio can be greater than one (hence no loss, but a surplus) because of the curvature of marginal productivities of capital. This surplus is however wasted in depreciation, as we will see in the consumption loss section.

Notice also that

$$\frac{Y^m}{Y^e} \ , \quad \frac{A^s}{A^i} \to \infty \ .$$

So that if the level of productivity of the superior technology is higher, production is very close.

In the unbalanced growth case, we have instead that the ratio is time varying, and that along any path

$$\frac{Y_t^m}{Y_t^e} \to 1 \ .$$

Consumption loss In order to study consumption, start again from the balanced growth case and recall that

$$C_t^e = c^e A_t^s = A_t^s[F(x^e, 1, 1) - (\delta^s + g)x^e] \ ,$$

and that

$$C_t^m = c^m A_t^s n_t^s = A_t^s n^s[F(x^s, 1, 1) - (\delta^s + g)x^s] + A_t^i n^i[G(x^i, 1, 1) - (\delta^i + g)x^i] \ .$$

But notice that assumption 1 implies that $A_t^s[F(x^s, 1, 1)-(\delta^s+g)x^s] < A_t^i[G(x^i, 1, 1)-(\delta^i + g)x^i]$, so that $C_t^m < C_t^e$ at all times.

Again $\frac{C_t^m}{C_t^e}$ is constant in balanced growth and with the property that $C_t^m \to C_t^s$ when $\frac{A^s}{A^i} \to \infty$.

In the unbalanced growth case, $\frac{C_t^m}{C_t^e}$ is time varying and along any path $C_t^m \to C_t^s$.

E Simulation Algorithm

This section describes in details the algorithm we use to simulate our economy in the case of exogenous labour supply. The simulation use the shooting algorithm proposed by Sargent and Stachursky (2024) applied to our model. The complicated part is solving efficiently the intratemporal side of the economy with the optimal allocation of factors of production between sectors, which we describe below.

We discretize time with steps dt and index steps by m, so that time at iteration m is $t = mdt$, and our model equations to obtain the following.

$$\begin{aligned}
A_m F_N(K_m^s, n_m^s, A_m^s) &= A_m^i G_N(K_m^i, 1 - n_m^s, A_m^i) - n_m^s G_{NN}(K_m^i, 1 - n_m^s, A_t^i) \\
F_K(K_m^s, n_m^s, A_m^s) - \delta^s &= G_K(K_m^i, 1 - n_m^s, A_m^i) - \delta^i \\
C_{m+1} - C_m &= C_m \frac{F_K(K_m^s, n_m^s, A_m^s) - \delta^s - \rho}{\gamma} dt \\
K_{m+1}^s + K_{m+1}^i &= K_m^s + K_m^i + \left[-C_m + F(K_m^s, n_m^s, A_m^s) - \delta^s K_m^s + G(K_m^i, 1 - n_m^s, A_m^i) - \delta^i K_m^i\right] dt
\end{aligned}$$

Define technology at iteration m as

$$A_m^s = A^s e^{g^s mdt} \ , \quad A_m^i = A^i e^{g^i mdt}$$

Notice that in continuous time and with no bounds on derivatives, K^s and K^i are optimally allocated at each instant, so we define $\chi = K^s + K^i$ as the variable to be carried between periods.

The simulation algorithm proceeds as follows.

1. Take as inputs χ_m and C_m.
2. Solve the intratemporal system

$$\begin{aligned} K^s_m + K^i_m &= \chi_m \\ F_N(K^s_m, n^s_m, A^s_m) &= G_N(K^i_m, 1-n^s_m, A^i_m) - n^s_m G_{NN}(K^i_m, , 1-n^s_m, A^i_t) \\ F_K(K^s_m, n^s_m, A^s_m) - \delta^s &= G_K(K^i_m, 1-n^s_m, A^i_m) - \delta^i \end{aligned}$$

subject to $K^s_m \geq 0$, $K^i_m \geq 0$ and $n^s_m \in [0,1]$, for K^s_m, K^i_m and n^s_m.

3. Obtain

$$\begin{aligned} C_{m+1} &= C_m \left[1 + \frac{F_K(K^s_m, n^s_m, A^s_m) - \delta^s - \rho}{\gamma} dt\right] \\ \chi_{m+1} &= \chi_m + \left[-C_m + F(K^s_m, n^s_m, A^s_m) - \delta^s K^s_m + G(K^s_m, 1-n^s_m, A^s_m) - \delta^i K^i_m\right] dt \end{aligned}$$

4. Check whether $mdt = T$, if not update to $m+1$ carrying forward C_{m+1} and χ_{m+1}.

We then obtain the optimal C_0 via a shooting algorithm using the terminal condition.

There are two transversality conditions in the model with two types of capital in finite time

$$\mu^{\mathbb{A}}_T K^s_T = 0 \ , \quad \mu^{K^i}_T K^i_T = 0$$

where $\mu^{\mathbb{A}}$ is the Lagrange multiplier of the budget constraint and μ^{K^i} is the multiplier of $\dot{K}^i_t = q^i_t$. But since the two multipliers must be equal along the solution path and since $K^s \geq 0$ and $K^i \geq 0$, we have

$$\mu^{\mathbb{A}}_T (K^s_T + K^i_T) = 0 \ ,$$

so that the transversality condition we use in the solution algorithm is

$$\mu^{\mathbb{A}}_T \chi_T = 0 \ .$$

For the shooting algorithm we also need an upper bound on the initial value of consumption, which is not trivial since the maximum productivity of the economy is the result of solving the system of three equations. To find an upper bound for given χ_0, solve

$$\begin{aligned} K^s_0 + K^i_0 &= \chi_0 \\ F_N(K^s_0, n^s_0, A^s_0) &= G_N(K^i_0, 1-n^s_0, A^s_0) - n^s_0 G_{NN}(K^i_0, 1-n^s_0, A^i_t) \\ F_K(K^s_0, n^s_0, A^s_0) - \delta^s &= G_K(K^i_0, 1-n^s_0, A^i_0) - \delta^i \end{aligned}$$

Then the upper bound is

$$C^{max}_0 = \frac{\chi_0}{dt} + F(K^s_0, n^s_0, A^s_0) - \delta^s K^s_0 + G(K^i_0, 1-n^s_0, A^i_0) - \delta^i K^i_0 \ .$$

However, solving numerically a system of three equations in three unknowns (subjects to four inequality constraints) can be costly and lead to errors, so we here derive one equation in one unknown and bounds that are sufficient to obtain all intertemporal values, under the Cobb-Douglas production assumption.

For sake of clarity, we suppress the index m. Denote $F(K,N,A) = K^\alpha (AN)^{1-\alpha}$ and $G(K,N,A) = K^{1-\epsilon}(AN)^\epsilon$. Define $x^s = \frac{K^s}{A^s n^s}$ and $x^i = \frac{K^i}{A^i(1-n^s)}$. Then our system reads

$$\begin{aligned} x^s A^s n^s + x^i A^i (1-n^s) = \chi \\ AF_N(x^s,1,1) = A^i G_N(x^i,1,1) - \frac{n^s}{1-n^s} A^i G_{NN}(x^i,1,1) \\ F_K(x^s,1,1) - \delta^s = G_K(x^i,1,1) - \delta^i \end{aligned}$$

subject to $x^s > 0$, $x^i > 0$, $n^s \in (0,1)$.

Notice that the third equation gives us x^i as a function of x^s and we will denote $x^i = x^i(x^s)$. The explicit expression is

$$x^i(x^s) = \left(\frac{1-\epsilon}{\alpha (x^s)^{\alpha-1} - \delta^s + \delta^i} \right)^{\frac{1}{\epsilon}} .$$

To simplify our algebra, define $\nu = \frac{n^s}{1-n^s}$ (and notice $n^s = \frac{\theta}{1+\theta}$ and $1 - n^s = \frac{1}{1+\theta}$) so that $\theta \in (0,\infty)$ and we just need to impose $\theta > 0$ together with $x^s, x^i(x^s) > 0$.

We can rewrite the first equation as

$$\theta x^s A^s + x^i(x^s) A^i = \chi(1+\theta)$$

so that

$$\theta(x^s) = \frac{\chi - x^i(x^s)A^i}{x^s A^s - \chi}$$

and we are left to solve

$$A^s F_N(x^s,1,1) = A^i G_N(x^i(x^s),1,1) - \theta(x^s) A^i G_{NN}(x^i(x^s),1,1) \ ,$$

which is one equation in one unknown. What are the right bounds? We need ti impose

$$\begin{aligned} x^s > 0 \\ x^i(x^s) > 0 \Longleftrightarrow \begin{cases} x^s < \left(\frac{\alpha}{\delta - \delta^h}\right)^{\frac{1}{1-\alpha}} & \delta > \delta^h \\ x^s < \infty & \text{otherwise} \end{cases} \\ \theta(x^s) > 0 \end{aligned}$$

The last constraint, being θ a ratio, is positive if numerator and denominator are of the same sign, this happens if

$$x^s A^s > \chi > x^i(x^s) A^i = \left(\frac{1-\epsilon}{\alpha (x^s)^{\alpha-1} - \delta^s + \delta^i} \right)^{\frac{1}{\epsilon}} A^i \ , \tag{31}$$

or

$$x^s A^s < \chi < x^i(x^s) A^i = \left(\frac{1-\epsilon}{\alpha (x^s)^{\alpha-1} - \delta^s + \delta^i} \right)^{\frac{1}{\epsilon}} A^i \ .$$

At all times, only one of the two chain of inequalities can be satisfied. In particular (31) is the one to consider if

$$\frac{\chi}{A^s} < \left(\frac{\alpha}{(1-\epsilon)\left(\frac{A^i}{\chi}\right)^{\epsilon} + \delta^s - \delta^i} \right)^{\frac{1}{1-\alpha}}$$

Hence Step 2 in the algorithm above is to be replaced with

2' Check if

$$\frac{\chi_m}{A^s_m} < \left(\frac{\alpha}{(1-\epsilon)\left(\frac{A^i_m}{\chi_m}\right)^{\epsilon} + \delta^s - \delta^i} \right)^{\frac{1}{1-\alpha}}$$

is satisfied.

a) If so, look for the zero of

$$A^s_m (1-\alpha)(x^s)^\alpha - A^h_m \epsilon (x^i(x^s))^{1-\epsilon}(1+\theta(x^s)(1-\epsilon)) = 0$$

in the interval

$$x^s \in \left(\frac{\chi_m}{A^s_m}, \left(\frac{\alpha}{(1-\epsilon)\left(\frac{A^i_m}{\chi_m}\right)^{\epsilon} + \delta^s - \delta^i} \right)^{\frac{1}{1-\alpha}} \right) \ .$$

b) If it is not satisfied, look for the zero of

$$A^s_m (1-\alpha)(x^s)^\alpha - A^h_m \epsilon (x^i(x^s))^{1-\epsilon}(1+\theta(x^s)(1-\epsilon)) = 0$$

in the interval

$$x^s \in \left(\left(\frac{\alpha}{(1-\epsilon)\left(\frac{A^i_m}{\chi_m}\right)^{\epsilon} + \delta^s - \delta^i} \right)^{\frac{1}{1-\alpha}}, \frac{\chi_m}{A^s_m} \right) \ .$$

2" Recover the values of K^s_m, K^i_m and n^s_m from x^s_m, x^i_m and θ_m.

F Growth with Endogenous Labor Supply

This appendix presents the model of monopsonistic growth with endogenous labour supply in details.

Setting The representative agent solves

$$\begin{aligned}
\max_{\{C_t,\mathbb{A}_t,K_t^i,h_t,n_t^s,n_t^i\}_{t\geq 0}} \quad & \int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma}-\psi\frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}\right]\,\mathrm{d}t \\
s.t. \quad & C_t+\dot{\mathbb{A}}_t+\dot{K}_t^i = w_t n_t + r_t\mathbb{A}_t+\Pi_t+\omega_t n_t^i+\xi_t K_t^i \\
& n_t^s+n_t^i = h_t\ ,\ n_t^s,n_t^i\geq 0\ ,\ h_t\leq 1\ ,\ \mathbb{A}_t\geq 0\ ,\ K_t^i\geq 0
\end{aligned}$$

In order to write the Hamiltonian, again define $\dot{K}_t^i = q_t^i$, so that we have the states $x_t = (\mathbb{A}_t, K_t^i)'$ and the controls $z_t = (C_t, n_t^s, h_t, q_t^i)'$. The current value Hamiltonian reads

$$\hat{H}(\mathbb{A}_t,K_t^i,C_t,n_t^s,q_t^i) = \frac{C_t^{1-\rho}}{1-\rho}+\mu_t^{\mathbb{A}}(-C_t-q_t^i+w_t^s n_t^s+r_t\mathbb{A}_t+\Pi_t+\omega_t^i(1-n_t^s)+\xi_t^i K_t^i)+\mu_t^{K^i}q_t^i$$

We maximise the Hamiltonian with two multipliers, but because of the FOC with respect to i_t^h we have that they must be equal, so that we obtain the system

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \frac{r_t-\rho}{\gamma} \\
\psi(h_t)^{\frac{1}{\theta}} &= C_t^{-\gamma}w_t \\
\psi(h_t)^{\frac{1}{\theta}} &= C_t^{-\gamma}\omega_t \\
r_t &= \xi_t \\
C_t+\dot{\mathbb{A}}_t^s+\dot{\mathbb{A}}_t^i &= w_t n_t^s + r_t\mathbb{A}_t^s+\Pi_t+(h_t-n_t^s)\omega_t+\xi_t\mathbb{A}_t^i \\
\lim_{t\to\infty} e^{-\rho t}C_t^{-\gamma}\mathbb{A}_t^s &= \lim_{t\to\infty} e^{-\rho t}C_t^{-\gamma}\mathbb{A}_t^i = 0
\end{aligned}$$

from which it immediately follows that

$$w_t = \omega_t\ .$$

The firm problem is the same as in the case with exogenous labour supply. The maximisation problem in extensive form is intratemporal and again reads

$$\max_{K^s,N^s}\Pi(N^s,K^s;G(\cdot),R_t,A_t^s,A_t^i,\overline{H}_t,K_t^i) = F(K^s,N^s,A_t^s)-G_N(K_t^i,\overline{H}_t-N^s,A_t^i)N^s-R_tK^s\ .$$

The first order conditions of the firm are

$$F_K(K_t^s, N_t^s, A_t^s) = R_t$$
$$F_N(K_t^s, N_t^s, A_t^s) = G_N(K_t^i, \overline{H}_t - N_t^s, A_t^i) - G_{NN}(K_t^i, \overline{H}_t - N_t^s, A_t^i)N_t^s$$

Equilibrium and the Fundamental System Imposing $R_t = r_t - \delta$ the firm's FOC are

$$F_K(K_t^s, n_t^s, A_t^s) = r_t - \delta^s \ ,$$
$$F_N(K_t^s, n_t^s, A_t^s) = G_N(K_t^i, h_t - n_t^s, A_t^i) - G_{NN}(K_t^i, h_t - n_t^s, A_t^i)n_t^s$$

and we also impose

$$\omega_t = G_N(K_t^i, n_t^i, A_t^i) = G_N(K_t^i, h_t - n_t^s, A_t^i)$$
$$\xi_t = G_K(K_t^i, n_t^i, A_t^i) - \delta^i = G_K(K_t^i, h_t - n_t^s, A_t^i) - \delta^i$$

The Fundamental system is

$$\frac{\dot{C}_t}{C_t} = \frac{F_K(K_t^s, n_t^s, A_t^s) - \delta^s - \rho}{\gamma}$$
$$F_N(K_t^s, n_t^s, A_t^s) = G_N(K_t^i, h_t - n_t^i, A_t^i) - G_{NN}(K_t^i, h_t - n_t^i, A_t^i)n_t^s$$
$$F_K(K_t^s, n_t^s, A_t^s) - \delta^s = G_K(K_t^i, h_t - n_t^i, A_t^i) - \delta^i$$
$$C_t + \dot{K}_t^s + \dot{K}_t^i = F(K_t^s, n_t^s, A_t^s) - \delta^s K_t^s + G(K_t^i, h_t - n_t^i, A_t^i) - \delta^i K_t^i$$
$$\psi(h_t)^{\frac{1}{\theta}} = C_t^{-\gamma} A_t^i G_N(K_t^i, h_t - n_t^i, A_t^i)$$

We now derive the system in efficiency units. Again define

$$x_t^s = \frac{K_t^s}{A_t^s n_t^s} \ , \quad x_t^i = \frac{K_t^i}{A_t^i n_t^i} = \frac{K_t^i}{A_t^i(h_t - n_t^s)} \ , \quad c_t = \frac{C_t}{A_t^s n_t^s}$$

and we also define some growth rates

$$g_t^h = \frac{\dot{h}_t}{h_t} \ , \quad g_t^{n^s} = \frac{\dot{n}^s}{n^s} \ , \quad g_t^{n^i} = \frac{\dot{n}^i}{n^i} = \frac{g_t^h h_t - g_t^{n^s} n_t^s}{h_t - n_t^s} \ .$$

The fundamental system in efficiency units for the model with endogenous labour supply is the following.

$$\begin{aligned}\frac{\dot c_t}{c_t} &= \frac{F_K(x_t,1,1)-\delta^s-\rho}{\gamma} - g^s - g_{n^s,t}\\ A_t^s F_N(x_t^s,1,1) &= A_t^i G_N(x_t^i,1,1) - A_t^i \frac{n_t^s}{h_t-n_t^s} G_{NN}(x_t^i,1,1)\\ \psi(h_t)^{\frac{1}{\theta}} &= c_t^{-\gamma}\frac{A_t^i}{(A_t^s n_t^s)^\gamma} G_N(x_t^i,1,1)\\ F_K(x_t^s,1,1)-\delta^s &= G_K(x_t^i,1,1)-\delta^i\\ c_t+\dot x_t^s+\dot x_t^i\frac{A_t^i(h_t-n_t^s)}{A_t^s n_t^s} &= F(x_t^s,1,1)-(\delta+g^s+g_{n^s,t})x_t^s + \left[G(x_t^i,1,1)-(\delta+g^i+g_{n^i,t})x_t^i\right]\frac{A_t^i(h_t-n_t^s)}{A_t^s n_t^s}\end{aligned}$$

Proof of Theorem 5 We look for a MBGP in which $\frac{\dot C_t}{C_t}$ and $\frac{\dot h_t}{h_t}$ are constant.

Proof. First of all notice that $\frac{\dot C_t}{C_t}$ being constant implies x^s must be constant, which through (25) implies x^i is constant, and both are determined according to the equation in the statement of the Theorem. But then (23) implies that $\nu = \frac{n_t^s}{h_t-n_t^s}$ is constant as well and

$$\nu = \frac{G_N(x^i,1,1)-\frac{A^s}{A^i}F_N(x^s,1,1)}{G_{NN}(x^i,1,1)} \ ,$$

and we can rewrite (24) as

$$\psi(h_t)^{\frac{1}{\theta}} = c_t^{-\gamma}\frac{A_t^i}{(A_t^s h_t)^\gamma}\left(\frac{1+\nu}{\nu}\right)^\gamma G_N(x^i,1,1) \ .$$

But since also $\frac{\dot h_t}{h_t}$ must be constant, and $\frac{\dot n_t^s}{n_t^s} = \frac{\dot h_t}{h_t} = \frac{\dot n_t^i}{n_t^i}$ must be constant as well, the budget constraint (26) implies that c is constant. This implies that we need to solve for g^C and g^h using

$$\psi(h_t)^{\frac{1}{\theta}} = c_t^{-\gamma}\frac{A_t^i}{(A_t^s h_t)^\gamma}\left(\frac{1+\nu}{\nu}\right)^\gamma G_N(x^i,1,1)$$

and

$$\frac{C_t}{A_t^s n_t^s} = c$$

and by differentiating with respect to time we find

$$\begin{cases}\left(\frac{1}{\theta}+\gamma\right)g^h = (1-\gamma)g\\ g^C - g - g^h = 0\end{cases}$$

which implies

$$g^C = \frac{\frac{1}{\theta}+1}{\frac{1}{\theta}+\gamma}g \ , \quad g^h = \frac{1-\gamma}{\frac{1}{\theta}+\gamma}g \ .$$

Since x^s and x^i are constant, it follows that K^s and K^i grow at rate g^C as well. The share of total production produced by sector s is

$$\frac{Y_t^s}{Y_t^s + Y_t^i} = \frac{A_t^s n_t^s F(x_,^s 1, 1)}{A_t^s n_t^s F(x_,^s 1, 1) + A_t^i n_t^i G(x_t^i, 1, 1)} = \frac{F(x^s, 1, 1)}{F(x^s, 1, 1) + \frac{A^i}{A^s \nu} G(x^i, 1, 1)}$$

hence constant, the risk free rate of return is function of x^s hence constant as well, wages

$$w_t^s = A_t^i G_N(x^i, 1, 1)$$

grow at rate g while profits

$$\Pi_t = -G_{NN}(x^i, 1, 1)\nu A_t^i n_t^s$$

grow at rate g^C. Notice that the labour share is constant since total wages grow at rate $g + g^h = g^C$.

□

Proof of Theorem 6 We look for a MUGP in which $\frac{\dot{C}_t}{C_t}$ and $\frac{\dot{h}_t}{h_t}$ converge to constants.

Proof. Notice that $\frac{\dot{C}_t}{C_t}$ converging to a constant implies that x_t^s converges to some x^s. Through (25) this implies that also x_t^i converges to an appropriate x^i. But then (23) implies that $\frac{n_t^s}{h_t - n_t^s} \to \infty$ or equivalently $\frac{n_t^s}{h_t} \to 1$, and that

$$\lim_{t\to\infty} \frac{\dot{n}_t^s}{n_t^s} = g_\infty^h \ , \quad \lim_{t\to\infty} \frac{\dot{n}_t^i}{n_t^i} = g_\infty^h + g^i - g^s \ .$$

We can then write in the limit

$$\psi(h_t)^{\frac{1}{\theta} + gamma} = c_t^\gamma \frac{A_t^i}{(A_t^s)^\gamma} G_N(x^i, 1, 1)$$

and then the budget constraint implies

$$c_t \to F(x^s, 1, 1) - (\delta^s + g^s + g_\infty^h) x^s$$

so that c_t converges to a constant c. This implies that we need to solve for g_∞^C and g_∞^h using

$$\psi(h_t)^{\frac{1}{\theta} + \gamma} = c^{-\gamma} \frac{A_t^i}{(A_t^s)^\gamma} G_N(x^i, 1, 1)$$

and

$$\frac{C_t}{A_t^s h_t^s} = c$$

and by differentiating with respect to time we find

$$g^h_\infty = \frac{g^i - \gamma g^s}{\frac{1}{\theta} + \gamma} \ , \quad g^C_\infty = \frac{\frac{1}{\theta} g^s - g^i}{\frac{1}{\theta} + \gamma} \ .$$

Since $x^s_t \to x^s$, $g^{Ks}_t \to g^s + g^h_\infty = g^C_\infty$ and since $x^i_t \to x^i$, $g^{Ki}_t \to g^i + g^h_\infty + g^i - g^s = g^C_\infty + 2(g^i - g^s)$. The share of production produced by sector s is

$$\frac{Y^s_t}{Y^s_t + Y^i_t} = \frac{A^s_t n^s_t F(x^s_, 1, 1)}{A^s_t n^s_t F(x^s_, 1, 1) + A^i_t n^i_t G(x^i_t, 1, 1)} \to$$

and the risk free rate is asymptotically constant as it is a function of x^s_t. Wages

$$w^s_t = A^i_t G_N(x^i_t, 1, 1)$$

asymptotically grow at rate g^i, while profits

$$\Pi_t = -G_{NN}(x^i, 1, 1) A^i_t \frac{(n^s_t)^2}{n^i_t}$$

grow at rate $g^i + 2g^h_\infty - g^h_\infty - g^i + g^s = g^C_\infty$. Notice that the wage share goes to zero as total wages asymptotically grow at rate $g^i + g^h_\infty < g^C_\infty$. $\square$

G Allocative Effects with Endogenous Labor Supply

GDP Loss As usual define $x^e = x^s$ such that $F_K(x^e, 1, 1) = \gamma g^s + \delta^s + \rho$ and x^i such that $G_K(x^i, 1, 1) = \gamma g^i + \delta^i + \rho$. Start from the balanced growth case $g^s = g^i = g$. Production in the optimal growth problem is

$$Y^e_t = F(K^e_t, h_t, A^s_t) = A^s_t h^e_t F(x^e, 1, 1)$$

while with monopsony

$$\begin{aligned} Y^m_t &= F(K^s_t, n^s_t, A^s_t) + G(K^i_t, n^i_t, A^i_t) = A^s_t n^s F(x^s, 1, 1) + A^i_t n^i G(x^i, 1, 1) = \\ &= \frac{\nu}{1+\nu} A^s_t h^m_t F(x^s, 1, 1) + \frac{1}{1+\nu} A^i_t h^m_t G(x^i, 1, 1) \ , \end{aligned}$$

where recall $\nu = \frac{n^s_t}{n^i_t}$. This implies

$$\frac{Y^m_t}{Y^e_t} = \frac{\nu}{1+\nu} \frac{h^m_t}{h^e_t} \left[1 + \frac{A^i}{A^s \nu} \frac{G(x^i, 1, 1)}{F(x^s, 1, 1)} \right] \ .$$

Notice that in balanced growth h_t^e and h_t^m grow at the same rate so that $\frac{Y_t^m}{Y_t^e}$ is constant and equal to

$$\frac{Y_t^m}{Y_t^e} = \frac{\nu}{1+\nu}\frac{h_0^m}{h_0^e}\left[1+\frac{A^i}{A^s\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right] .$$

Now $h_0^e = (c^e)^{-\gamma}(A^s)^{1-\gamma}F_N(x^e,1,1)$ where $c^e = F(x^e,1,1)-(\delta^s+g)x^e$, while $h_0^m = (c^m)^{-\gamma}\frac{A^i}{(A^s)^\gamma}(\frac{1+\nu}{\nu})^\gamma G_N(x^i,1,1)$ where $c^m = F(x^e,1,1)-(\delta^s+g)x^s+\frac{A^i}{A^s\nu}[G(x^i,1,1)-(\delta^i+g)x^i]$. This implies

$$\begin{aligned}
\frac{Y_t^m}{Y_t^e} &= \left[\frac{\nu}{1+\nu}+\frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right]\frac{h_0^m}{h_0^e} = \\
&= \left[\frac{\nu}{1+\nu}+\frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right]\frac{(c^m)^{-\gamma}\frac{A^i}{(A^s)^\gamma}(\frac{1+\nu}{\nu})^\gamma G_N(x^i,1,1)}{(c^e)^{-\gamma}(A^s)^{1-\gamma}F_N(x^e,1,1)} = \\
&= \left[\frac{\nu}{1+\nu}+\frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right]\frac{\left(F(x^e,1,1)-(\delta^s+g)x^s+\frac{A^i}{A^s\nu}[G(x^i,1,1)-(\delta^i+g)x^i]\right)^{-\gamma}\frac{A^i}{(A^s)^\gamma}(\frac{1+\nu}{\nu})^\gamma G_N(x^i,1,1)}{(F(x^e,1,1)-(\delta^s+g)x^e)^{-\gamma}F_N(x^e,1,1)} = \\
&= \frac{A^i}{A^s}\left(\frac{1+\nu}{\nu}\right)^\gamma\left[\frac{\nu}{1+\nu}+\frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right]\underbrace{\frac{\left(F(x^e,1,1)-(\delta^s+g)x^s+\frac{A^i}{A^s\nu}[G(x^i,1,1)-(\delta^i+g)x^i]\right)^{-\gamma}G_N(x^i,1,1)}{(F(x^e,1,1)-(\delta^s+g)x^e)^{-\gamma}F_N(x^e,1,1)}}_{<1}
\end{aligned}$$

but we have no way to bound the left part of the expression above, so it can be that instead of an output loss there is an output surplus, yet this brings anyway a consumption loss under assumption 1 because of depreciation. However, differently from the fixed labour supply case, when $\frac{A^s}{A^i}\to\infty$, we have

$$\frac{Y_t^m}{Y_t^e} = \underbrace{\frac{A^i}{A^s}}_{\to 0}\underbrace{\left(\frac{1+\nu}{\nu}\right)^\gamma}_{\to 1}\underbrace{\left[\frac{\nu}{1+\nu}+\frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)}{F(x^s,1,1)}\right]}_{\to 1}\underbrace{\frac{\left(F(x^e,1,1)-(\delta^s+g)x^s+\frac{A^i}{A^s\nu}[G(x^i,1,1)-(\delta^i+g)x^i]\right)^{-\gamma}}{(F(x^e,1,1)-(\delta^s+g)x^e)^{-\gamma}}}_{\to 1}\frac{G_N(x^i,1,1)}{F_N(x^s,1,1)}$$

so that the output loss goes to one because the presence of monopsony distorts the overall labor supply from agents.

Consumption Loss First of all notice that in unbalanced growth consumption growth at a faster rate in the optimal Ramsey problem, so that the $C_t^m/C_t^e \to 0$ along any path.

In balanced growth consumption grows at the same rate in monopsonistic and optimal problem, so the ratio is constant an equal to time zero. Now

$$C_0^e = c^e h_0^e A^s = A^s h_0^e[F(x^e,1,1)-(\delta^s+g)x^e]$$

and

$$C_0^m = c^m A^s n_0^s = A^s n_0^s[F(x^s,1,1)-(\delta^s+g)x^s]+A^i n_0^i[G(x^i,1,1)-(\delta^i+g)x^i]$$

so that

$$\frac{C_t^m}{C_t^e} = \underbrace{\frac{A^i}{A^s}}_{\to 0} \underbrace{\left(\frac{1+\nu}{\nu}\right)^{\gamma}}_{\to 1} \underbrace{\left[\frac{\nu}{1+\nu} + \frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)-(\delta^i+g)x^i}{F(x^s,1,1)-(\delta^s+g)x^s}\right]}_{\to 1}$$

$$\underbrace{\frac{\left(F(x^e,1,1)-(\delta^s+g)x^s+\frac{A^i}{A^s\nu}[G(x^i,1,1)-(\delta^i+g)x^i]\right)^{-\gamma}}{(F(x^e,1,1)-(\delta^s+g)x^e)^{-\gamma}}}_{\to 1} \frac{G_N(x^i,1,1)}{F_N(x^s,1,1)} =$$

$$= \frac{A^i}{A^s}\left[\frac{\nu}{1+\nu} + \frac{A^i}{A^s}\frac{1}{1+\nu}\frac{G(x^i,1,1)-(\delta^i+g)x^i}{F(x^s,1,1)-(\delta^s+g)x^s}\right]^{1-\gamma} \frac{G_N(x^i,1,1)}{F_N(x^s,1,1)}$$

but this is less than one as all terms in the multiplication above are less than one. This implies that $\frac{C^m}{C^e} \to 0$ as $\frac{A^s}{A^i} \to \infty$, so that the consumption loss goes to one again because of monopsony distorting the total amount of labor employed in production.

Notice that for low values of this ratio can be greater than one (hence no loss, but a surplus) because of the curvature of marginal productivities of capital. But this is not what is relevant, what is relevant is consumption loss

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A Figures

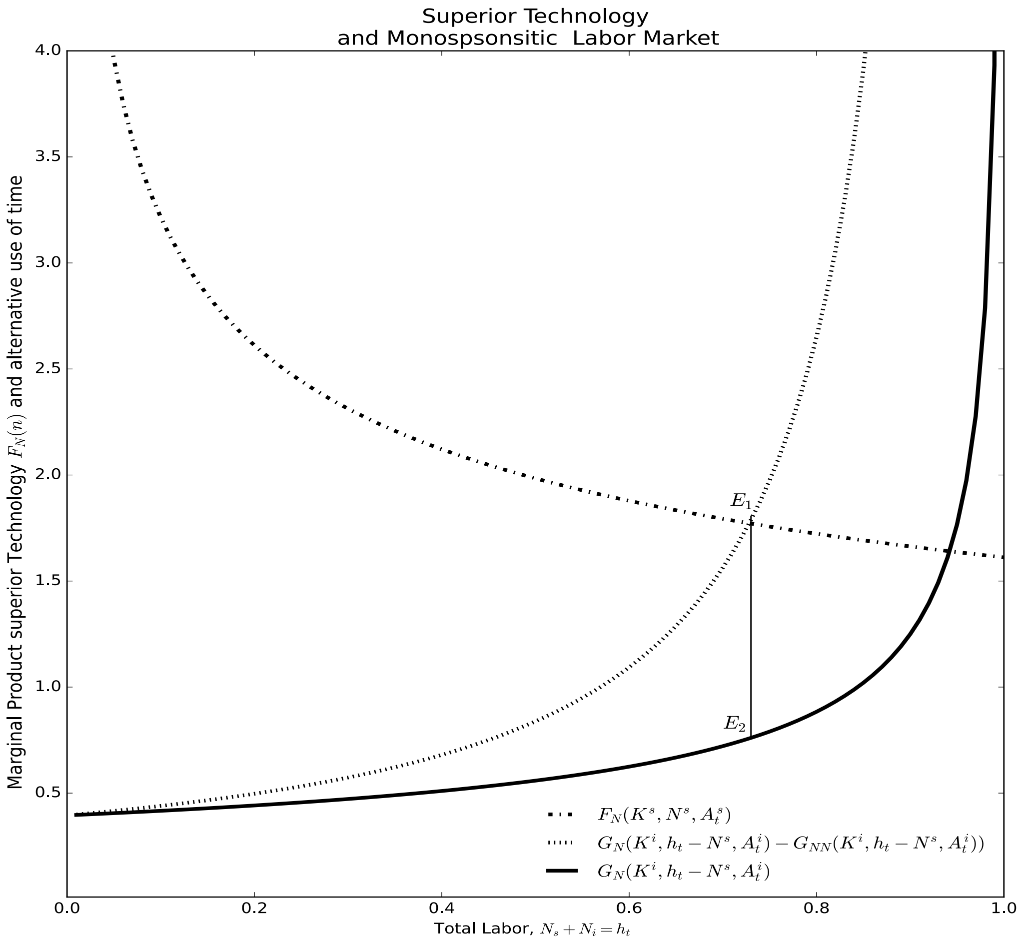

Figure 1: Monopsonistic Labor Market with the Superior Technology

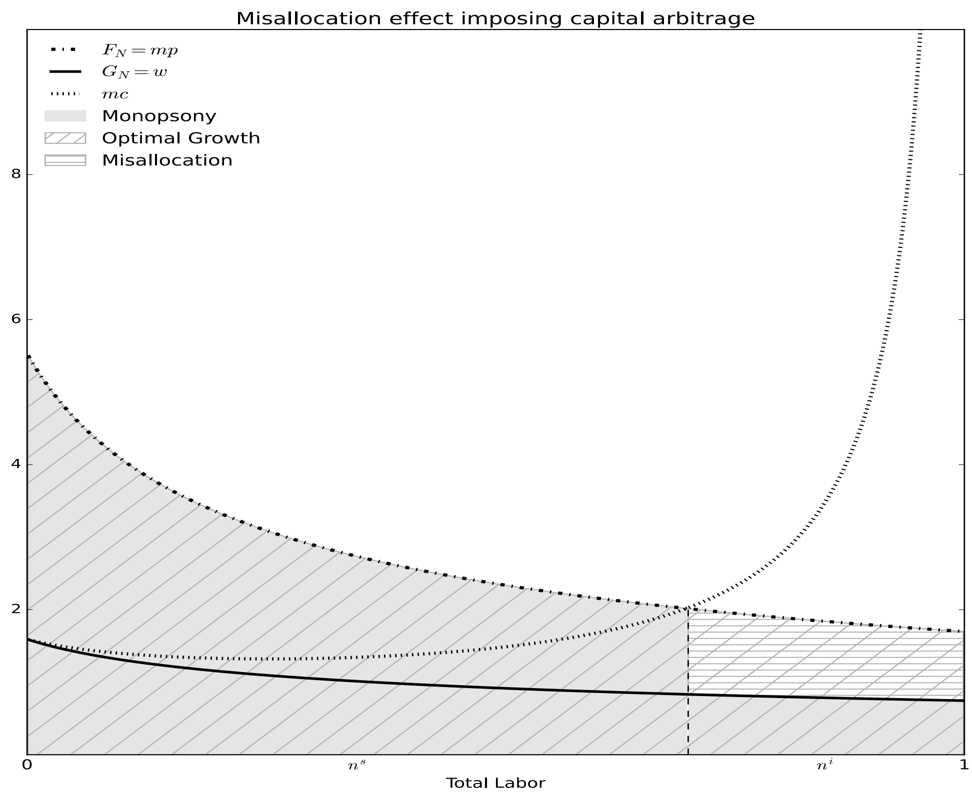

Figure 2: Illustration of misallocation

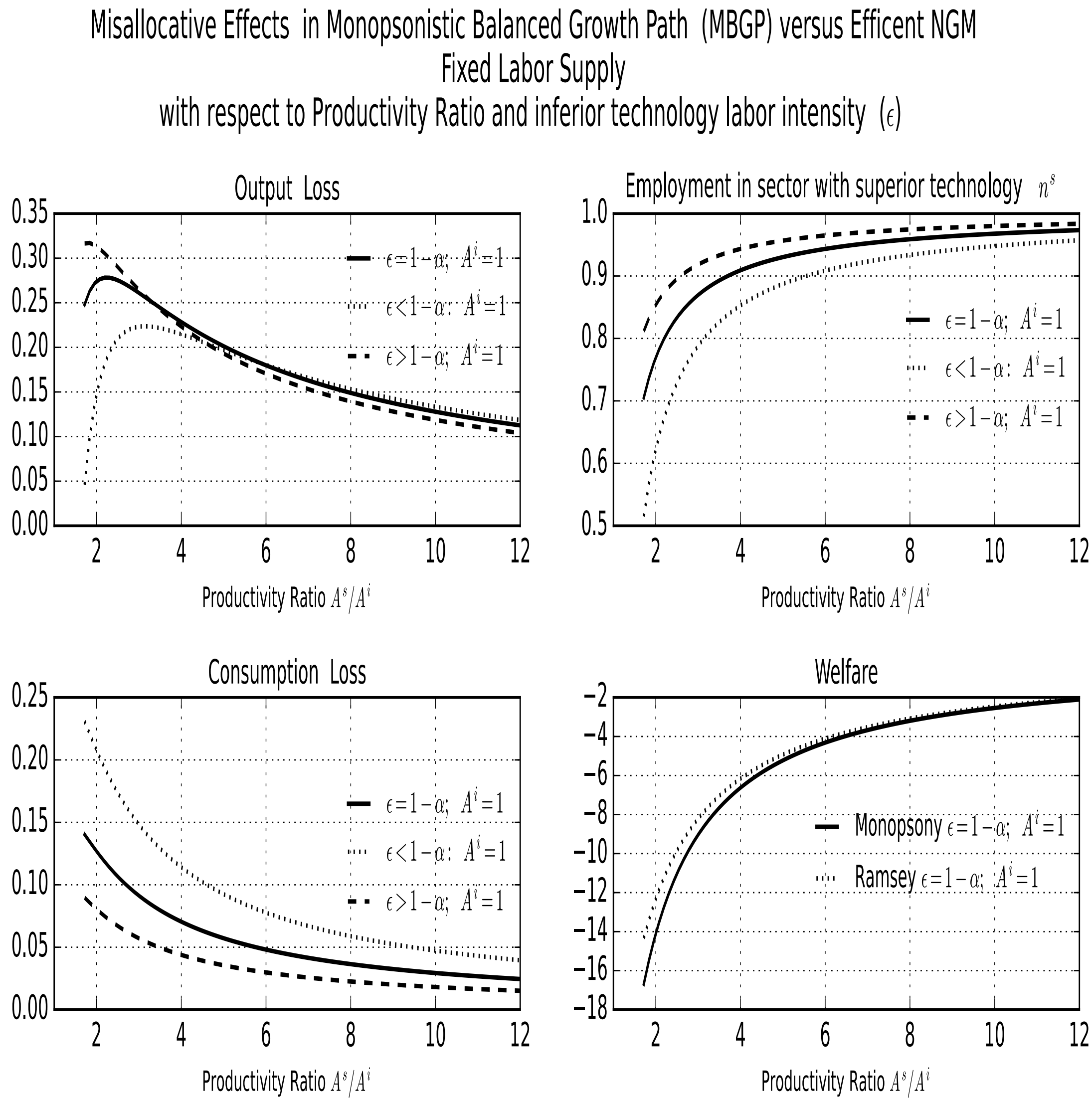

Figure 3: Misallocative Effects in Monopsonistic Balanced Growth Path with respect to Optimal Growth

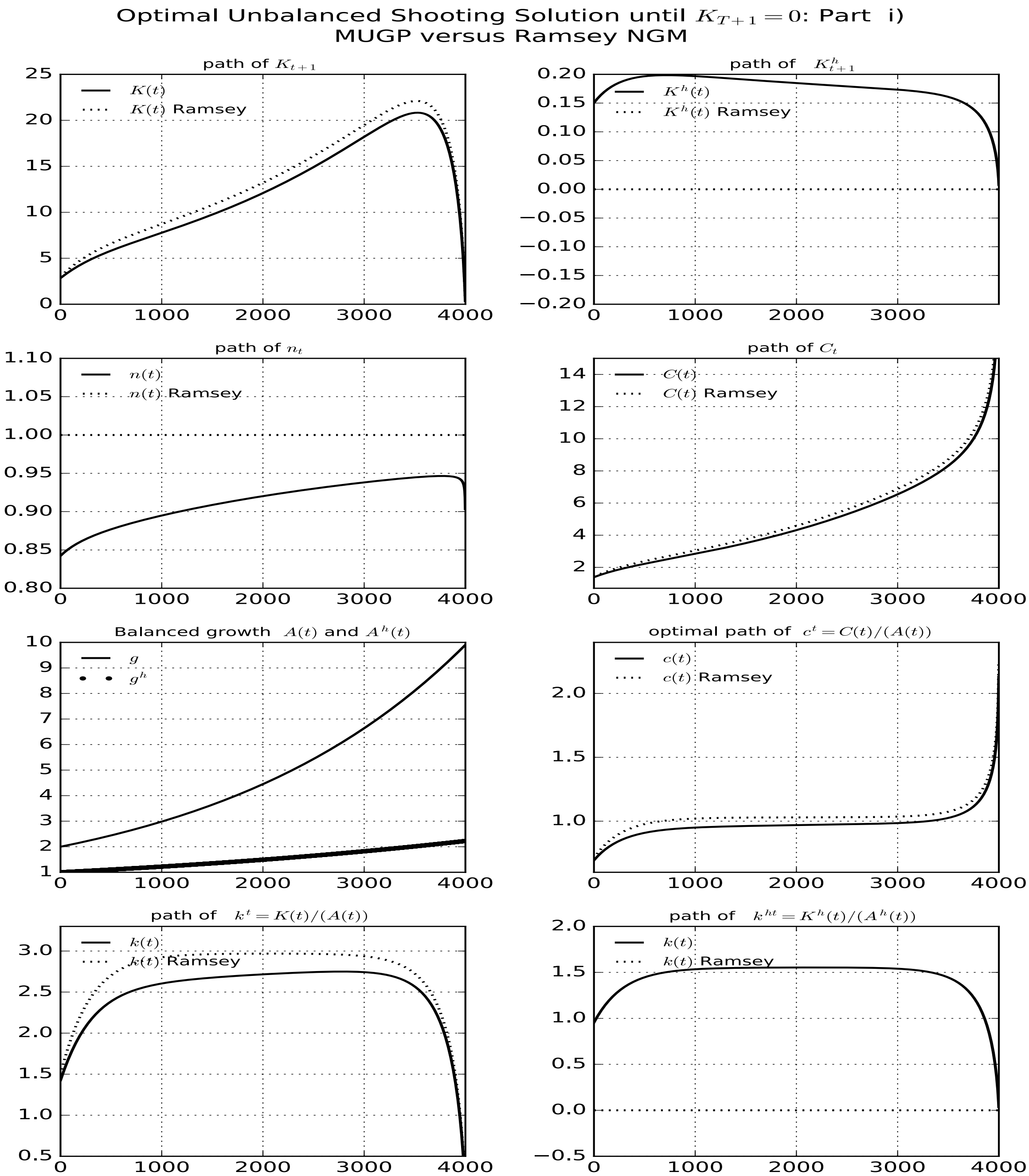

Figure 4: Simulation of path outside MBGP with balanced growth

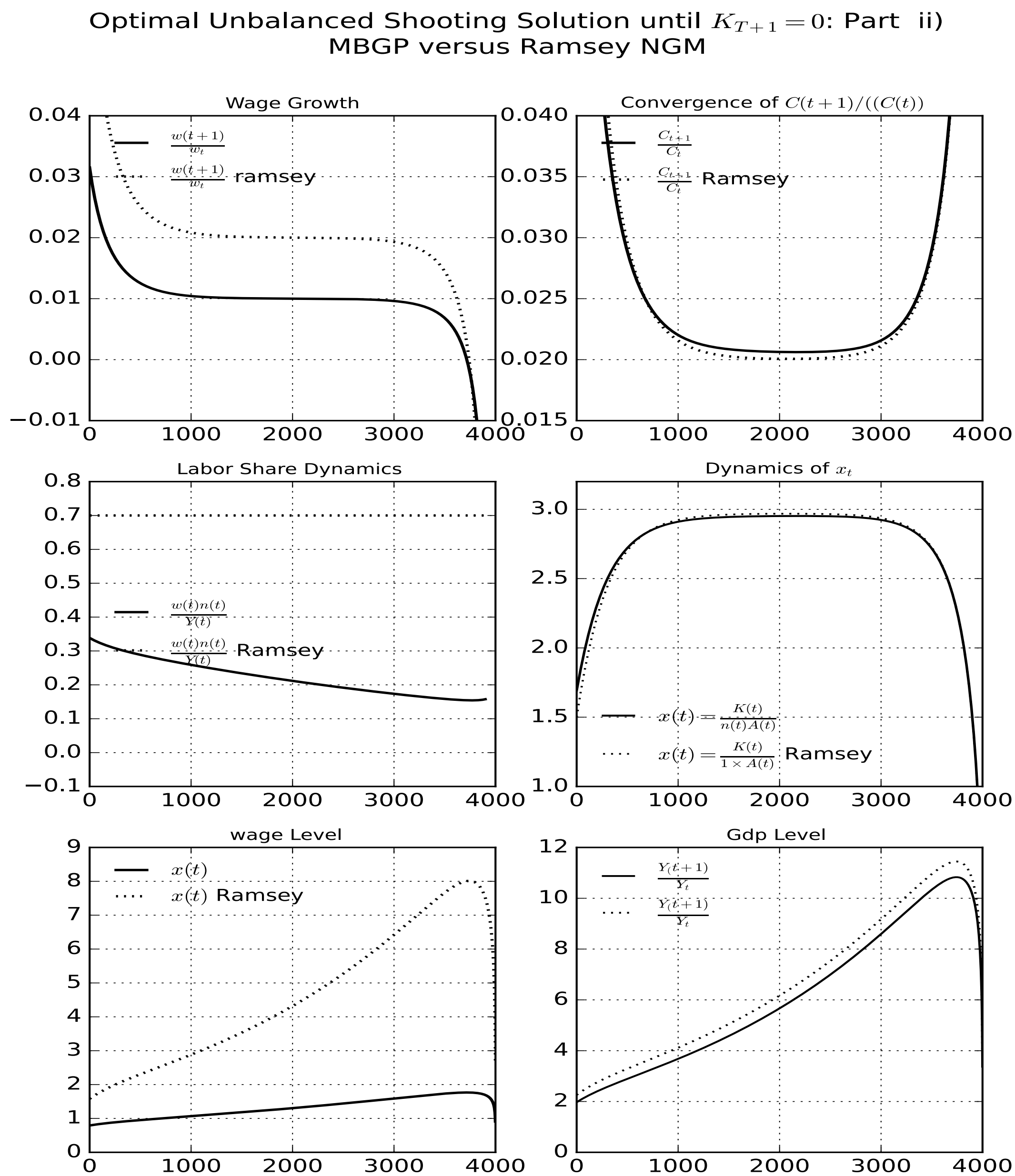

Figure 5: Simulation of path outside MBGP with balanced growth, continued

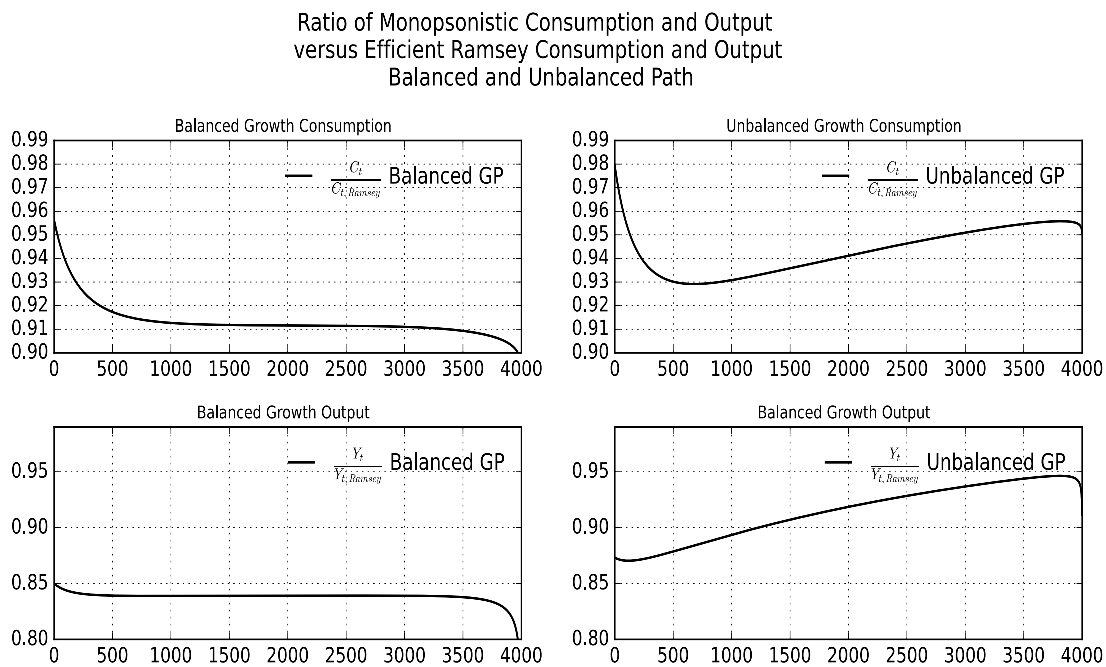

Figure 6: Consumption And Output in Monopsony versus Consumption and Output in Optimal Growth. Balanced and Unbalanced Path

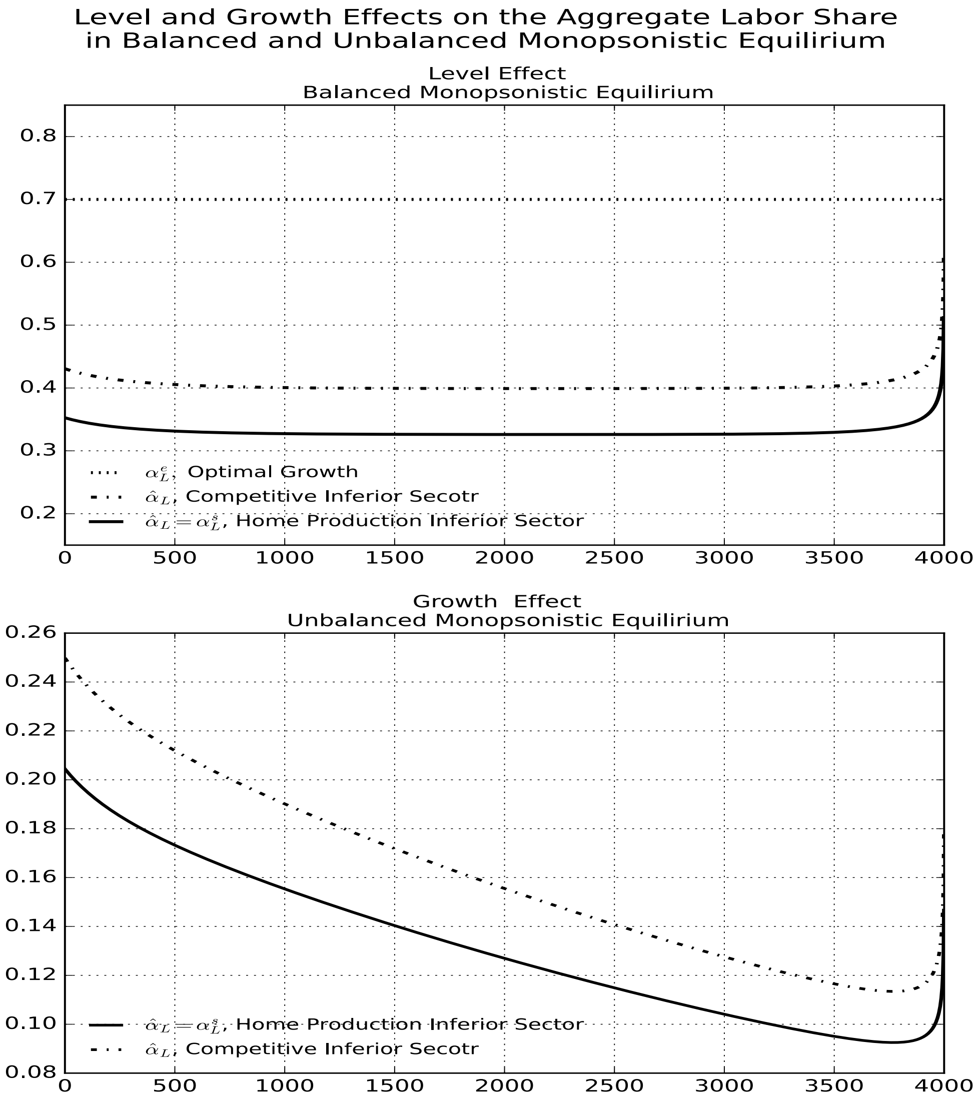

Figure 7: Growth and Level Effects on the Aggregate Labor Share

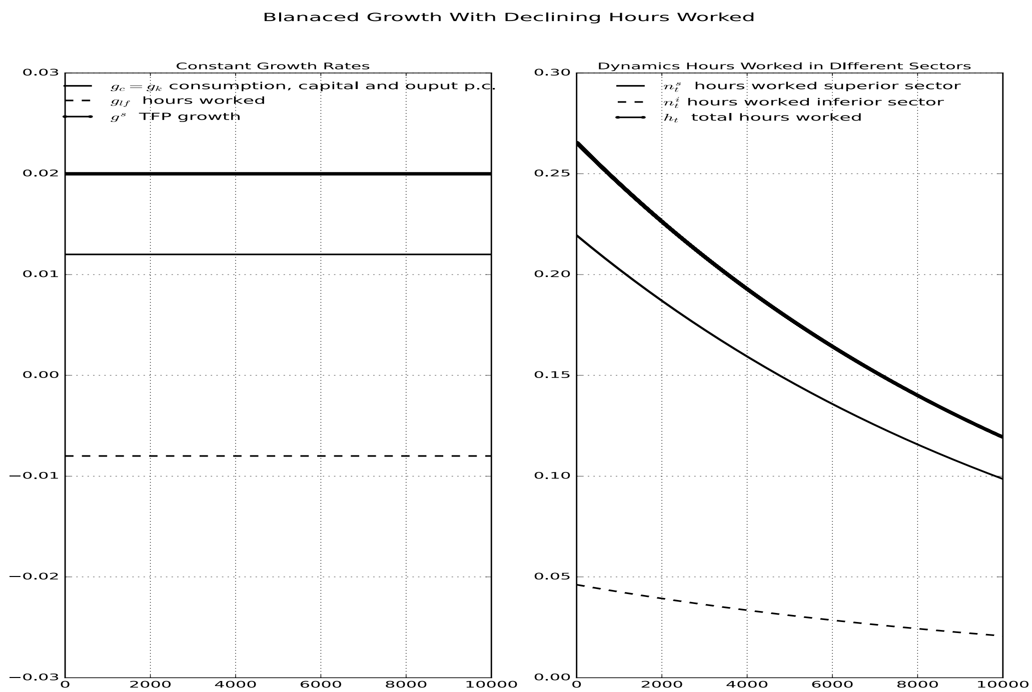

Figure 8: A Balanced Growth Path with Declining Hours Worked

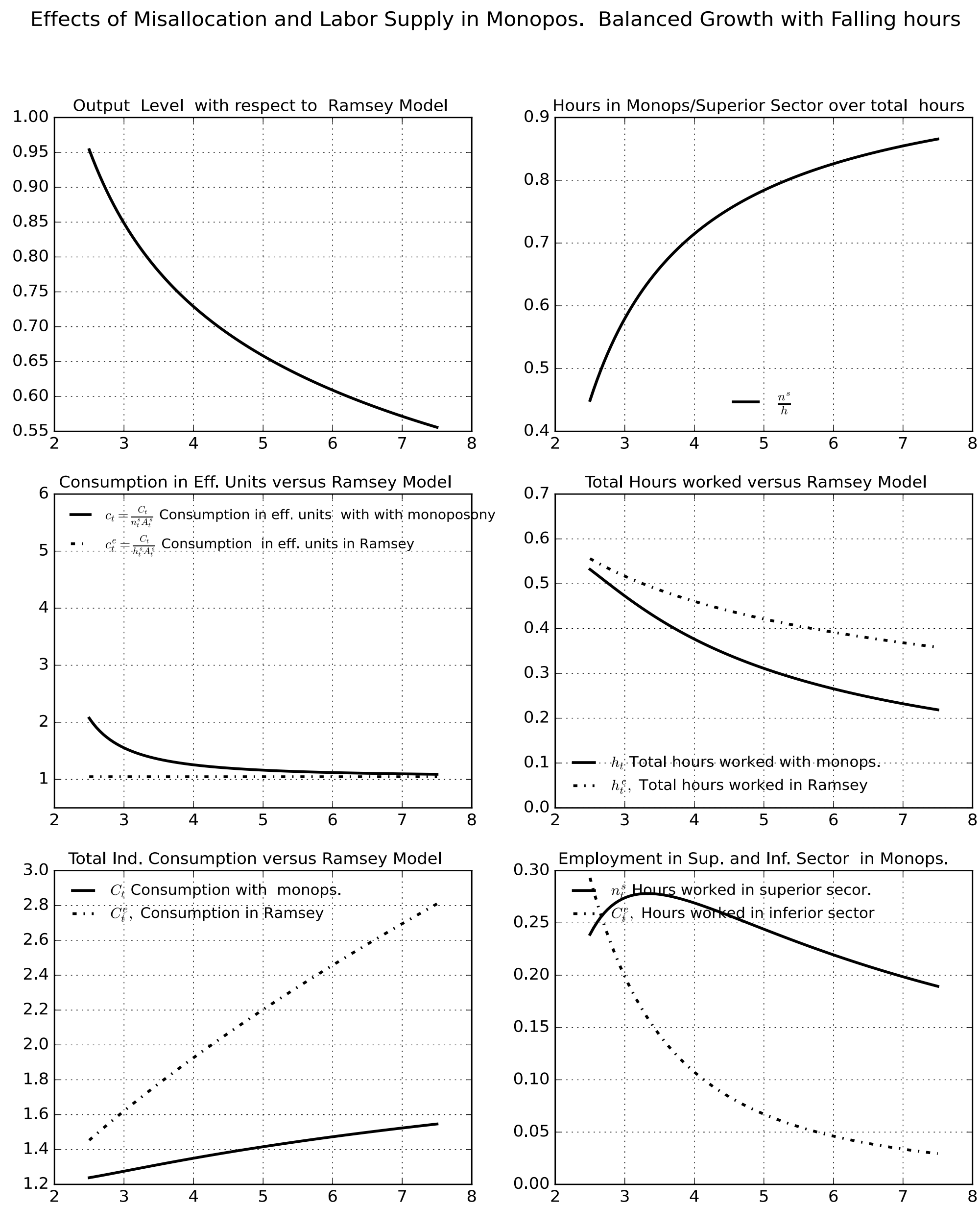

Figure 9: Welfare Loss and Labor Supply in Monopsonistic Equilibrium With Declining Hours Worked